Radial Basis Function-generated Finite Differences (RBF-FD): Basic concepts and some applications

Bengt Fornberg, University of Colorado

The RBF-FD method can be seen as a major generalization of classical finite differences. By supplementing multivariate polynomials with RBFs (or using RBFs only) when generating the stencil weights, structured grids become unnecessary. When instead using scattered nodes, the geometric flexibility becomes vastly improved, which greatly simplifies both local refinement and the effective handling if curved interfaces and boundaries.
It has recently been found that polynomials of relatively high orders together with a (constrained set of) polyharmonic splines (such as cubic or quintic radial functions) are particularly effective in this context. The resulting linear systems for calculating the RBF-FD stencil weights will then feature good numerical stability even in high-accuracy regimes. When applied to PDEs of different types, these stencils prove to be very cost effective in a wide range of cases. Among some applications, we will in particular note seismic exploration, which requires the elastic wave equation to be solved in media with large numbers of curved interfaces.