Cameron Gordon problems

Exercises.

(1) Show that $\Delta(m_1/l_1, m_2/l_2) = |m_1 l_2 - m_2 l_1|$.

(2) Show that the homeomorphism type of $M(\alpha)$ depends only on the isotopy class of α.

(3) Let M be a compact, orientable, irreducible 3-manifold with ∂M an incompressible torus. Suppose M contains an essential annulus. Show that M either contains an essential torus or is Seifert fibered.

(4) Show that pq-Dehn surgery on the (p,q)-torus knot is a connected sum of two lens spaces.

(5) Show that ± 4-Dehn surgery on the figure eight knot contains a Klein bottle.

Steve Boyer Problems

Character Varieties, Surfaces and Applications to Surgery Theory

July 15, 2019

Exercises for Lecture 1

1. Let T be a tree and $\mathrm{Aut}(T)$ the group of simplicial automorphisms of T without inversions[1].
   (a) Suppose that $\varphi_1, \varphi_2 \in \mathrm{Aut}(T)$ satisfy $\mathrm{Fix}(\varphi_1) \neq \emptyset, \mathrm{Fix}(\varphi_2) \neq \emptyset$, $\mathrm{Fix}(\varphi_1) \cap \mathrm{Fix}(\varphi_2) = \emptyset$. Show that $\varphi_1 \circ \varphi_2$ has no fixed points in T.
   (b) If X, Y, Z are connected subtrees of T which are not pairwise disjoint, then $X \cap Y \cap Z \neq \emptyset$.
   (c) Prove that a finitely generated group acts non-trivially on a tree if and only if some element of the group acts fixed point freely on the tree.
2. Let v be a discrete valuation on a field $\mathbb{F}$
   (a) $\mathcal{O}(v)$ has a unique maximal ideal $\mathfrak{M}(v)$ given by $\mathfrak{M}(v) = \{a \in \mathbb{F} : v(a) \geq 1\}$.
   (b) The group of units of the ring $\mathcal{O}(v)$ is $\mathcal{O}(v) \setminus \mathfrak{M}(v)$.
3. Let v be a discrete valuation on a field $\mathbb{F}$ and $T(v)$ the associated tree.
   (a) Show that the stabilizer of a vertex of $T(v)$ is conjugate to $SL(2, \mathcal{O}(v))$. In particular, the trace of such an element is contained in $\mathcal{O}(v)$.
   (b) Show that an element $A \in SL(2, \mathbb{F})$ fixes a vertex of $T(v)$ iff $\mathrm{tr}(A) \in \mathcal{O}(v)$.
   (c) **Challenge**: Show that if $A \in SL(2, \mathbb{F})$ fixes an edge e of $T(v)$, it fixes the vertices incident to e.

0.1 References for Lecture 1

1. J.-P. Serre, **Trees**, Springer-Verlag Berlin Heidelberg New York 1980.
2. P. B. Shalen, *Representations of 3-manifold groups*, in **Handbook of Geometric Topology**, R. Daverman and R. Sher, eds. North-Holland, Amsterdam, 2002, pp. 955–1044.

[1] An *inversion* is a simplicial automorhism of T which fixes the midpoint of some edge but not the edge.