Entropy stable high order discontinuous Galerkin methods for hyperbolic conservation laws

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It is well known that semi-discrete high order discontinuous Galerkin (DG) methods satisfy cell entropy inequalities for the square entropy for both scalar conservation laws and symmetric hyperbolic systems, in any space dimension and for any riangulations. However, this property holds only for the square entropy and the integration in the DG methods must be exact. It is significantly more difficult to design DG methods to satisfy entropy inequalities for a non-square convex entropy, and / or when the integration is approximated by a numerical quadrature. In this talk, we report on our recent development of a unified framework for designing high order DG methods which will satisfy entropy inequalities for any given single convex entropy, through suitable numerical quadrature which is specific to this given entropy. Our framework applies from one-dimensional scalar cases all the way to multi-dimensional systems of conservation laws. For the one-dimensional case, our numerical quadrature is based on the methodology established in the literature, with the main ingredients being summation-by-parts (SBP) operators derived from Legendre Gauss-Lobatto quadrature, the entropy stable flux within elements, and the entropy stable flux at element interfaces. We then generalize the scheme to two-dimensional triangular meshes by constructing SBP operators on triangles based on a special quadrature rule. A local discontinuous Galerkin (LDG) type treatment is also incorporated to achieve the generalization to convection-diffusion equations. Numerical experiments will be reported to validate the accuracy and shock capturing efficacy of these entropy stable DG methods.

This is a joint work with Tianheng Chen.
We will also report the generalization to MHD in the symmetrizable Godunov form as a joint work with Yong Liu and Mengping Zhang.