One parameter families of Calabi–Yau threefolds with trivial monodromy

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Explicit Methods for Modularity of K3 Surfaces and Other Higher Weight Motives

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Differential operators of Calabi–Yau type

Picard–Fuchs operator of one parameter family of Calabi–Yau threefolds is the order four differential operator anihilating the period integral. We shall write differential operator in the following way $D := \sum_{i=0}^{r} t^i P_i(\Theta)$, where P_i is a polynomial of degree at most 4 and $\Theta := t\frac{d}{dt}$ is the logarithmic derivation.

Goal: Classify (make list of) them.

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Abstract version *Calabi–Yau type operators*

- has a maximal unipotent monodromy point at 0, i.e. $P_0(\Theta) = \Theta^4$,
- there is a holomorphic solution $\phi(t) \in \mathbb{Z}[[t]]$ around $t = 0$,
- instanton numbers are integral
- . . . .

Hypergeometric operators

$$D = \Theta^4 - \mu t(\Theta + \lambda_1)(\Theta + \lambda_2)(\Theta + \lambda_3)(\Theta + \lambda_4)$$

with $\lambda_1 + \lambda_4 = \lambda_2 + \lambda_3 = 1$

$$\begin{aligned}(\lambda_1, \lambda_2, \lambda_3, \lambda_4; \mu) = {} & (\tfrac{1}{5}, \tfrac{2}{5}, \tfrac{3}{5}, \tfrac{4}{5}; 5^5), (\tfrac{1}{6}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{5}{6}; 2^5 3^6), \\ & (\tfrac{1}{8}, \tfrac{3}{8}, \tfrac{5}{8}, \tfrac{7}{8}; 2^{18}), (\tfrac{1}{10}, \tfrac{3}{10}, \tfrac{7}{10}, \tfrac{9}{10}; 2^9 5^6), \\ & (\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3}; 3^6), (\tfrac{1}{4}, \tfrac{2}{4}, \tfrac{2}{4}, \tfrac{3}{4}; 2^{10}), (\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{2}{3}; 2^4 3^3), \\ & (\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}; 2^8), (\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{3}{4}, \tfrac{3}{4}; 2^{12}), (\tfrac{1}{6}, \tfrac{1}{6}, \tfrac{5}{6}, \tfrac{5}{6}; 2^8 3^6), \\ & (\tfrac{1}{4}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{3}{4}; 2^6 3^3), (\tfrac{1}{6}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{5}{6}; 2^8 3^3), \\ & (\tfrac{1}{6}, \tfrac{1}{4}, \tfrac{3}{4}, \tfrac{5}{6}; 2^{10} 3^3), (\tfrac{1}{12}, \tfrac{5}{12}, \tfrac{7}{12}, \tfrac{11}{12}; 12^6)\end{aligned}$$

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The first and most famous is

$$D = \Theta^4 - 5t(5\Theta+1)(5\Theta+2)(5\Theta+3)(5\Theta+4).$$

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$$D = \Theta^4 - 5t(5\Theta + 1)(5\Theta + 2)(5\Theta + 3)(5\Theta + 4).$$

Can we have more then 500 examples?

Double octic Calabi–Yau threefolds

Double octic is a double cover X of $\mathbb{P}^3$ branched over a surfaces of degree 8, it can be define as

$$u^2 = f_8(x, y, z, t), \qquad \text{in } \mathbb{P}(1^4, 4),$$

where f_8 is the equation of octic surface $D_8 = \{f_8 = 0\} \subset \mathbb{P}$.
If D_8 is non–singular then X is a Calabi–Yau threefolds with Hodge numbers

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C. Meyer found 11 rigid examples and 63 one–parameter families.

Fiber products

Let Y, Y' be rational elliptic surfaces over $\mathbb{P}^1$. If the positioins of singular fiber for Y and Y' are disjoin then the fiber product

$$X = Y \times_{\mathbb{P}^1} Y'$$

is a non–singular Calabi–Yau 3–fold. A pair of singular points of both factors over the same point in $\mathbb{P}^1$ introduce a singular point in the fiber product, when both fibers are semi–stable we get A_1 singularities.

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(+) well understood resolution of singularities, easy to compute Hodge numbers

(−) often non–projective

Conifold expansion

If we can identify a vanishing cycle for the family of Calabi–Yau threefolds, we might be able to find an expansion of the period integral.

Consider a family of double octics with a *vanishing tetrahedron*

$$u^2 = xyz(t - x - y - z)P_t(x, y, z), \qquad (P_0(0, 0, 0) \neq 0)$$

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Expanding $\frac{1}{\sqrt{P_t(tx,ty,tz)}} = \displaystyle\sum_{iklm} C_{iklm}x^ky^lz^mt^i$

we conclude $\phi(t) = A_0 + A_1t + A_2t^2 + \dots$, with

$$A_i = 2\pi^2 \sum_{klm} \frac{(2k)!(2l)!(2m)!}{4^{k+l+m}k!l!m!(k+l+m+1)!} C_{iklm}$$

Conifold expansion

Once we have sufficiently many terms of the powerseries expansion of $\phi(t) := A_0 + A_1 t + A_2 t^2 + \dots$ we find the operator that anihilates it by finding a polynomial recursion for the coefficients sequence A_i.

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- computing high powers of polynomials

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Fiber product case: We can identify vanishing cycle as a collision of fibers, this requires reparametrisation and can lead to computation in a number field.

Griffiths–Dwork method

Griffiths isomorphism (for a smooth degree d hypersurface in $\mathbb{P}^n$) $R_{d(k+1)-(n+1)} \equiv H^{n-1-k,k}_{\text{prim}}$ gives a basis $(\omega_1, \dots, \omega_r)$ of differential forms over the field $\mathbb{C}(t)$. We seek for a matrix A with entries in $\mathbb{C}(t)$ such that

$$\frac{d}{dt}\begin{pmatrix}\omega_1\\ \omega_2\\ \dots\\ \omega_r\end{pmatrix} = A(t)\begin{pmatrix}\omega_1\\ \omega_2\\ \dots\\ \omega_r\end{pmatrix}$$

This approach was implemented by Pierre Lairez in `Magma` ([*Computing periods of rational integrals* to appear in Mathematics of Computation].)

Can be given by the equation

$$w^2 = xyz(x+y+z-t)(1-x-y)(1-x)(1-z)(1+(t-2)x-y-z)$$

Expansion of the period function

$$f(t) = \pi^2 t(1 + t + \tfrac{43}{48}\, t^2 + \tfrac{19}{24}\, t^3 + \tfrac{10811}{15360}\, t^4 + \tfrac{9713}{15360}\, t^5 + \tfrac{987877}{1720320}\, t^6 + \tfrac{45289}{86016}\, t^7 + \tfrac{643165307}{1321205760}\, t^8 + \tfrac{598883431}{1321205760}\, t^9 + \tfrac{8976341483}{21139292160}\, t^{10} + \tfrac{4226087513}{10569646080}\, t^{11} + \tfrac{14631865278341}{38693360369664}\, t^{12} + \tfrac{69456457818479}{193466801848320}\, t^{13} + \tfrac{1058574187899337}{3095468829573120}\, t^{14} + \tfrac{126412705017457}{386933603696640}\, t^{15} + \tfrac{16862193585453801821}{538859213852088729600}\, t^{16} + \tfrac{32376611121257838 73}{107771842770417745 92}\, t^{17} + \tfrac{364181902092303090331}{1260101546238730567680}\, t^{18} + \tfrac{228154815542838809 1907}{819066005055174868 9920}\, t^{19} + \tfrac{16585324384187926301579}{61670852145330813665280}\, t^{20} + \tfrac{24784674044822741750011}{95309498770056712028160}\, t^{21} + \tfrac{176634921143827769536 7267}{701477910947617400527 2576}\, t^{22} + \tfrac{2354817323785002831732 4039}{9645321275529739257249 7920}\, t^{23} + \tfrac{117037050768319617164138590309}{493840449307122649971189350400}\, t^{24} + \tfrac{113736181591496323010991037217}{493840449307122649971189350400}\, t^{25} + \tfrac{3123996063897641413702576 03231}{1394373033337758070506887 577600}\, t^{26} + \tfrac{272168842371363178977922 18783}{124759692456536248413774 151680}\, t^{27} + \tfrac{93537919709575701641477646 19780999}{43995257947872942640633316 848435200}\, t^{28} + \tfrac{91235813820022693551062385 38236841}{43995257947872942640633316 848435200}\, t^{29} + \tfrac{4417349794747462016583628183774235833}{21821647942144979549754125156823859200}\, t^{30} + \tfrac{2074577473581544738956162453 6360677}{1049117689526200939892025247 92422400}\, t^{31} + \tfrac{3141369811969198016390031692597725 13227}{1625117635836833386105325393497282 314240}\, t^{32} + \tfrac{15363788271585525873295389704778 31678567}{81255881791841669305266269674864 11571200}\, t^{33} + \tfrac{11506866865856494689556282468568848080179}{62178413892887538250986362881635149414400}\, t^{34} + \dots)$$

Arr. No. 36: Picard–Fuchs operator

The operator is determined by the first 34 terms of the expansion and reads

$$32\,\theta\,(\theta-2)\,(\theta-1)^2 - 16\,t\theta\,(\theta-1)\left(9\,\theta^2-13\,\theta+8\right) + 8\,t^2\theta\left(33\,\theta^3-32\,\theta^2+38\,\theta-10\right)$$
$$-t^3(252\,\theta^4+104\,\theta^3+304\,\theta^2+76\,\theta+20) + t^4(132\,\theta^4+224\,\theta^3+292\,\theta^2+160\,\theta+38)$$
$$-t^5(36\,\theta^4+104\,\theta^3+140\,\theta^2+88\,\theta+21) + 4\,t^6\,(\theta+1)^4$$

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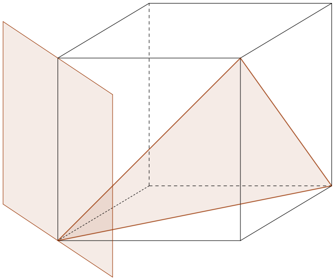
The Riemann symbol of this operator (table of indicial powers) is

$$\left\{\begin{array}{cccc} 0 & 1 & 2 & \infty \\ \hline 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 2 & 1 \end{array}\right\}$$

$t=0$ a *conifold* point
$t=1,\infty$ points of *maximal unipotent monodromy* (**MUM**).
Instanton numbers: 32, -96, 1440, 19704, -14496, 15837984, ...
It is a pullback of a simpler (known) operator.

Arrangement No. 70



Plane containning triangle is fixed, the other plane rotates. It can be given by the equation

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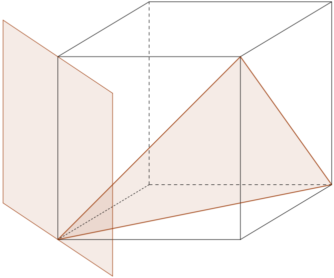

Plane containning triangle is fixed, the other plane rotates. It can be given by the equation

$$w^2 = xyz(x+y+z-t)(2-x-y-z)(1-x-y-z)(1-x)(1-z)$$

Period function expansion

$$\pi^2 t(1 + \frac{13}{16}\, t + \frac{485}{768}\, t^2 + \frac{12299}{24576}\, t^3 + \frac{534433}{1310720}\, t^4 + \frac{21458473}{62914560}\, t^5 + \frac{411317647}{1409286144}\, t^6 + \frac{7652032023}{30064771072}\, t^7 + \frac{3903778335439}{17317308137472}\, t^8 + \frac{280153481542507}{1385384650997760}\, t^9 + \frac{193501181678449}{1055531162664960}\, t^{10} + \frac{373735472718085 37}{2229281815548395 52}\, t^{11} + \frac{1432318881322834 3115}{9273812352681325 3632}\, t^{12} + \frac{1652393035078076 38355}{1154074426111453 822976}\, t^{13} + \frac{739240634523953 2133129}{553955724533497 83502848}\, t^{14} + \frac{886035323107919692670363}{709063327402877228836 4544}\, t^{15} + \frac{54931691647352511741987219}{46755206073595783331761 3568}\, t^{16} + \frac{8202282228571703962371574 45}{7398626016041530549201797 12}\, t^{17} + \frac{306854463222459126482688014641}{292393700153961287304455022 1824}\, t^{18} + \frac{1195123256995062146490000 4530361}{1199563898067533486377251 3730560}\, t^{19} + \frac{7644107868343839518295644 96337869}{8061069395013825028455129 226936320}\, t^{20} + \dots)$$

Arr. No. 70: Picard–Fuchs operator

$$16\,\theta\,(\theta-2)\,(\theta-1)^2-2\,t\theta\,(\theta-1)\left(24\,\theta^2-24\,\theta+13\right)+t^2\theta^2\left(52\,\theta^2+25\right)$$
$$-2\,t^3\left(3\,\theta^2+3\,\theta+2\right)(2\,\theta+1)^2+t^4\,(2\,\theta+1)\,(\theta+1)^2\,(2\,\theta+3)$$

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It has *no point of maximal unipotent monodromy*!
The first examples of families Calabi-Yau manifolds without MUM-point were described by J. Rohde and studied further by A. Garbagnati and B. van Geemen. It should be pointed out that in those cases the associated Picard-Fuchs operator was of second order, contrary to the above fourth order operator.

Arrangement No. 254

The equation

$$w^2 = xyz(x+y+z-t)(1+t-t^2x+tz-5tx+z-2y-4x)\times$$
$$\times(1-z+tx-2x)(1-tx+z)(1-3z+t-t^2x+tz-tx-2y)$$

Period function expansion

$$\pi^2 t(1 + 1/2\,t + \frac{37}{24}\,t^2 + \frac{41}{16}\,t^3 + \frac{13477}{1920}\,t^4 + \frac{14597}{768}\,t^5 + \frac{2075481}{35840}\,t^6 + \frac{5636567}{30720}\,t^7 + \frac{893398711}{1474560}\,t^8 +$$
$$\frac{4716401057}{2293760}\,t^9 + \frac{589476222067}{82575360}\,t^{10} + \frac{4167958565669}{165150720}\,t^{11} + \frac{5704625497323833}{62977474560}\,t^{12} +$$
$$\frac{151925391248597}{461373440}\,t^{13} + \frac{365832470577260891}{302291877888}\,t^{14} + \frac{4524231452313355151}{1007639592960}\,t^{15} +$$
$$\frac{27621035540417445960079}{1644467815710720}\,t^{16} + \frac{12245144172376534851791}{193466801848320}\,t^{17} + \frac{6667183616265713579789083}{27773234220892160}\,t^{18} +$$
$$\frac{24095673115566438209932751}{26311485051371520}\,t^{19} + \frac{14045196683497951603695570139}{3999345727808471040}\,t^{20} +$$
$$\frac{2775173016132463606951929553}{205094652708126720}\,t^{21} + \frac{12843502522939166762719901631967}{245293204638919557120}\,t^{22} +$$
$$\frac{13015849757510516084180565576689}{63989531644935536640}\,t^{23} + \frac{1353220534793078387748925126232239}{1706387510531614310400}\,t^{24} +$$
$$\frac{146097218869365109917818799125118583}{47096295290672554967040}\,t^{25} + \frac{1810216509017249913008015659593304169}{148725143023176489369600}\,t^{26} +$$
$$\frac{10024664203687239476444697147866644679}{209316867958544688742400}\,t^{27} +$$
$$\frac{19056304743608321498688012784533187444097}{100858527760947994637107200}\,t^{28} +$$
$$\frac{67567359797197607793832710351319623963917}{90424886958091305536716800}\,t^{29} +$$
$$\frac{13957968678274871488244522723737286415005099}{4712578166685454126232371200}\,t^{30} +$$
$$\frac{3290947423891634237542898440747295439160537}{279714316990362438460243968}\,t^{31} +$$
$$\frac{20516955224186198115500109575917307927642307841}{438120951243851903609308446720}\,t^{32} +$$
$$\frac{1036347299803173376615524311049830613193375010129}{5549532049088790779051240325120}\,t^{33} +$$
$$\frac{1737080902805317220929804544746994058532851716791}{2328474985631660466734786150400}\,t^{34} + \ldots)$$

Arr. No. 254: Picard–Fuchs operator

The operator is very complicated and has the following Riemann symbol:

$$\left\{\begin{array}{ccccccccc} \alpha_1 & \alpha_2 & 0 & \rho_1 & \rho_2 & \rho_3 & -1 & 1 & \infty \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3/2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 3/2 \\ 1 & 1 & 1 & 3 & 3 & 3 & 0 & 0 & 3/2 \\ 2 & 2 & 2 & 4 & 4 & 4 & 0 & 0 & 3/2 \end{array}\right\}$$

where at 0 and $\alpha_{1,2} = -2 \pm \sqrt{5}$: conifold points,

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where at 0 and $\alpha_{1,2} = -2 \pm \sqrt{5}$: conifold points,
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where at 0 and $\alpha_{1,2} = -2 \pm \sqrt{5}$: conifold points,
at the $\rho_{1,2,3}$, roots of the cubic equation $2t^3 - t^2 - 3t + 4 = 0$: apparent singularities (singularities of the operator, not of the solutions/family)
at $-1, 1, \infty$: MUM points.

Operator No. 500

Consider fiber product of rational elliptic surfaces with matching of fiber described in the table

E_1	I_8	I_1	I_1	I_1	I_1	$-$
E_2	I_6	I_3	I_2	$-$	$-$	I_1

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In fact there are three possibilities, two correspondes to the surface with fibers I_8, I_1, I_1, I_1, I_1 admitting 2–isogeny (to make it more complicated one case has a MUM point, the other not). Here we are interested in the case of the other surface with this fibers.

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In fact there are three possibilities, two correspondes to the surface with fibers I_8, I_1, I_1, I_1, I_1 admitting 2–isogeny (to make it more complicated one case has a MUM point, the other not). Here we are interested in the case of the other surface with this fibers.

The operator:

$$7^4 19^4 31^4 \theta^4 +$$
$$-2\cdot 3\cdot 7^3 19^3 31^3 x(12369 + 948290\theta^2 + 535990\theta + 824600\theta^3 + 2859450\theta^4)$$

$$+2^2 3^2 7^2 19^2 31^2 x^2(4811788380+290877773690\theta^2+180462266950\theta+235960238290\theta^3+397090658910\theta^4)$$

$$-3^3 7\cdot 19\cdot 31 x^3(7088213400460185+3505936400534543 5\theta^2+23755786502915950\theta+26419894748007670\theta^3$$
$$+2848844027798513 9\theta^4)$$

$$+3^4 x^4(16669233206377318387 38 + 690261982508687548727 6\theta^2 + 508332978768525121338 4\theta$$
$$+481106790003602222980 8\theta^3 + 369986673458618390921 7\theta^4)$$

$$-3^6 x^5(228789441024214325103 33 + 805809444024535953050 08\theta^2 + 642108592876303252424 52\theta$$
$$+516977149135597761577 16\theta^3 + 298795595006241190137 32\theta^4)$$

The operator

$$+3^7x^6(2202889010094755347359202\theta^2+1892481190613499528853468\theta+1293501725762844344767340\theta^3$$
$$+5767457700522526227081540\theta^4 + 7269883105826698093391670)$$
$$-3^8x^7(4886266489034846284175044 7\theta^2 + 451392560118280057014943820\theta$$
$$+2607046825174606846517361 8\theta^3+9050437481180008738264233\theta^4+1858971734944421921543003 1)$$
$$+3^9x^8(902588336515869339418933478\theta^2 + 895347229504501318909874088\theta$$
$$+4333783073421920729580961 92\theta^3+1164268496675364223957248 55\theta^4+3936976552450626638642821 71)$$
$$-2\cdot 3^{11}x^9(117518548820843343847795624 8 + 2356939329671016907479676972\theta^2$$
$$+2510152456766418955780795298\theta + 1004806105892486887374384408\theta^3$$
$$+2035479241185899739958814441\theta^4)$$
$$+2^23^{12}x^{10}(158693267688736667484974751 44\theta^2 + 181657711641709168572260332 87\theta$$
$$+588984819200221330605426905 9\theta^3 + 844656419818843478563506775\theta^4$$
$$+904001913259313266219945872 0)$$
$$-3^{13}x^{11}(741596059269587712890172488575\theta^2 + 914807847672640269768371591718\theta$$
$$+232425308924740126592983581366\theta^3 + 199859022683113217781784796 75\theta^4$$
$$+483612218931173921195884630506)$$
$$+3^{15}x^{12}(2521264699802723582500677029 192\theta^2 + 336614940778801826527849686267 2\theta$$
$$+633753919484213408908097856112\theta^3 + 145369852601399999508003963 97\theta^4$$
$$+189134727418846688326135486665 0)$$

The operator (2)

$$+3^{16}x^{13}(-2252407857249907835752002235876 4\theta^2 - 32765908586332134223915248566088\theta$$

$$-4106576803328584575949620643140\theta^3 + 2599832780922224404338683758400\theta^4$$

$$-19600741058443711138837662826473)$$

$$-3^{17}x^{14}(-1762625151764868169176686644830 02\theta^2 - 28224089806357124503103008682067 6\theta$$

$$-17953565538966823768732472544516\theta^3$$

$$-18032384378987977592252270296183 5 + 4547203073217102524579048448806\theta^4)$$

$$+3^{19}x^{15}(-40108528129151201211392875027702 3\theta^2 - 71824320011238778920521813789633 4\theta$$

$$-8139125984353981269589133719914\theta^3 + 1502593351820816269826358904741 9\theta^4$$

$$-4925833674688431059704215821164 53)$$

$$-3^{21}x^{16}(-7876934781756592367773764848025 98\theta^2 - 16190003054741094853371916535677 52\theta$$

$$-12010742628433691697744042580763 43 + 1048005436051051643275252406997 76\theta^3$$

$$+3660519281449004908690210731059 4\theta^4)$$

$$+2\cdot 3^{23}x^{17}(-6523774586887253856854727919564 33\theta^2 - 16115568460973588002865334795069 05\theta$$

$$-13082755521379196056322358663478 62 + 2219512390508263054142816203624 48\theta^3$$

$$+3357181160702105061363956637275 4\theta^4)$$

$$-2\cdot 3^{25}x^{18}(-8649038605625888153231193237402 00\theta^2 - 28178661438527904716917733700071 24\theta$$

$$-25469076110559872129459844208867 33 + 6294504574735696001857761230480 24\theta^3$$

$$+4006206956394870890220217568287 8\theta^4)$$

The operator (3)

$$-3^{27}x^{19}(886480038051104405732843084319563\,3 + 15714829920761441144203017676318876\theta^2$$

$$+858699193548612783983864798091165 6\theta - 28309280811877132357119022920046664\theta^3$$

$$+6336728164512179621222958378060\theta^4)$$

$$+2\cdot 3^{29}x^{20}(6915040270710610189230809080736771 + 11373870574569408091614087125063 6\theta^2$$

$$+56615739126218678425681269811402292\theta - 26342172295695813753650106911059952\theta^3$$

$$+1832692293337563602778882317173 20\theta^4)$$

$$-2^2 3^{31}x^{21}(4888337794108661218150256149106629 - 5896205243919325229311650370274443\theta^2$$

$$+3268371403300315225872581136619579\theta - 2028274974450924631561054453857272\theta^3$$

$$+322086219382312676091893891620092\theta^4)$$

$$+2^2 3^{33}x^{22}(-1110824540573595549669458712652674\theta^2+3635613036534328688914086430194332\theta$$

$$-2468423982346106154901624398721648\theta^3 + 786281269788490413359473764346392\theta^4$$

$$+6459005897430192114633490759862673)$$

$$-2^4 3^{35}x^{23}(-655805530419664652116987378896 11\theta^2 + 1270057367020607251211055169563163\theta$$

$$-4870413845716520544246849368721 98\theta^3 + 3932255937186582871379398361389 97\theta^4$$

$$+21296644421009722574486900310921 03)$$

$$+2^3 3^{37}x^{24}(20693456405481461326191735260813 90\theta^2 + 511922459355354061666542159443694 8\theta$$

$$+2184561964203388845185350002929 92\theta^3 + 136127051882267182811448882182164 4\theta^4$$

$$+611739020865131476932541240707 8111)$$

The operator (4)

$$-2^4 3^{39} x^{25}(4937489097320581807566576545061121 + 369227437726081053060775886332200 0\theta^2$$
$$+577007479680876804588343174498038 2\theta + 137163234025852226100183500204086 4\theta^3$$
$$+104534815119606937861749302775759 6\theta^4)$$
$$+2^4 3^{41} x^{26}(840871786374382512459290702764063 4\theta^2 + 118920179946352509604433101865621 56\theta$$
$$+335639010854501672622990838764462 4\theta^3 + 144414610794636343125874167184226 0\theta^4$$
$$+84118448894454502485865638859149 65)$$
$$-2^4 3^{43} x^{27}(151311751836219089333136120597651 64\theta^2 + 213112480121416507363548282412214 60\theta$$
$$+587551752871484152271621315721562 4\theta^3 + 180893896718354023273231394783297 2\theta^4$$
$$+13894609676879798849776717677383057)$$
$$+2^6 3^{45} x^{28}(574559345271115547083065797714735 7\theta^2 + 830290221843849753133223123580578 8\theta$$
$$+212079274586098825449827158915188 8\theta^3 + 516008810259505706182452243432468\theta^4$$
$$+52840856839955334693602768802925 53)$$
$$-2^6 3^{47} x^{29}(758468083417968379356570227219491 1\theta^2 + 113514939086408146847905453446101 03\theta$$
$$+264702454005298719596518915420631 2\theta^3 + 537638382335309145020522182243132\theta^4$$
$$+72453796897979785048927186142602 56)$$
$$+2^7 3^{49} x^{30}(441665401362782624842373194222197 8\theta^2 + 686081414349133841117116360808561 6\theta$$
$$+145778503657613683710710080943404 0\theta^3 + 255966557717869256869328092628764\theta^4$$
$$+44462624222362278366934509505036 83)$$

The operator (5)

$$
\begin{gathered}
-2^{8}3^{52}x^{31}(7623677390691789343433430267882710\theta^{2}+12285404044211891047125282028032600\theta \\
+81269023047939440994431261474909 5+2385350184262353069501975243182340\theta^{3} \\
+371039564968528737601403316816930\theta^{4}) \\
+2^{9}3^{55}x^{32}(189520189452461807982118953681 61+1678228517180367744346591878004 20\theta^{2} \\
+280124278197910073618892961154980\theta+4992894053838582782538395395777 60\theta^{3} \\
+7006468914229894890993434547790\theta^{4}) \\
-2^{9}3^{58}x^{33}(38649199990566729362293446357173+3235689099942487567993004871343 90\theta^{2} \\
+5583661914333525434174921093348 90\theta+9183216593608902341326083682448 0\theta^{3} \\
+11791065228055685506304134659660\theta^{4}) \\
+2^{10}3^{60}x^{34}(15043776981734011582307619067599+11936205876394259855750736802267 0\theta^{2} \\
+21252791859162824572455391643360\theta+32420575315917635568446579102920\theta^{3} \\
+38520961579282201487258728820 90\theta^{4}) \\
-2^{8}3^{62}x^{35}(41597911782330322795778551064505+31366316012198451410032859468084 0\theta^{2} \\
+57513938769686120994722586807344\theta+81787198360043760438404257550800\theta^{3} \\
+9076005821551415161987450610680\theta^{4}) \\
+2^{9}3^{64}x^{36}(127103229315809844791316865841 49+91340742664640344891809940400000\theta^{2} \\
+17216126002956627415257401506844\theta+22930688868638950843920080639680\theta^{3} \\
+23949808861562619047392739092 0\theta^{4})
\end{gathered}
$$

The operator (6)

$$-2^{10}3^{67}x^{37}(781207400195805669338257543931\theta^2 + 15109355597875874043238849724473\theta$$
$$+18933427701260883700567759452 0\theta^3 + 18733218074474616769641246996\theta^4$$
$$+113744335604260772840564682243 7)$$
$$+2^{10}3^{70}x^{38}(1167896772232807028764005708850\theta^2 + 2314122378220094623303353247724\theta$$
$$+2739522215736304674043106305 6\theta^3 + 2582080789707259688566846448 8\theta^4$$
$$+1774480989335616484803807807 83)$$
$$-2^{12}3^{73}x^{39}(833487076320411314054818506\theta^2 + 1725538986110401836219276140\theta$$
$$+1844092465447749061847202256\theta^3 + 1599226668805967460965204 4\theta^4$$
$$+1368558959871613702634342645)$$
$$+2^{11}3^{76}x^{40}(833487076320411314054818506\theta^2 + 1725538986110401836219276140\theta$$
$$+1844092465447749061847202256\theta^3 + 1599226668805967460965204 4\theta^4$$
$$+1368558959871613702634342645)$$
$$-2^{12}3^{79}x^{41}(386104137622055638831513 16\theta^2 + 815314147559102358762027 94\theta$$
$$+832304800657123068132419 2\theta^3 + 69642667291538512019941 2\theta^4$$
$$+65664272701049831836088309)$$
$$+2^{12}3^{82}x^{42}(2938286001939816703619430\theta^2 + 6319679623505894502290660\theta$$
$$+6183806426845941646451 36\theta^3 + 50097096544749271926676\theta^4$$
$$+51631288561380047218622 23)$$

The operator (7)

$$-2^{12}3^{85}x^{43}(17858308967551896493843 6\theta^2 + 3906344753536776522010 12\theta$$

$$+3677052936909503050712 8\theta^3 + 289371857789414694920 4\theta^4$$

$$+3233794682887700700726 75)$$

$$+2^{14}3^{89}x^{44}(696233800511834479005\theta^2+1546225920586853622280\theta+140577918974065655216\theta^3$$

$$+107831269857580538 64\theta^4 + 1295296520603062067994)$$

$$-2^{14}3^{91}x^{45}(716637538073170336673\theta^2 + 1612514186241845216 53\theta + 1422808365348625 3256\theta^3$$

$$+106775077590942264 4\theta^4 + 136473985594950642858)$$

$$+2^{17}3^{98}x^{46}(5075270918978910+2615318616990976\theta^2+5947358126269313\theta+512217554716360\theta^3$$

$$+377666229465592\theta^4)$$

$$-2^{18}3^{106}x^{47}(812684982237\theta^2+1863178224226\theta+157504145520\theta^3+11455915528\theta^4+160007328132)$$

$$+2^{20}3^{112}x^{48}(\theta+3)(89296\theta^3 + 972536\theta^2 + 3533657\theta + 4280248)$$

$$-2^{20}3^{119}x^{49}(\theta+4)(\theta+3)(2\theta+7)^2$$

Riemann Symbol

$$\left\{\begin{array}{r|cccc} 0 & 0 & 1 & 2 & 3 \\ -485/112045044870668655830016 & 0 & 0 & 0 & 0 \\ -8003/1792720717930698493280256 & 0 & 1/2 & 1/2 & 1 \\ -4123/896360358965349246640128 & 0 & 0 & 0 & 0 \\ \deg 2 & 0 & 2 & 3 & 5 \\ \deg 2 & 0 & 1 & 1 & 2 \\ \deg 2 & 0 & 0 & 1 & 1 \\ \deg 2 & 0 & 2 & 3 & 5 \\ \deg 6 & 0 & 1 & 3 & 4 \\ \deg 10 & 0 & 1 & 1 & 2 \\ \infty & 3 & 7/2 & 7/2 & 4 \end{array}\right\}$$

$t^2 + 3961/448180179482674623320064t +$
$31385603/1606923786248979515117300060960621361393479712768,$
$t^2 + 8003/896360358965349246640128t +$
$16016923/803461893124489757558650030480310680696739856384$
$t^2 + 8003/896360358965349246640128t +$
$32053529/1606923786248979515117300060960621361393479712768$
$t^2 + 2021/224090089741337311660032t +$
$32682089/1606923786248979515117300060960621361393479712768$

Riemann Symbol (2)

$t^6 + 8003/298786786321783082213376t^5 +$

$640434163/2142565048331972686823066747947495148524639617024t^4 +$

$5124659498927/2880765563744186843537527894536004723716407549075502863613941509005508608t^3+$
$20501911523330909/3442938739750341341599283860082387732685893996171781723961198670451200\backslash$
$916966662345141317042962432t^2 +$
$512592770441645123/48220528197786298659753720395648040442982892474584139284012961625605250\backslash$
$572410456269732077309626636447062661462558244864t +$
$8201613496192790062043/1037351279156795329019371621444598195797802667331924201527944351393\backslash$
$262543380811684913759909974201664702026539922150015728781301902006161095262208,$

$t^{10} + 40015/896360358965349246640128t^9 +$

$40030279/44636771840249430975480557248906148927596658688t^8 +$

$5125846095143/480127593957364473922921315756000787286067924845917143935656918167584768t^7 +$
$35894670420487721/43036734246879266769991048251029846658573674952147271549514983380640011\backslash$
$4620832793142664630370304t^6 +$
$114906320754209306725/2571761503881935928520198421101228823625754265311154095147357953365\backslash$
$61336386189100105237745651342061051000861133643972608t^5 +$
$574743018942870668017709/3457837597189317763397905404815327319326008891106414005093147837\backslash$
$97754181126937228304586636658067221567342179974050005242927100634002053698420736t^4 +$
$5256654542137255055236347677/123978741994419903345028962906983188064176681450860721247436\backslash$
$0041769260953327919866295008331065710801727470977032175437915428400008722052047984314\ 121\backslash$
$791694072910899576832t^3 +$
$5258422218124573779192644871521/7408641978546042916558640851239518815470108390713064584\backslash$
$26705448914160453254471503225018606970163233260397530171\ 51432153451772844814421208903962\backslash$
$0128980867794194506079528439667357865728660209664t^2 +$
$5260074447836475186988476201822353/7470914606252197088383931984591316806027519550154686\backslash$
$843959757718718646980344876045571120784240826043341157661760851445252343680435927898963\backslash$
$13012926778116830791429620874266113490587681621406904932920771312630540271616t +$
$263079464111906022723475220035269967/8370789622824612759110832356677192166969731115133648\backslash$
$44028816091141000107091943724157223388176636936520770513320182634358882216054324561645\backslash$
$6199194394457640107682448862446249811369833971763072237699511914457029342561077060313958791\backslash$
0565625856

Picard–Fuchs operators of double octics

Operators or order < 4:

In the list there are seven examples with operator of order two, one of them (Arr. No. 13) is birational to the Borcea–Voisin Calabi–Yau threefold.

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30 families, 17 operators

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30 families, 17 operators
Operators with multiple MUM points:
There are five families (four operators) with two MUMs and a single operator with three MUM points.
Operators without MUM points:
Finally there are 20 families without a MUM point (orphants)

Finite monodromy

There are seven one parameter families of double octics with a singular element and finite local monodromy at that point

No.	equation	B/A	rigid
96	$xyzt(x+y)(x+y-z+t)\times$ $\times(Ax-By+Bz+At)(Ay+Bz+At)$	-2	32
100	$xyzt(x+y-z+t)(Ax+Ay+Bz)\times$ $\times(Ay+Bz+At)(By-Bz-At)$	$-\frac{1}{2}$	69
153	$xyzt\,(x+y+z)\,(y+z+t)\times$ $\times\,(Ax-By+At)\,(Ax-By+Az+At)$	-2	93
155	$xyzt(Ax+By+Az)(Ax+(A+B)y-Bz+At)\times$ $\times(Ax-Bz-Bt)(Ax+By+Az+At)$	$\frac{-1\pm\sqrt{-3}}{2}$	93(3)
197	$xyzt(x-y-z+t)(Ax+By+Bz)\times$ $\times(By+Bz+At)(Ax+Bz+At)$	$-\frac{1}{2}$	93
199	$xyzt(x+y+z)(y+z+t)\times$ $\times(Ax+By+(A-B)z)(Ax+By+Az+Bt)$	1	1
200	$xyzt(x+y+z+t)(Ax+Ay-Bz-Bt)\times$ $\times(Ay-Bz+At)(Ax-By-Bt)$	$\frac{-1\pm\sqrt{-3}}{2}$	93(3)

Arr. No. 197

$$u^2 = xyz\,(x+1-y-z)\,(ty+zt+1)\,(zt+x+1)\,(zt+ty+x)$$

Riemann Symbol:

$$\left\{\begin{array}{r|ccccc} 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} & \frac{3}{2} & 2 \\ -1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ \infty & 1 & \frac{3}{2} & \frac{3}{2} & 2 \end{array}\right\}$$

Arr. No. 197

$u^2 = xyz\,(x+1-y-z)\,(ty+zt+1)\,(zt+x+1)\,(zt+ty+x)$

Riemann Symbol:

$$\left\{\begin{array}{c|cccc} 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} & \frac{3}{2} & 2 \\ -1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ \infty & 1 & \frac{3}{2} & \frac{3}{2} & 2 \end{array}\right\}$$

This double octic is birational to the Kummer fibration associated to the fiber product

∞	0	1	$-B/A$
I_2	I_2	$-$	I_2^*
I_2	I_2	I_0^*	I_2

Arr. No. 197

$u^2 = xyz\,(x+1-y-z)\,(ty+zt+1)\,(zt+x+1)\,(zt+ty+x)$

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∞	0	1	$-B/A$
I_2	I_2	$-$	I_2^*
I_2	I_2	I_0^*	I_2

What happens at $(B:A) = (-1:2)$ – **Nothing**