Thermoacoustics and the Spherical Radon Transform

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Consider a spherical Radon transform $$ Rf({\bf x},t)= \int_{|x-y|=\nu_s t} f(y) dS_y $$ where ${\bf x}$ represents the location of an ultrasound transducer, t is time, and $\nu_s \sim 1.5 mm/\mu s$ is propagation speed. Ultrasound reflection tomographyÄù motivated Norton and Linzer to derive series solutions for f for specific measurement geometries \cite{Norton80, NL79, NL81}. Concurrent experimental research on thermoacoustic phenomena neglected the mathematics of image reconstruction, although the idealized mathematical models were essentially identical \cite{Bowen81, Caspers82, Lin84, Nasoni84, Nasoni85}. Early thermoacoustic tomography systems reconstructed via filtered backprojection, applying the filter and weights of the standard (planar) Radon transform but accounting for the spherical integration surface during backprojection \cite{Kruger}. This approach causes low-frequency shading across the image volume, but was sufficiently accurate near the origin.

Mathematically exact approaches to image reconstruction from a wide range of measurement geometries have been developed, primarily by members of the mathematical community. Image reconstruction is not currently a limiting factor in the development of thermoacoustic imaging. Understanding the thermoacoustic contrast mechanisms is required before thermoacoustic techniques can be translated into the clinic. For instance, just as xray CT projections are highly dependent upon the energy of the irradiating xrays, thermoacoustic signal production is as a function of irradiation frequency. Additionally, clinical ultrasound arrays rarely provide sufficient coverage to collect mathematically complete data. Finally, experimental constraints cause measured data to deviate from that modeled by the spherical Radon transform.

The bibliography:
{JS_Mu90} M.~Muster, \textit{Computing certain invariants of topological spaces of dimension three}, Topology \textbf{32} (1990), 100--120.

{JS_Mu90a} M.~Muster, \textit{Computing other invariants of topological spaces of dimension three}, Topology \textbf{32} (1990), 120--140.

{Norton80} NORTON, S. 1980. \textit{Reconstruction of a two-dimensional reflecting medium over a circular domain: exact solution,} Journal of the Acoustical Society of America, \textbf{67}, 1266-1273.

{NL79} NORTON, S. and LINZER, M. 1979. \textit{Ultrasonic Reflectivity Imaging in Three Dimensions: Reconstruction with Spherical Transducer Arrays,} Ultrasonic Imaging, \textbf{1}, 210-239.

{NL81} NORTON, S. J. and LINZER, M. 1981. \textit{Ultrasonic Reflectivity Imaging in Three Dimensions: Exact Inverse Scattering Solutions for Plane, Cylindrical, and Spherical Apertures,} IEEE Transactions on Biomedical Engineering, BME-\textbf{28}, 202-220.

{Bowen81} BOWEN, T., NASONI, L., PIFER, A. E. and SEMBROSK, G. H. \textit{Some Experimental Results on the Thermoacoustic Imaging of Tissue Equivalent Phantom Materials.} IEEE Ultrasonics Symposium 2. Chicago, IL, (1981).

{Caspers82}CASPERS, F. and CONWAY, J. \textit{Measurement of Power Density in a Lossy Material by means of Electromagnetically induced acoustic signals for non-invasive determination of spatial thermal absorption in connection with pulsed hyperthermia.} 12th European Microwave Conference Helsinki, (1982).

{Lin84}LIN, J. C. and CHAN, K. H. \textit{Microwave Thermoelastic Tissue Imaging--System Design,} IEEE Transactions on Microwave Theory and Techniques, \textbf{32}, (1984), 854-860.

{Nasoni84}NASONI, R. L., EVANOFF, G. A., HALVERSON, P. G. and BOWEN, T. 1984. \textit{Thermoacoustic Emission by Deeply Penetrating Microwave Radiation,} In: MCAVOY, B. R. (ed.) IEEE 1984 Ultrasonics Symposium. Dallas, TX.

{Nasoni85}NASONI, R., LIEW, S., HALVERSON, P. and BOWEN, T. 1985. \textit{Thermoacoustic Images Generated by a 2450 MHz Portable Source and Applicator.} IEEE Ultrasonics Symposium.

{Kruger} KRUGER, R. A., LIU, P., FANG, Y. R. and APPLEDORN, C. R. 1995. Photoacoustic ultrasound (PAUS) - Reconstruction Tomography. Medical Physics, \textbf{22}, 1605-1609.