Accuracy Controlled Computation - the Merit of Residuals

Wolfgang Dahmen, University of South Carolina

The efficient approximate solution of partial differential equations and how to
estimate the accuracy of a numerical result is by far best understood for elliptic problems and their close relatives. But what about assessing accuracy when dealing with ""less friendly"" problem types such as non-coercive, singularly perturbed, or transport dominated problems? In this talk we propose a general ""roadmap"" towards error controlled computation beyond the linear elliptic case. A central ingredient is to devise a possibly unsymmetric stable variational formulation which then entails tight error-residual relations. This means that errors in a given trial-norm are bounded from above and below by corresponding residuals in an appropriate dual test norm. Since residuals contain only known quantities this may serve as the starting point for deriving reliable and efficient a posteriori error bounds. We briefly indicate the relevance of such bounds far beyond adaptive mesh refinement, for instance, in the context of generating certified reduced bases and their role in Uncertainty Quantification. We then indicate realizations of the ""roadmap"" for two different problem scenarios, namely a kinetic model of radiative transfer type, and the p-Poisson problem with $1<p<2$.