The geometry of nilpotent orbits via subbundles of the cotangent bundle

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The Springer resolution is the map from the cotangent bundle of the flag variety G/B of a simple algebraic group G to the nilpotent cone in the Lie algebra $\mathfrak{g}$. The fibers of this map, and the representations of the Weyl group on the cohomology of these fibers, play an important role in the representation theory of G over various fields.

Viewing the cotangent bundle as $G \times^B \mathfrak{n}$, where $\mathfrak{n}$ is the nilradical of the Lie algebra of B, one can also consider maps $G \times^B I \to \mathfrak{g}$ where $I \subset \mathfrak{n}$ is a B-stable subspace. The image of such a map is always the closure of a nilpotent orbit. These vector bundles (over G/B) and the corresponding maps to $\mathfrak{g}$ are important for understanding the geometry of nilpotent orbits, their closures, and their covers, and related questions in representation theory. We discuss some of these questions in this talk.