The Sum-of-Squares Approach to Disentangling Non-Spherical Gaussian Mixtures

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Sum-of-Squares (SoS) is a systematic proof system for reasoning about real solutions to systems of multivariate polynomial inequalities over the reals. In the last few years, this proof system has been used to develop a principled method for designing efficient algorithms for average-case algorithmic problems - problems where the inputs are chosen according to some natural probability distribution. By relying on the relationship between sum-of-squares proofs and semidefinite programming, this method reduces designing an efficient algorithm for a given problem to giving a low-degree sum-of-squares certificate of the correctness of a purported solution.

In this talk, I'll describe an algorithm that uses this approach to cluster a mixture of k arbitrary Gaussians whenever they are information-theoretically clusterable. A similar result for the special case when all components have spherical covariances was shown earlier in 2018. This result works for arbitrary non-spherical (and potentially rank deficient covariances).

The most exciting aspect of this algorithm is showing identifiability of Gaussian clusters relying on sum-of-squares certificates of hypercontractivity of degree 2 polynomials and anti-concentration of linear projections of Gaussians. Both of these tools have already found new interesting applications.

The talk should be accessible to a broad audience and should not require any prior knowledge of the problems/techniques. The talk is based on joint works with Ainesh Bakshi (CMU) (https://arxiv.org/abs/2005.02970).