Computing Multiple Solutions of Partial Differential Equations

Patrick Farrell, University of Oxford

Nonlinear PDEs typically support multiple solutions, and computing them is difficult. This problem arises in the context of liquid crystals when solving the first-order optimality conditions to compute energy minimizers; distinct minimizers correspond to different solutions of the Euler--Lagrange equations. In this talk I will present a new technique, deflation, for this task.

Deflation has three advantages. First, it is capable of computing disconnected bifurcation diagrams; previous approaches only aimed to compute the subset of a bifurcation diagram continuously connected to the initial data. Second, its implementation is extremely simple: it only requires a minor modification to an existing Newton-based solver. Third, it can scale to very large discretizations if a good preconditioner is available, such as a multigrid method.

We will present the application of deflation for identifying previously unknown solutions to various problems in nematic, cholesteric and smectic-A liquid crystals, including the Oseen--Frank, Landau--de Gennes, and Pevnyi--Selinger--Sluckin models.