Rigidity of Intermingled Point and Sphere Configurations

Graham, Opal- Florida State University

The rigidity of collections of points and the rigidity of collections of circles in the extended complex plane is proven by Beardon and Minda using a maximal amount of conformal invariant information. Crane and Short generalize this work to collections of points and collections of spheres in higher dimensions. Both use a notion of the cross-ratio for 4-tuples of points, while for circles and spheres, the inversive distance between pairs is used. When a collection involves intermingled points and spheres, these conformal invariants can no longer be used. Working in Lorentz space, we develop a notion of an invariant known as the Lorentz ratio, which corresponds to a conformal invariant of a point and two spheres. The Lorentz setting allows for an understanding of the correspondences between several geometries. Here, the rigidity of intermingled points and spheres can be seen as part of the more general statement of the rigidity of intermingled points, ideal points, and hyperplanes of hyperbolic (n+1)-space. This rigidity statement is accomplished via the new conformal invariant.