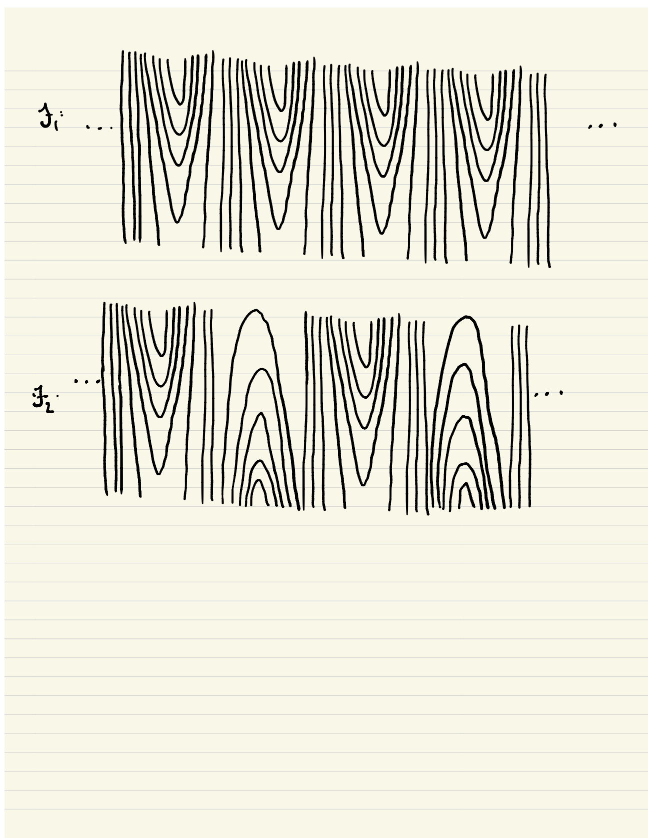
CODIMENSION ONE FOLIATIONS - PROBLEM SET 2
ICERM - JULY 17, 2019

(1) Let $\mathcal{H}$ be the foliation of $\mathbb{R}^3$ by horizontal planes. Draw a smooth embedding of S^2 in $\mathbb{R}^3$ such that $\mathcal{H} \cap S^2$ has exactly one saddle singularity, together with extrema. Describe an isotopy of the embedded sphere to an embedded sphere S_1^2 so that $S_1^2 \cap \mathcal{H}$ has no saddle singularities.

(2) Consider $M = T^2 \times [0,1]$. For different choices of orientations on the surfaces $T^2 \times \{0\}, T^2 \times \{1\}$, and $C \times [0,1]$, perform staircase cut and paste with $\gamma_1 = T^2 \times \{0,1\}$ and $\gamma_2 = C \times [0,1]$, where C is a fixed essential simple closed curve in T^2. What do you get?

(3) Consider the foliations $\mathcal{F}_1$ and $\mathcal{F}_2$ of $\mathbb{R}^2$ given in Figure 3.
   (a) Prove that $\mathcal{F}_1$ and $\mathcal{F}_2$ are not isotopic.
   (b) Prove that the foliations $\mathcal{F}_1 \times \mathbb{R}$ and $\mathcal{F}_1 \times \mathbb{R}$ of $\mathbb{R}^3$ are isotopic.



(4) Suppose $\mathcal{F}$ is a codimension one foliation of T^2 such that all leaves are smoothly immersed, and L is a compact leaf of $\mathcal{F}$. Cover L by foliation charts and suppose A is a very small smoothly embedded regular neighbourhood of L contained in the union

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of these charts. What can you say about $\mathcal{F} \cap A$? (You may assume the fact that A can be chosen so that each component of ∂A can be chosen to either lie in a leaf of $\mathcal{F}$ or else to be everywhere transverse to $\mathcal{F}$.)

(5) Suppose $\mathcal{F}$ is a codimension-one foliation of T^3 such that all leaves are smoothly immersed, and $L = T^2 \times \{1\}$ is a leaf of $\mathcal{F}$. Suppose N is a very small smoothly embedded regular neighbourhood of L. What can you say about $\mathcal{F} \cap N$? (You may assume the Roussarie-Thurston result.)

(6) Suppose $\mathcal{F}$ is a codimension-one foliation of a closed oriented 3-manifold such that all leaves are smoothly immersed, and $L = S^2$ is a leaf of $\mathcal{F}$. Suppose N is a very small smoothly embedded regular neighbourhood of L. What can you say about $\mathcal{F} \cap N$? (Again, you may assume the Roussarie-Thurston result.)

(7) Suppose that $\mathcal{F}$ is a co-oriented taut foliation of $S^1 \times S^2$ that has at least one compact leaf. Prove that $\mathcal{F}$ is isotopic to the product foliation of $S^1 \times S^2$ with leaves $\theta \times S^2$, $\theta \in S^1$. (Again, you may assume the Roussarie-Thurston result.)