

Computing huge subspaces of polynomials: Symmetries to the rescue!

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Last spring, I visited François Bergeron and we worked on his favorite objects: the spaces $H(n,k)$ of diagonal harmonic polynomials in k sets of n variables. Those spaces connect to many areas of algebraic combinatorics, representation theory and beyond, and the case $H(n,2)$ became famous a decade ago with the $n!$ conjecture and its proof by Haiman.

To fuel his ongoing studies François needed to compute the structure of $H(5,6)$. This is a space of dimension $6 \cdot 10^5$ made of polynomials in 30 variables of degree up to 15, each having thousands of terms.

In this talk, I'll explain how the calculation can now be completed in 45 minutes with a dozen cores and ~ 15 Go of memory. This exploits a combination of strategies (symmetries, representation theory of the symmetric and general linear group, ...), each of which reduces the complexity in time and memory by one or two orders of magnitude.

There will be little prerequisites and I'll focus on the code behind the scene to start a discussion on some pieces of it that are general purpose and could be merged in SageMath.