High-order compact schemes for Navier-Stokes Equations

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We are interested in high-order discretizations of the Navier-Stokes equations. The Navier-Stokes equations play a central role in modeling fluid flows. Here we focus on incompressible flows. It is well-known that this system may be represented in pure streamfunction formulation (due to Lagrange , 1871), $$\partial_t \Delta \psi + \nabla ^\perp \psi \cdot \nabla \Delta \psi - \nu \Delta^2 \psi = f(x,y,t),$$ where $\nabla^\perp \psi = (-\partial_y \psi, \partial_x \psi)$ is the velocity vector. These equations are supplemented with the no-slip boundary condition and with an initial condition.

To understand the nature of the scheme, we first describe the high-order compact approximation for a one-dimensional time-independent scheme $\partial_x^4 \psi=f$ on the interval $[0,1]$, where boundary conditions on psi and ψ' . We prove that the discrete approximation of the problem converges to the exact solution, and that the error is bounded by $C h^4$, where h is the mesh size [1]. This result is extended to a more general time-independent fourth-order one-dimensional problem and it is shown that also in this case the scheme exhibits fourth-order convergence. We then consider a one-dimensional time-dependent fourth-order equation. Almost optimal convergence is proved for this case. Numerical results show optimal (fourth-order) convergence.

In the second part of the talk the two-dimensional biharmonic problem and the Navier-Stokes system are considered. We describe a fourth-order compact scheme for regular domains in 2D. Convergence was proved for a second–order scheme approximating the full Navier-Stokes equations. We then proceed to irregular domains. A scheme is constructed for the biharmonic equation via two-dimensional polynomials on irregular elements. A fourth-order compact scheme is presented for the Navier-Stokes equations using one-dimensional discrete differentiation operators near the irregular boundary. Numerical results demonstrate fourth-order accuracy of the scheme for irregular 2D domains as well.

Joint work with Matania Ben-Artzi and Jean-Pierre Croisille

[1] M. Ben-Artzi and J.-P. Croisille, “Navier-Stokes Equations in Planar Domains”, 2013, Imperial College Press.