From the master equation to mean field game limits, fluctuations, and large deviations

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A mean field game (MFG) is a stochastic differential game with a continuum of players, describing the limit as n tends to infinity of Nash equilibria of certain n-player games, in which agents interact symmetrically through the empirical measure of their state processes. One way to understand a mean field game (MFG) is through its "master equation," an infinite-dimensional PDE for the value function. A solution of this equation can be used, for instance, to construct a solution of the original mean field game or to prove convergence of n-player equilibria to the MFG. This talk shows how to use a sufficiently smooth solution to answer several open questions about the limit theory for MFGs. In particular, we derive for the first time a central limit theorem and a large deviations principle for the n-player empirical measure (in equilibrium), as well as non-asymptotic concentration estimates. The key idea is to use the master equation to quantitatively relate the n-player equilibrium to a McKean-Vlasov system of interacting diffusions. Joint work with Francois Delarue and Kavita Ramanan.