

Hodge theory and combinatorics

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A conjecture of Read predicts that the coefficients of the chromatic polynomial of any graph form a log-concave sequence. A related conjecture of Welsh predicts that the number of linearly independent subsets of varying sizes form a log-concave sequence for any configuration of vectors in a vector space. All known proofs use Hodge theory for projective varieties, and the more general conjecture of Rota for possibly 'nonrealizable' configurations is still open. In this talk, I will argue that two main results of Hodge theory, the Hard Lefschetz theorem and the Hodge-Riemann relations, continue to hold in a realm that goes far beyond that of Kahler geometry. This cohomology theory gives strong restrictions on numerical invariants of tropical varieties, and in particular those of graphs and matroids. Joint work with Karim Adiprasito and Eric Katz.