Structure Preserving Schemes for Complex Nonlinear Systems

Jie Shen, Purdue University

Many complex nonlinear systems have intrinsic structures such as energy dissipation or conservation, and/or positivity/maximum principle preserving. It is desirable, sometimes necessary, to preserve these structures in a numerical scheme.

I will present the scalar auxiliary variable (SAV) approach and its variants to deal with nonlinear terms in a large class of dissipative/conservative systems. The SAV approach is not restricted to specific forms of the nonlinear part of the free energy or Hamiltonian. It leads to linear and unconditionally energy stable schemes which only require solving decoupled
linear equations with constant coefficients. Hence, these schemes are extremely efficient and very accurate when combined with higher-order BDF or diagonally implicit Runge-Kutta schemes.

As a specific example of application, we shall use the SAV approach to construct a simple and robust numerical scheme for the Q-tensor model for rod-like liquid crystals.