Bound-preserving high order schemes for hyperbolic equations: survey and recent developments

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Solutions to many hyperbolic equations have convex invariant regions, for example solutions to scalar conservation laws satisfy maximum principle, solutions to compressible Euler equations satisfy positivity-preserving property for density and internal energy, etc. It is however a challenge to design schemes whose solutions also honor such invariant regions. This is especially the case for high order accurate schemes. In this talk we will first survey strategies in the literature to design high order bound-preserving schemes, including the general framework in constructing high order bound-preserving finite volume and discontinuous Galerkin schemes for scalar and systems of hyperbolic equations through a simple scaling limiter and a convex combination argument based on first order bound-preserving building blocks, and various flux limiters to design high order bound-preserving finite difference schemes. We will then discuss a few recent developments, including high order bound-preserving schemes for relativistic hydrodynamics, high order discontinuous Galerkin Lagrangian schemes, high order discontinuous Galerkin methods for radiative transfer equations, high order discontinuous Galerkin methods for MHD, and implicit bound-preserving schemes. Numerical tests demonstrating the good performance of these schemes will be reported.