

Finding diagrams

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Using SnapPy and KLO, we will explore a few Dehn surgery problems: (1) Given a SnapPy manifold with an S^3 filling, we'll find a knot diagram. (2) Starting from Motegi's asymmetric seiferter, we find many more. (3) Given a SnapPy manifold with some exceptional fillings and a strong inversion, we'll find a tangle diagram for its quotient.

Excercises on Finding Diagrams using SnapPy and KLO

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1 Example 1

Recall that we have

```
M=Manifold('t12533')
M01=M.drill(0).drill(1)
```

1. Browse M and poke around.
2. Check that filling the last two cusps of M01 along the meridians of the drilled geodesics produces a manifold isometric to M.
3. Check that M is asymmetric. Is M01 asymmetric?
4. Find knot diagrams for the following manifolds:
't12681', 'o9_38928', 'o9_39162', 'o9_40363',
'o9_40487', 'o9_40504', 'o9_40582', 'o9_42675'
These census manifolds are complements of asymmetric L-space knots, found by Nathan Dunfield.
5. Which are closures of positive braids? (Up to mirroring.)
6. Use KLO to find the Alexander polynomials.
7. Use SnapPy within Sage to find their Alexander polynomials.

2 Example 2

1. Draw the (2,6) torus link in SnapPy. Find the cabling slope on one component. For each solid torus filling of one component, find the further corresponding S^3 fillings of the other component.
2. Generalize DMM's other seiferters of $P(-3, 3, 5)$.
3. In our example, what goes wrong when $n = 2$?
4. Find a link diagram for our example with $n = 3$.
5. From KLO, export a link diagram for use in SnapPy. Load in PLink and send to SnapPy. Is it the manifold you expected?
6. What Seifert fibered spaces are obtained from our examples? Use KLO!

3 Example 3

In §4.2 of "A census of exceptional Dehn fillings," Dunfield identifies the following hyperbolic, one-cusped manifolds as having four toroidal fillings.

```
's772', 's778', 's911', 'v2640', 't08282', 't11538', 't12033', 't12035', 't12036', 't12041', 't12043', 't12045',  
't12050', 't12548', 't12648', 'o9_35259', 'o_936732', 'o9_37030', 'o9_38039', 'o9_39094', 'o9_40054', 'o9_41000',  
'o9_41004', 'o9_41006', 'o9_41007', 'o9_41008', 'o9_43799'.
```

Are there infinitely many such examples?

By visualizing their quotients, we may be able to discern relationships among them that generalize to infinite families. Pick a manifold and try the following:

1. Show the manifold is strongly invertible or freely invertible.
2. Find a tangle diagram for the quotient.
3. Observe how essential tori arise in the double branched covers of the associated fillings.

Basic Notes on SnapPy

- `M=Manifold()` opens PLink sketcher to input link complement
- `M=Manifold('blahblah')` loads the known manifold “blahblah”
- Press `Tab` to complete partially typed function or get list of possible completions
- Press `↑` to cycle through previously typed commands. If you’ve already typed something, it will cycle through commands that complete what you’ve typed.
- Don’t forget `()` even if there is no argument for the function!
- `M.solution_type()` tells you if SnapPy has found a hyperbolic structure. When it finds one, you’ve got it. When it doesn’t, maybe it was a funny triangulation or maybe it’s really not hyperbolic.
- `M.identify()` tells you if the manifold is in the census
- `M.browse()` opens a clickable interface. It’ll give you a link diagram if it knows it is a link exterior.
- Cusps
 - `M.cusp_info()` tells you about the cusps and their shape. Numbering starts at 0.
 - Unless M was given as a link exterior from PLink, it will use the two shortest slopes for a basis. Roughly speaking, any non-hyperbolic filling is short.
 - If M is a link exterior from PLink, then its cusps have the standard meridian-longitude basis. So $(1, 0)$ is the meridian and $(0, 1)$ is the longitude.
- Dehn filling
 - `M.dehn_fill([(a,b), (c,d), (e,f)])` fills the cusps of M with the specified slopes. Numbering starts at 0, so the 1st cusp is filled with slope (c, d) .
 - Filling with slope (p, q) when $\gcd(p, q) > 1$ gives an orbifold. So $(2, 0)$ filling a knot complement
 - Use `(0, 0)` to keep the cusp unfilled.
- `M.drill(n)` gives the complement of the n th shortest geodesic. Numbering starts at 0.
- `M.fundamental_group()` and `M.homology()` give the fundamental group and 1st homology of M . These work even if M is not hyperbolic.
- `M.volume()` gives the volume of the hyperbolic manifold M .
- `M.symmetry_group()` returns the symmetry group of the manifold M .
- `M.is_isometric_to(N)` tells you if the hyperbolic manifolds M and N are isometric.
- `M.is_isometric_to(N, return_isometries=True)` gives information on the action of the symmetries on the cusps

Basic Notes on KLO

- When drawing, hold **Shift** to crouch.
- When Processing a diagram, click crossings to correct.
- Focus on component with **Ctrl**-click, **Cmd**-click, or right-click. Also hold **Shift** to add more components to focus.
- Focus on a single component and then click the little colored box to change the color of the component.
- Click on region for Reidemeister I, II, III simplification.
- Drag over/under strand to adjacent crossing.
- Rolfsen Twists (twisting about a circle)
 - Remove self-crossings from unknot component
 - Click disk to eliminate interior crossings
 - **P** Parallelize strands through circle
 - **T** Twist about circle, enter number
 - It can be useful to simplify the diagram before twisting.
- Click ∞ -framed component to remove it.
- Click Δ to draw in ∞ -framed unknots for twisting.
- From the menu, “Convert to Knot” to calculate invariants of the underlying link.