Kernel methods for partial differential equations with random input data

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In this talk, we consider partial differential equations (pdes) with random input data.
We show how reproducing kernels can be used in the reconstruction of quantities of interest of the parametric pde. Here, one needs to consider regularized reconstruction methods since the evaluation of the quantity of interest involves a numerical solution to the pde and hence is corrupted by some deterministic noise.
We show how sampling inequalities can be used for an a priori error analysis which also allows to determine the accuracy of the pde-solver which is needed not to spoil the reconstruction error due to the finite amount of samples.
It is important to have an a priori analysis here, since the pde-solver accuracy has to be determined before the data is generated.
Another application of kernel methods in the context of parametric pdes concerns the fact that if the pde contains a random field its solution is usually also a random field. If all random fields are Gaussian and the pde is linear, the theory of Gaussian process regression can be applied. It is well established that this theory is closely linked to reproducing kernels and generalized reconstruction problems. We will also outline here, how sampling inequalities can be used to derive error estimates.