Higher Order Globally Constraint-Preserving FVTD and DGTD Schemes for Time-Dependent Computational Electrodynamics

By Dinshaw S. Balsara (dbalsara@nd.edu), University of Notre Dame

The numerical solution of Maxwell's time-dependent equations plays a very useful role in electrical engineering, physical chemistry and photonics at the nanoscale. Maxwell's equations constitute an involution-constrained PDE system. The equations can be cast as a regular hyperbolic system of conservation laws and many innovations that have been developed in that area of study can be used to advantage even for computational electrodynamics (CED). However, the structure of Maxwell's equations is inherently very different from that of other conservation laws, with Faraday's and Ampere's laws having a curl-type update. This curl-type update also ensures that Gauss' laws for charge and magnetic charge are naturally satisfied. The highly popular and successful Finite Difference Time Domain (FDTD) scheme can mimetically fulfil the goals of global constraint preservation, treatment of perfectly matched layer (PML) boundary conditions and inclusion of auxiliary differential equations (ADEs) for dispersive media. However, FDTD does not extend seamlessly to higher orders. Finite Volume Time Domain (FVTD) and Discontinuous Galerkin Time Domain (DGTD) schemes have been designed for CED and they do extend to higher orders. However, versions of these methods that are in the literature do not simultaneously satisfy the global constraints, and present a seamless path for the inclusion of PML and ADEs. Clearly, a synthesis is needed where the best aspects and versatility of FDTD are retained, while rethinking from the ground-up the best aspects of FVTD and DGTD schemes for CED.

The goal of this talk is to present recently innovated FVTD and DGTD schemes that are indeed the closest analogues of FDTD. In fact, these novel methods are built on the same foundation provided by the Yee-type mesh; thereby retaining many of the advantages of FDTD. However, they also incorporate recent innovations from the CFD and MHD fields. They represent a confluence of three leading-edge innovations that have been pioneered by the author:- **1)** The new methods use WENO and DG reconstruction methods, but only with a highly innovative recasting of the reconstruction, so as to satisfy Gauss' laws (Balsara 2001, 2004, 2009, Balsara *et al*. 2017, 2018). **2)** The methods also utilize multidimensional Riemann solvers so that the global constraints are satisfied on the same control volume (Balsara 2010, 2012, 2014, Balsara &

Dumbser 2015, Balsara *et al*. 2017, 2018). **3)** To treat stiff source terms, PML and ADEs we also recast the ADER methods (Dumbser *et al*. 2008, 2013, Balsara *et al*. 2009, 2013, Balsara *et al*. 2017, 2018a,b). A full von Neumann stability analysis of multidimensional, globally constraint-preserving DGTD schemes for CED is also presented (Balsara and Käppeli 2018) and it is shown that DGTD and PNPM schemes that result from our analysis have superior wave propagation properties as well as preservation of electromagnetic energy at higher orders of accuracy. The PNPM schemes also offer the advantage of large CFL number relative to DGTD schemes.

As the focus in engineering and the sciences moves to uncertainty quantification, machine learning and verification and validation of simulated data, a new generation of high accuracy schemes is crucial to those broader goals. The proposed new techniques lay a firm foundation for the entire discipline of 21st century CED, leading to advances across multiple disciplines.