Heights on stacks
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Here are two popular questions in number theory:

1. How many degree-d number fields are there with discriminant at most X?
2. How many rational points are there on a cubic surface with height at most X?

Our expectations about the first question are governed by Malle's conjecture; about the second, by the Batyrev-Manin conjecture. The forms of the conjectures are very similar, predicting in both cases an asymptotic of the form c X^a (log X)^b, and this is no coincidence: I will explain how to think of both questions in a common framework, that of counting points of bounded height on an algebraic stack. A serious obstacle is that there is no definition of the height of a rational point on a stack. I will propose a definition and try to convince you it's the right one. If there's time, I'll also argue that when we talk about heights with respect to a line bundle we have always secretly meant "vector bundle," or should have.

(joint work with Matt Satriano and David Zureick-Brown)