

Combinatorics of punctured surface intersections

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If M is a compact 3-manifold with torus boundary whose interior has a complete hyperbolic structure, Thurston showed that all but finitely many Dehn fillings $M(\alpha)$ are hyperbolic. Many of the exceptions can be characterized as manifolds containing interesting (embedded) surfaces of low genus, for example an essential sphere or torus, or a Heegaard sphere or torus. If $M(\alpha)$ and $M(\beta)$ are two such exceptional fillings, the surfaces in $M(\alpha)$ and $M(\beta)$ give rise to punctured surfaces in M with boundary-slopes α and β , respectively. By studying the intersection of these surfaces one can get information about the topology of the situation, for example upper bounds on the intersection number $\Delta(\alpha, \beta)$ between the slopes α and β .

In the lectures we will briefly summarize known results on the various types of pairs of exceptional Dehn fillings, and then we will describe some of the combinatorial techniques that have been developed in this context. As an illustration we will apply these to the situation where $M(\alpha)$ and $M(\beta)$ are reducible; in particular we will show that here $\Delta(\alpha, \beta) = 1$. We will also discuss several open questions about this case.