Towards Error Bounds for Localized Kernel-Based Meshless Methods

Robert Schaback, Georg-August-Universität Göttingen

Much has been done in recent years for the error analysis of nonlocal kernel-based meshless methods, and this talk will focus on the localized case. Any error analysis must deal with consistency and stability. Consistency of localized methods is well understood now, including optimality (https://arxiv.org/abs/1611.04750) and minimality (https://arxiv.org/abs/1611.05001, both with Oleg Davydov) of localized approximations to differential operators, but stability is a serious open problem because it is of nonlocal nature, based on the proper selection of sufficiently many test equations using localized approximations covering the whole domain and the boundary. An experimental approach (Lecture Notes in Computational Science and Engineering 115, 2017, p. 117-143) can in certain situations let the computer confirm the stability of a localized method, and therefore there is hope for a good theory of stability. For the nonlocal case, well-posedness of the PDE problem together with oversampling on the test side turned out to be sufficient for stability (Num. Math. 132, 2016, 597-630), and this can also be expected for the much more difficult localized situation. The talk will extend the latter approach to the localized case and present preliminary results together with promising failures.