Ergodic Stochastic Differential Equations and Sampling: A numerical analysis perspective

Konstantinos Zygalakis, University of Edinburgh

Understanding the long time behaviour of solutions to ergodic stochastic differential equations is an important question with relevance in many field of applied mathematics and statistics. Hence, designing appropriate numerical algorithms that are able to capture such behaviour correctly is extremely important. A recently introduced framework [1,2] using backward error analysis allows us to characterise the bias with which one approximates the invariant measure (in the absence of the accept/reject correction). Using this framework we will analyse splitting [3] and stochastic gradient algorithms [4] arising in molecular dynamics and machine learning respectively. These ideas will also be used to design numerical methods exploiting the variance reduction of recently introduced nonreversible Langevin samplers [5].

[1] K.C. Zygalakis. On the existence and applications of modified equations for stochastic differential equations. SIAM J. Sci. Comput., 33:102-130, 2011.

[2] A. Abdulle, G. Vilmart, and K. C. Zygalakis. High order numerical approximation of the invariant measure of ergodic sdes. SIAM J. Numer. Anal., 52(4):1600-1622, 2014.

[3] A. Abdulle, G. Vilmart, and K.C. Zygalakis, Long time accuracy of Lie-Trotter splitting methods for Langevin dynamics . SIAM J. Numer. Anal., 53(1):1-16, 2015.

[4] S. J. Vollmer, K.C. Zygalakis and Y. W. Teh, Exploration of the (Non-)asymptotic Bias and Variance of Stochastic Gradient Langevin Dynamics. J. Mach. Learn. Res., To appear, (2016).

[5] A. Duncan, G. A. Pavliotis and T. Lelievre, Variance Reduction using Nonreversible Langevin Samplers J. Stat. Phys. 163(3) pp 457-491, (2016).

Stochastic integrators for multiscale and ergodic dynamical systems

Assyr Abdulle, École Polytechnique Fédérale de Lausanne

We discuss recent advances in the design of stochastic integrators for stiff and ergodic stochastic differential equations (SDEs). For SDEs with multiple scale, stability constraints usually prevent an explicit solver to access the coarse levels of a hierarchical sampling such as the multi-level Monte Carlo (MLMC) method. We then explain how this issue can be overcome by using appropriate stabilization procedures. In the second part of the talk we show that ideas coming from geometric integration, such as backward error analysis and modified equations, allow to construct new numerical integrators capable of capturing high order approximation of the invariant measure of ergodic SDEs.

This talk is based on joint works with various collaborators [1, 2, 3, 4, 5].

References:

[1] A. Abdulle, D. Cohen, G. Vilmart, and K. C. Zygalakis, High order weak methods for stochastic differential equations based on modified equations, SIAM J. Sci. Comput., 34 (2012), pp. 1800--1823.

[2] A. Abdulle and A. Blumenthal, Stabilized multilevel Monte Carlo method for stiff stochastic differential equations, J. Comput. Phys., 251, (2013), 445--460.

[3] A. Abdulle, G. Vilmart, and K. C. Zygalakis, High order numerical approximation of the invariant measure of ergodic SDEs, SIAM J. Numer. Anal., 52 (2014), pp. 1600--1622.

[4] A. Abdulle and A. Blumenthal, Improved stabilized multilevel Monte Carlo method for stiff stochastic differential equations, Lecture Notes in Computational Science and Engineering, 103 (2015), 537--545.

[5] A. Abdulle, G. Vilmart, and K. C. Zygalakis, Long time accuracy of Lie-Trotter splitting methods for Langevin dynamics , SIAM J. Numer. Anal., 53 (2015), pp. 1--16.