Rigid Continuation Paths II: Structured Polynomial Systems

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We design a probabilistic algorithm that, on input a polynomial system F given by black-box evaluation functions and e>0, outputs a (validated) approximate zero of F, with probability at least 1-e. When applying this algorithm to u . F, where u is uniformly random in the product of unitary groups, the algorithm performs at most
poly(n,d) L(F) (Gamma(F) log Gamma(F) + log log 1/e)
operations on average. Here n is the number of variables, d the maximum degree, L(F) denotes the evaluation cost of F, and Gamma(F) reflects a numerical aspect of F. Moreover, we prove that for inputs given by random Gaussian algebraic branching programs, the algorithm runs on average in time polynomial in n and d.
Our result may be interpreted as a first step towards providing an affirmative answer to a refined version of Smale's 17th question, concerned with structured systems of polynomial equations.

This is joint work with Felipe Cucker and Pierre Lairez. It is a continuation of Pierre Lairez's paper "Rigid Continuation Paths I" (J. AMS 33.2, 2020). We hope to finalize a version for the arXiv soon.