Connections

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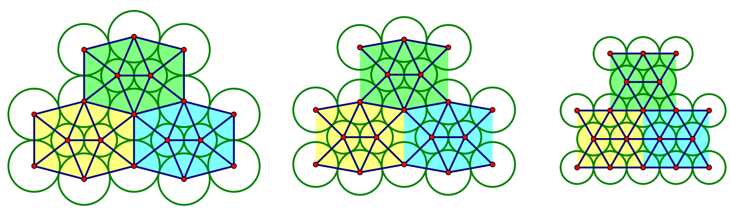
There are all kinds of circle packing problems in the plane. They are all connected:

1. László Fejes Tóth gave a rigorous proof that the most dense packing of equal circles in the plane is the usual one, where each circle touches 6 others as in the figure on the right.



   What about packings with different sizes of disks? Suppose the radii r_i are constrained so that $\rho \leq r_i \leq 1$. For a given ρ he had series of best guesses as the figure on the left when the graph is triangulated, and the middle, where it "flows" away from the one on the left.

2. Sándor Fekete and others have algorithmic methods of packing different sized circles and other shapes in a circle and other shapes, when conceivable with respect to density.

3. Thomas Fernique has an example of a triangulated circle packing in the plane, with different sizes of circles, that has a ρ larger than any of Fejes Tóth's, improving Fejes Tóth's best guesses.

4. In the talk by Steven Gortler it is shown that the flip-and-flow motion of Fejes Tóth (seen between the first two figures above) is so powerful that it achieves any triangulated packing of circles in the sphere, proving a classic theorem in KAT theory.

5. Rigidity theory can apply to the theory of "sticky disks" as in the talk by Louis Theran to show that locally jammed packings correspond to the standard generic rigidity of bar frameworks. This is motivated by a previously unproved assumption in the theory of packings of granular materials.

I propose these as counterexamples to the thought that the theory of classical density oriented circle packings are disjoint from the packings of KAT theory.