Efficient and accurate numerical methods for solving fractional PDEs

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We present efficient and accurate numerical methods for fractional Laplacian equations and for time-fractional diffusion equations.

For fractional Laplacian problem in bounded domains, we adopt the Caffarelli-Silvestre extension which transforms the fractional Laplacian equation in d-dimension into an equivalent system with local derivatives in (d+1)-dimension. We develop an efficient numerical method based on the generalized Laguerre approximation in the extended direction and usual (FEM or spectral) approximation in the original domain. Moreover, we enrich the spectral approximation space by using leading singular functions associated with the extended y-direction so that high-accuracy can be achieved despite the singularity of the extended problem at $y=0$.

For time-fractional diffusion equations, we can adopt a similar approach used for the extended problem of the fractional Laplacian. However, an essential difficulty arises as the time-fractional operator is not self-adjoint which makes the diagonalization process very ill conditioned. We shall propose a novel approach to overcome this difficulty.