Title: Fractional Partial Differential Equations for Conservation Laws and Beyond

Presenter: George Em Karniadakis, Division of Applied Mathematics, Brown University

Abstract

Fractional partial differential equations (FPDEs) are emerging as a powerful tool for modeling challenging multiscale phenomena including overlapping microscopic and macroscopic scales, anomalous transport, and long range time memory or spatial interactions. Compared to integer-order PDEs, the fractional order of the derivatives in FPDEs may be a function of space and time or even a distribution, opening up great opportunities for modeling and simulation of multi-physics phenomena, e.g. seamless transition from wave propagation to diffusion, or from local to non-local dynamics. In addition, data-driven fractional differential operators may be constructed to fit data from a particular experiment or specific phenomenon, including the effect of uncertainties, in which the fractional orders are determined directly from the data, and introducing nonlinearities leading to more complex operators, with one or more fractional orders, capable to model less typical phenomena (such as, for instance, wave propagation in heterogeneous systems).

We will review the theory and stochastic interpretation of some prototype fractional PDEs and subsequently we will introduce new algorithms for their numerical solution. Specifically, we will introduce different classes of regular and singular fractional Sturm-Liouville (FSLPs) problems with an important extension that leads to tempered FSLPs. We first obtain explicitly the eigenvalues and eigenfunctions of these problems and show that they have the form of Poly-Fractonomials which coincide with the standard spectral polynomial when the fraction order approaches the integer order. They are best-suited to approximate singular solutions. Hence, we employ them as new basis/test functions in developing Petrov-Galerkin spectral methods for fractional elliptic problems, followed by stability and error analysis. The same eigenfunction basis can be used both in collocation and Petrov-Galerkin projections to approximate distributed-order fractional PDEs, which can model non-local phenomena but also account for uncertainties.