

Nonlocal Diffusion Operators for Normal and Anomalous Dynamics

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The Laplacian is the infinitesimal generator of isotropic Brownian motion, being the limit process of normal diffusion, while the fractional Laplacian serves as the infinitesimal generator of the limit process of isotropic Lévy process. Taking limit, in some sense, means that the operators can approximate the physical process well after sufficient long time. We introduce the nonlocal operators (being effective from the starting time), which describe the general processes undergoing normal diffusion. For anomalous diffusion, we extend to the anisotropic fractional Laplacian and the tempered one in \mathbb{R}^n . Their definitions from different ideas are proved to be equivalent. Based on these new diffusion operators, we further derive the deterministic governing equations of some interesting statistical quantities of the very general jump processes with multiple internal states. Finally, we consider the associated initial and boundary value problems and prove their well-posedness of the Galerkin weak formulation in \mathbb{R}^n . To obtain the coercivity, we claim that the probability measure function should be nondegenerate.