Bayesian and Game theoretical numerical method for multiscale PDEs

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Multiscale problems arise naturally from many scientific and engineering areas such as geophysics, material sciences and biology. Numerical homogenization concerns the (coarse) finite dimensional approximation of the solution space of, for example, divergence form elliptic equation with rough coefficients which allows for nonseparable scales. Based on a Bayesian reformulation of numerical homogenization, we propose a class of numerical homogenization methods which allow for exponential decaying bases, localization, as well as optimal convergence rates. Furthermore, multi-resolution ""gamblet"" decomposition and corresponding multigrid method can be constructed using the game theoretical approach. Those gamblet based methods have rigorous a priori error bounds, bounded condition number at each subband, and solve boundary value problems in near-linear complexity. The method can be generalized to time dependent problems such as wave propagation in heterogeneous media, and multi-scale eigenvalue problems.