Boundary Conditions for Two-Sided Fractional Diffusion

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This talk develops appropriate boundary conditions for the two-sided fractional diffusion equation, where the usual second derivative in space is replaced by a weighted average of positive and negative fractional derivatives. Mass preserving, reflecting boundary conditions for two-sided fractional diffusion involve a balance of left and right fractional derivatives at the boundary. Stable, consistent explicit and implicit Euler methods are detailed, and steady state solutions are derived. Steady state solutions for two-sided fractional diffusion equations using both Riemann-Liouville and Caputo flux are computed. For Riemann-Liouville flux and reflecting boundary conditions, the steady-state solution is singular at one or both of the end-points. For Caputo flux and reflecting boundary conditions, the steady-state solution is a constant function. Numerical experiments illustrate the convergence of these numerical methods. The influence of the reflecting boundary on the steady-state behavior subject to both the Riemann-Liouville and Caputo fluxes is analyzed, and extensions to tempered fractional derivatives are discussed. If time permits, an application to source identification in hydrology will be presented.