A fractional phase-field model using an infinitesimal generator of α stable L\'{e}vy process

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In this talk, we study a space-fractional Allen-Cahn phase-field model in which the fractional Laplacian is considered to be an infinitesimal generator of an α-stable L\'{e}vy process with $0 < \alpha < 2$. This model shares the same advantages as the Riemann-Liouville (R-L) fractional model that: (i) it can control the sharpness and the decaying behavior of the interface via the tunable order parameter α ; (ii) as $\alpha \rightarrow 2^{-}$, it provides good approximations to the classical Allen-Cahn model. Additionally, it enjoys the following advantages: (i). it generates numerical solutions with sharper interfaces than the R-L fractional model for $\alpha \in (1,2)$; furthermore, it can generate numerical solutions with even sharper interfaces by reducing the fractional order to $\alpha \in (0,1]$; (ii). it is rotationally invariant, i.e., the numerical simulation results are independent of the orientation of the physical problem or the computational mesh, which is a desired property physically; (iii). it exhibites a much slower mass decaying rate than the R-L fractional model, especially for smaller α. We also propose and analyze a first order in time and second order in space energy stable finite difference scheme.