Quadratic differentials and deformation space of circle patterns on surfaces

Lam, Wai Yeung- University of Luxembourg

William Thurston proposed regarding the map induced from two circle packings with the same tangency pattern as a discrete holomorphic function. A discrete analogue of the Riemann mapping is deduced from Koebe-Andreev-Thurston theorem. A natural question is how to extend this theory to Riemann surfaces and relate classical conformal structures to discrete conformal structures.

Since circles are preserved under complex projective transformations, one can consider circle packings on surfaces with complex projective structures. Kojima, Mizushima and Tan conjectured that for a given combinatorics the deformation space of circle packings is diffeomorphic to the Teichmueller space via a natural projection.

In this talk, we report progress on the torus case. A tool to study the deformation space is a notion of quadratic differentials on graphs, which is related to self-stresses in rigidity theory.