Flip Paths, Farey Sequences and Material Defects

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The problem of finding a path between two triangulations via a sequence of diagonal flips has been studied for nearly a century in the combinatorial (topological) and geometric settings. Finding the shortest flip path in the geometric setting is NP-complete. However, when restricted to lattice triangulations, i.e. triangulations of a subset of the integer lattice bounded by a simple, closed polygon, the problem has a polynomial time algorithm. Lattice triangulations have been studied for their uses in discriminant theory, Hilbert's 16th problem, toric varieties, and quantum spin systems. We significantly improve the complexity of algorithms for variants of flip path problems over lattice triangulations, by leveraging the structural relationship between their edges and Farey sequences. Our proofs use elementary number theory and geometry. Additionally, given two sets of edges we characterize which pairs of lattice triangulations respectively containing the two edge sets, minimize the length of the shortest flip path between the triangulations. The key result is an explicit, structural characterization of a unique minimal partially ordered set of diagonal flips that is necessary and sufficient for a flip path between an equilateral lattice triangulation to (any) lattice triangulation containing a given edge. Using this result, we give a simpler alternative proof that there is a unique minimal partially ordered set of diagonal flips (hence unique shortest flip path up to reordering) between two given lattice triangulations. Finally, we investigate the application of flip paths between lattice triangulations to so-called Stone-Wales defects and crack propagation in crystalline structures and other materials. Joint work with William Sims.