Energy-Minimization, Finite Elements, and Multilevel Methods for Nematic and Cholesteric Liquid Crystals

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The main focus of this talk will be on the computational modeling of equilibrium configurations for nematic and cholesteric liquid crystals influenced by elastic and electric effects. Thus, the method targets minimization of the system free energy based on the Frank-Oseen free-energy model, subject to the unit-length constraint of the director. We consider an energy-minimization finite-element approach to discretize the constrained optimization problem along with Newton's method to linearize the system. We are able to show that solutions to the intermediate discretized linearizations exist generally and are unique under certain assumptions. Numerical experiments are performed for problems with a range of Frank elastic constants as well as simple and patterned boundary conditions. The resulting algorithm accurately handles heterogeneous Frank constants and efficiently resolves configurations resulting from complicated boundary conditions relevant in ongoing research. Additionally, we present techniques to handle situations in which multiple equilibrium states can arise. This involves the incorporation of multilevel and nonlinear deflation techniques to solve the resulting linear systems.