

Energy Stable Discontinuous Galerkin Methods for Maxwell's Equations in Nonlinear Optical Media

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The propagation of electromagnetic waves in general media is modeled by the time-dependent Maxwell's partial differential equations (PDEs), coupled with constitutive laws that describe the response of the media. In this work, we focus on nonlinear optical media whose response is modeled by a system of first order nonlinear ordinary differential equations (ODEs), which include a single resonance linear Lorentz dispersion, and the nonlinearity comes from the instantaneous electronic Kerr response and the residual Raman molecular vibrational response. We apply high order discontinuous Galerkin discretizations in space to the hybrid PDE-ODE Maxwell system with several choices of numerical fluxes, and the resulting semi-discrete methods are shown to be energy stable. Under some restrictions on the strength of the nonlinearity, error estimates are also established. We propose novel strategies to treat the nonlinearity in our model within the framework of the second-order leap-frog and implicit trapezoidal time integrators. The performance of the overall algorithms are demonstrated through numerical simulations of kink and antikink waves, and third-harmonic generation in soliton propagation.