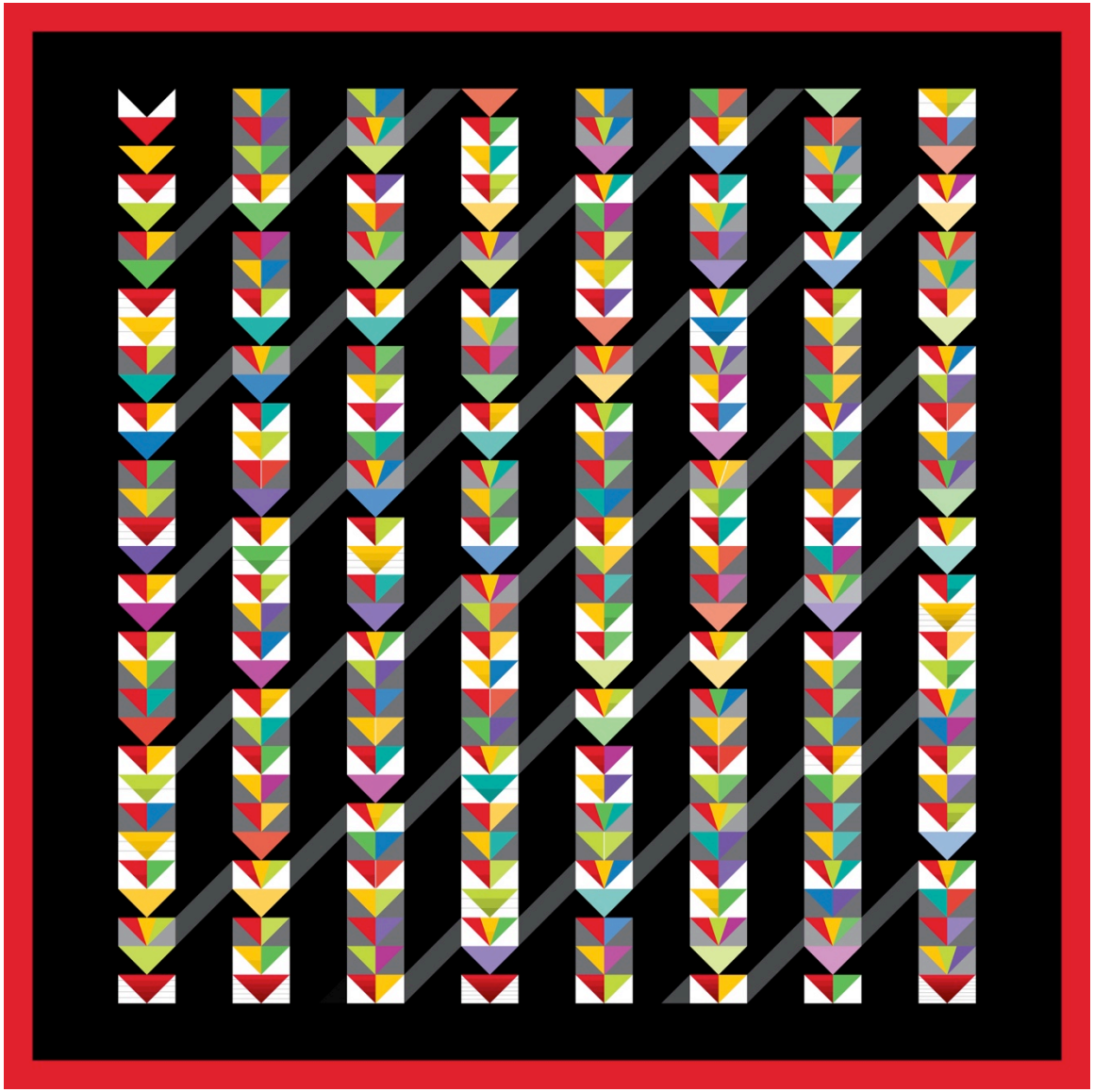
Illustrating Number Theory and Algebra
ICERM Workshop
23 October 2019

Numbers that Inspire

Margaret Kepner
mekvisysuals.yolasite.com
renpek1010@gmail.com

Prime Goose Chase

Bridges 2011



Prime Goose Chase

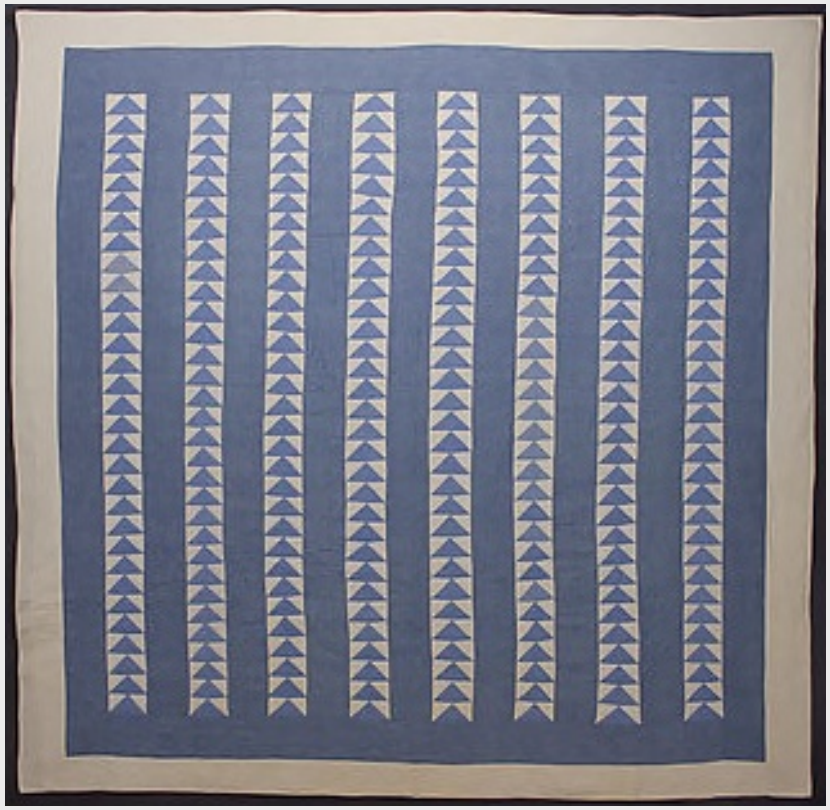
1880 [Stella Ruben Antiques]

1800-1830 [Newark Museum]

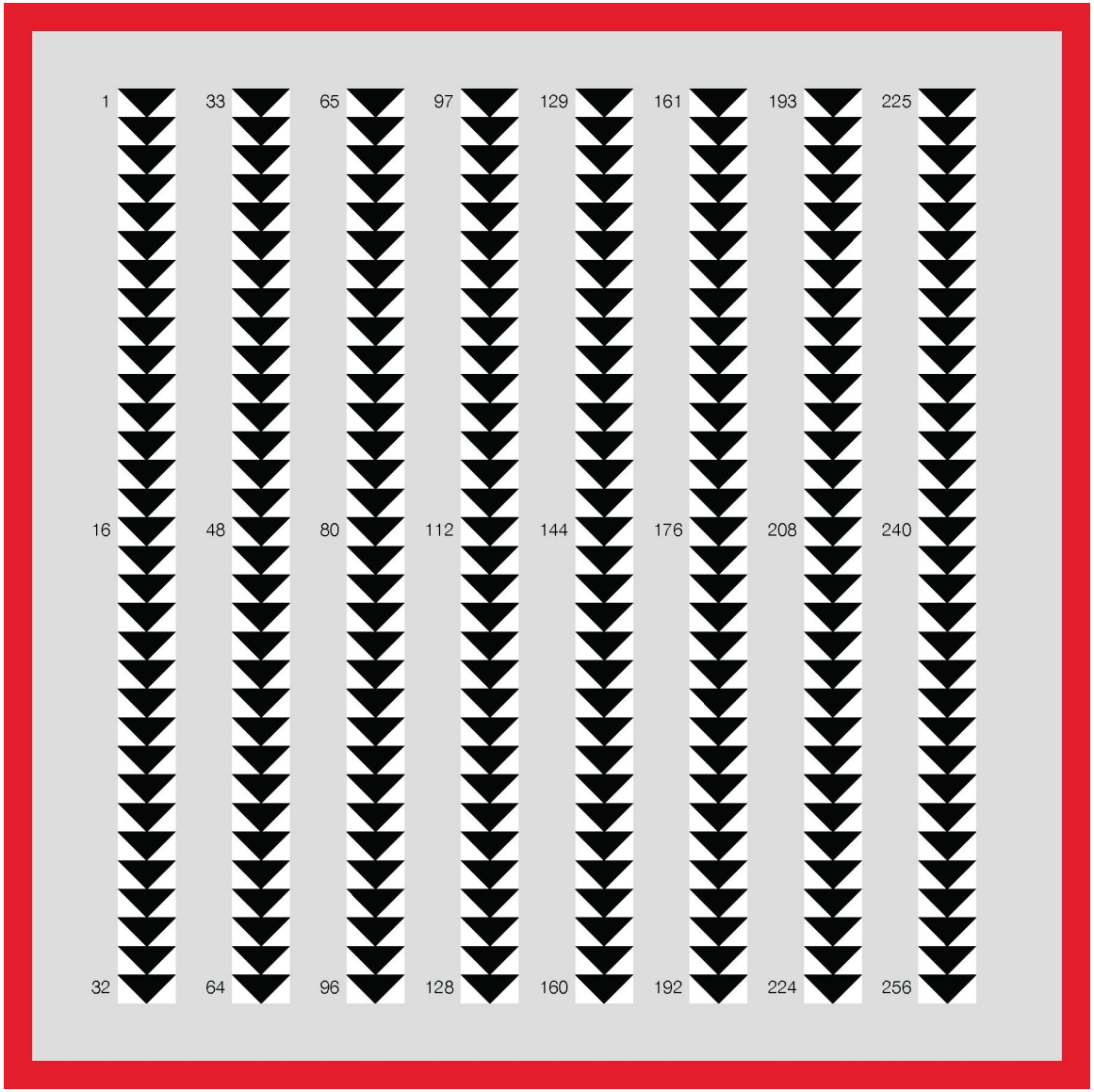
1838 [The Museum of Florida History]

Chika Goto, 2006 [Quilter’ s Connection]

Mary Johnson, 1890 [Rocky Mountain Quilt Museum]



1920 [Stella Rubin Antiques]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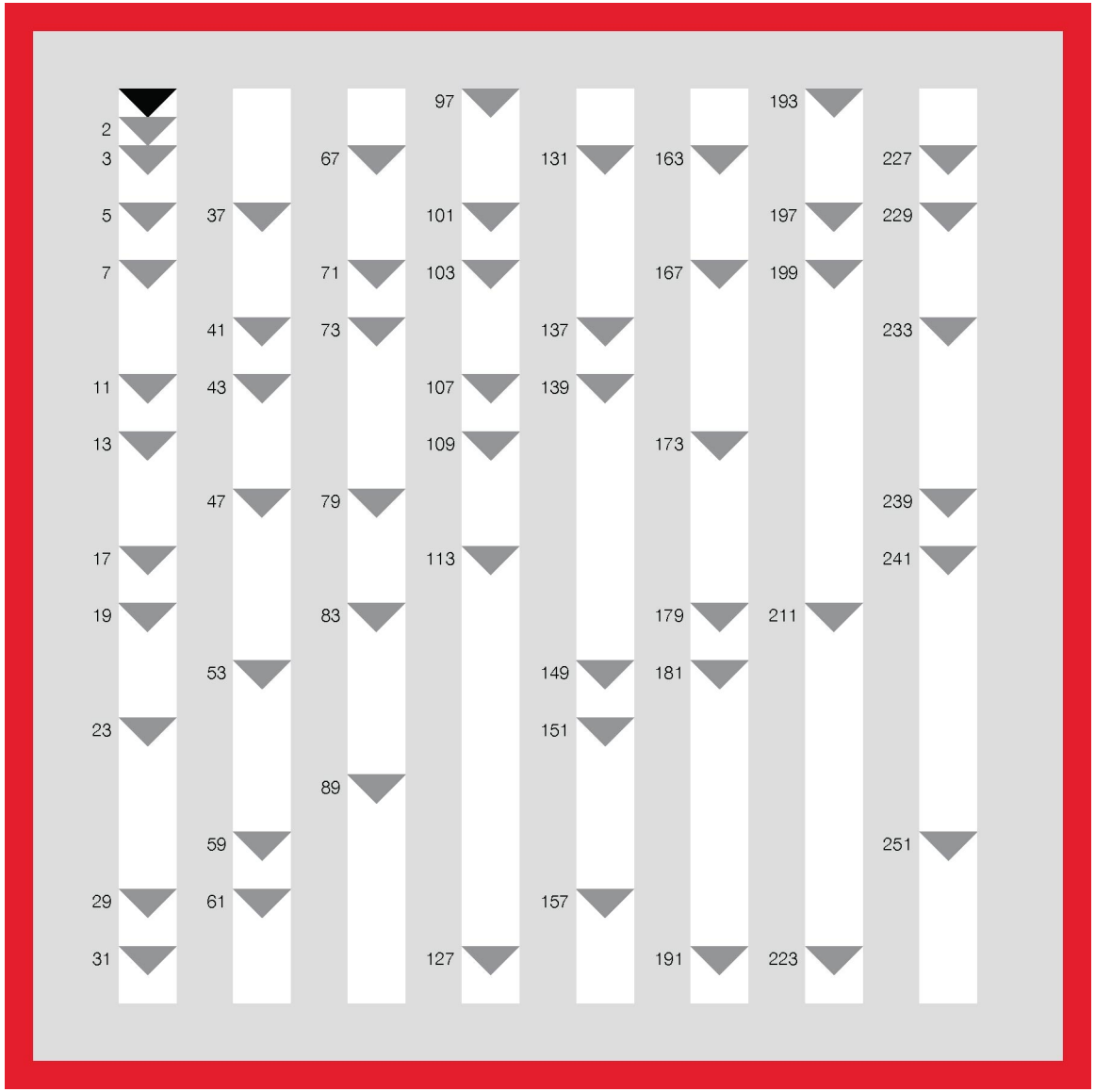
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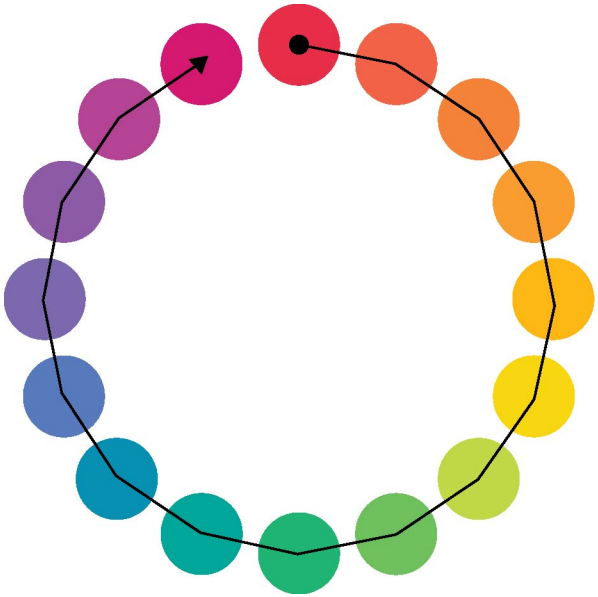

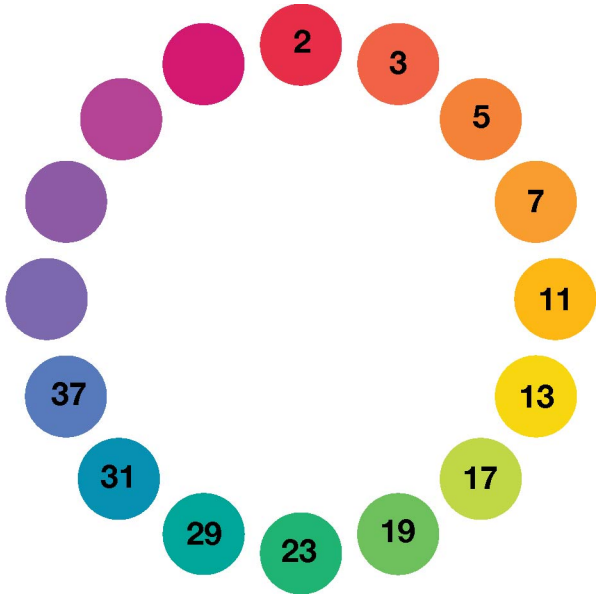
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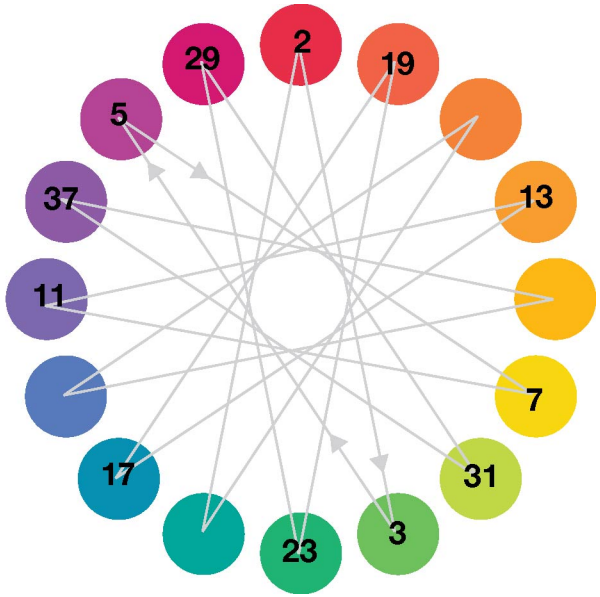
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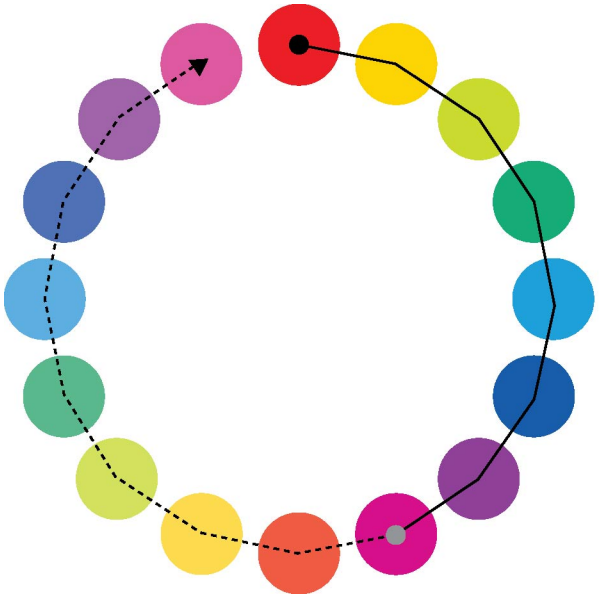

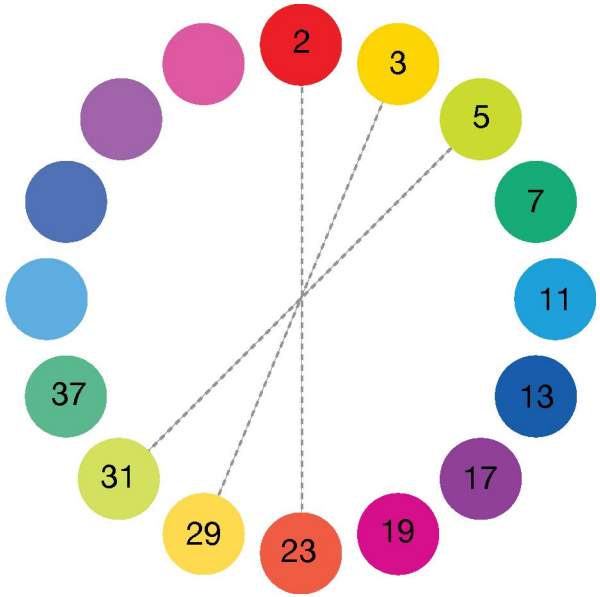
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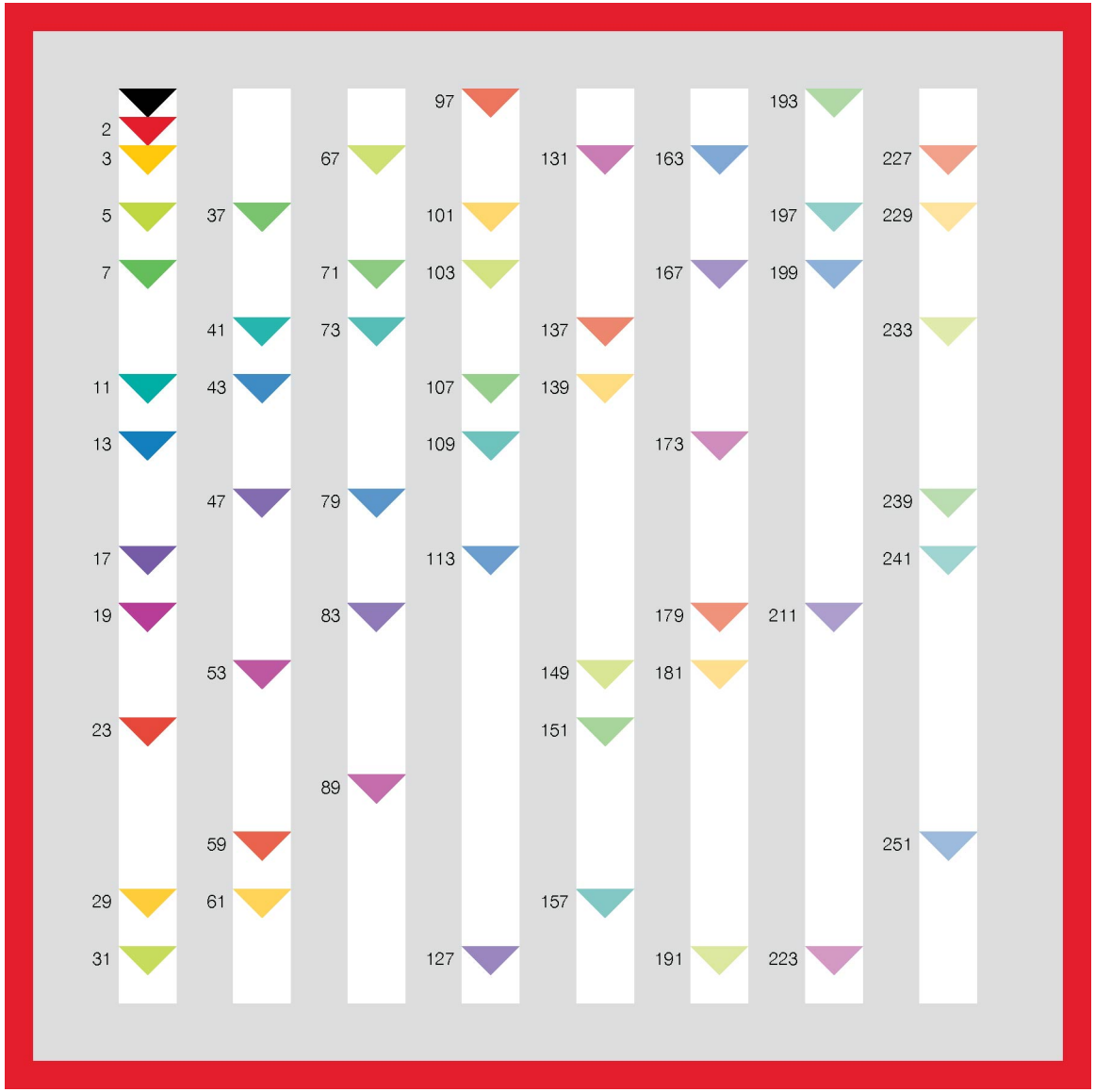
Primes Up to 256

Cycle								Percent White
0	**2**	3	5	7	11	13	17	19	0
1	23	29	31	37	41	43	47	53	12.6
2	59	61	67	71	73	79	83	89	23.6
3	97	101	103	107	109	113	127	131	33.3
4	137	139	149	151	157	163	167	173	41.7
5	179	181	191	193	197	199	211	223	49.1
6	227	229	233	239	241	**251**	*257*	*263*	55.5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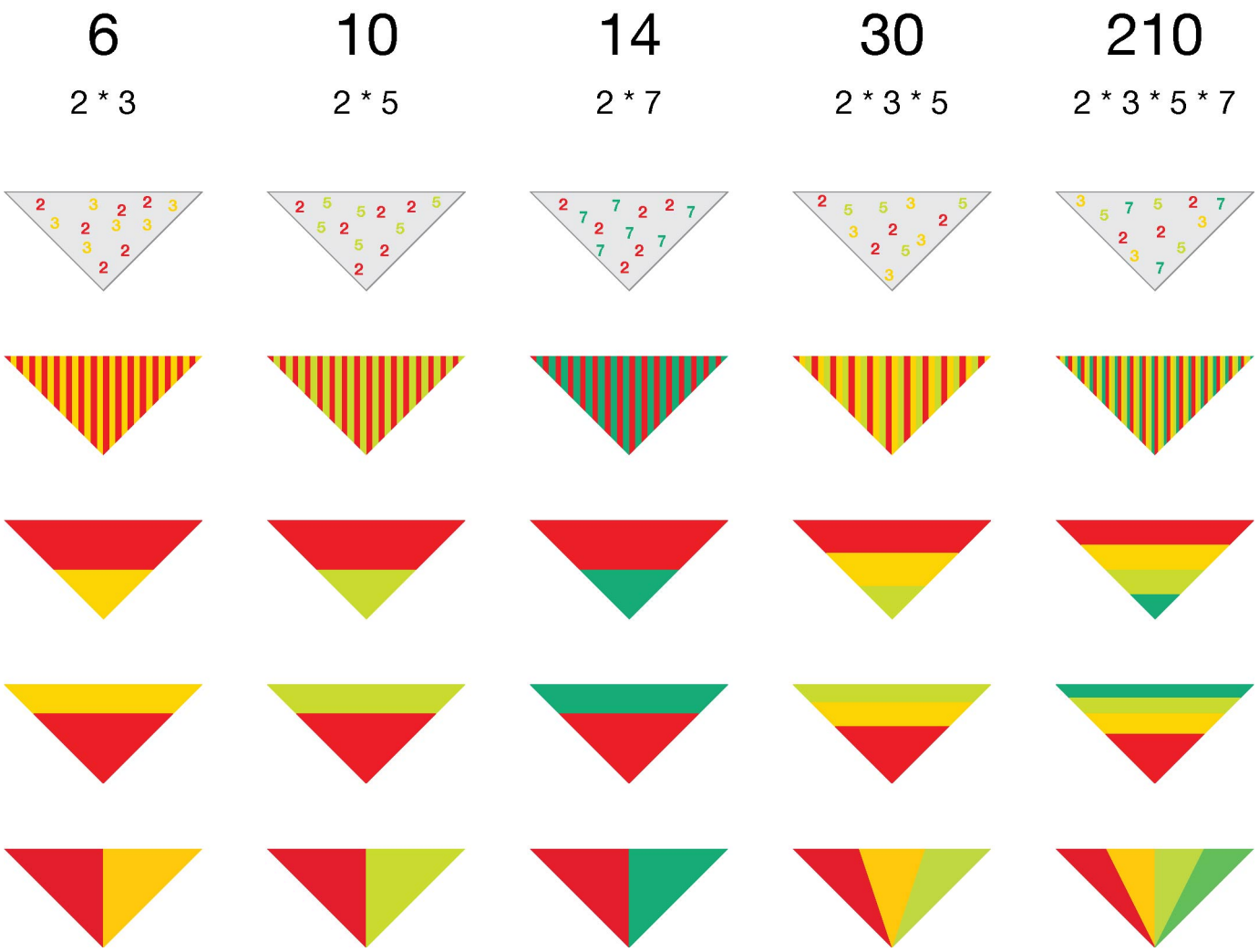
6
2 * 3
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2 * 5
14
2 * 7
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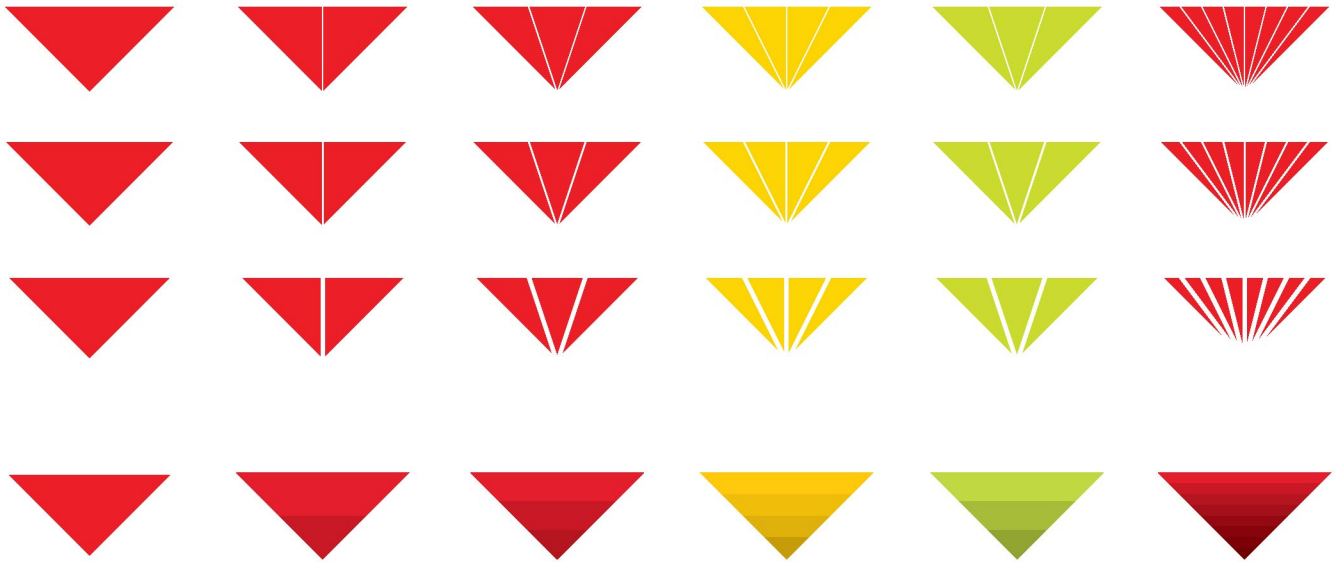
2 * 3 * 5
210
2 * 3 * 5 * 7

2	4	8	81	125	256
	2 * 2	2 * 2 * 2	3 * 3 * 3 * 3	5 * 5 * 5	2 ^ 8



Powers Up to 256

Base\Exp	1	2	3	4	5	6	7	8	Percent Black
2	2	4	8	16	32	64	128	256	0 - 61.1
3	3	9	27	81	243	*729*			0 - 36.6
5	5	25	125	*625*					0 - 28.6
7	7	49	*343*						0 - 16.8
11	11	121	*1331*						0 - 18.5
13	13	169	*2197*						0 - 20.3

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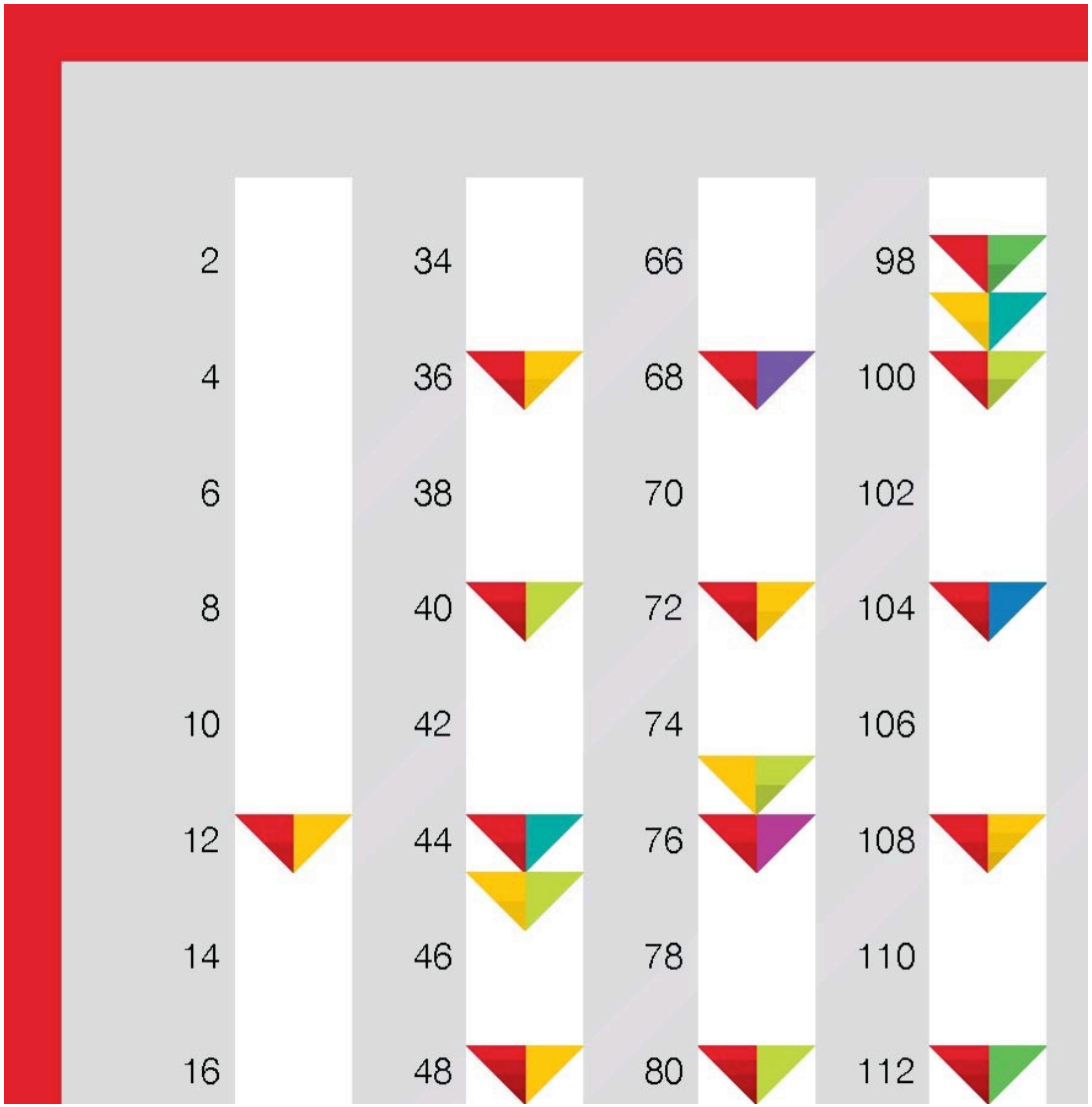
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106
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112

Primes Up to 256

Cycle									Percent White
0	**2**	3	5	7	11	13	17	19	0
1	23	29	31	37	41	43	47	53	12.6
2	59	61	67	71	73	79	83	89	23.6
3	97	101	103	107	109	113	127	131	33.3
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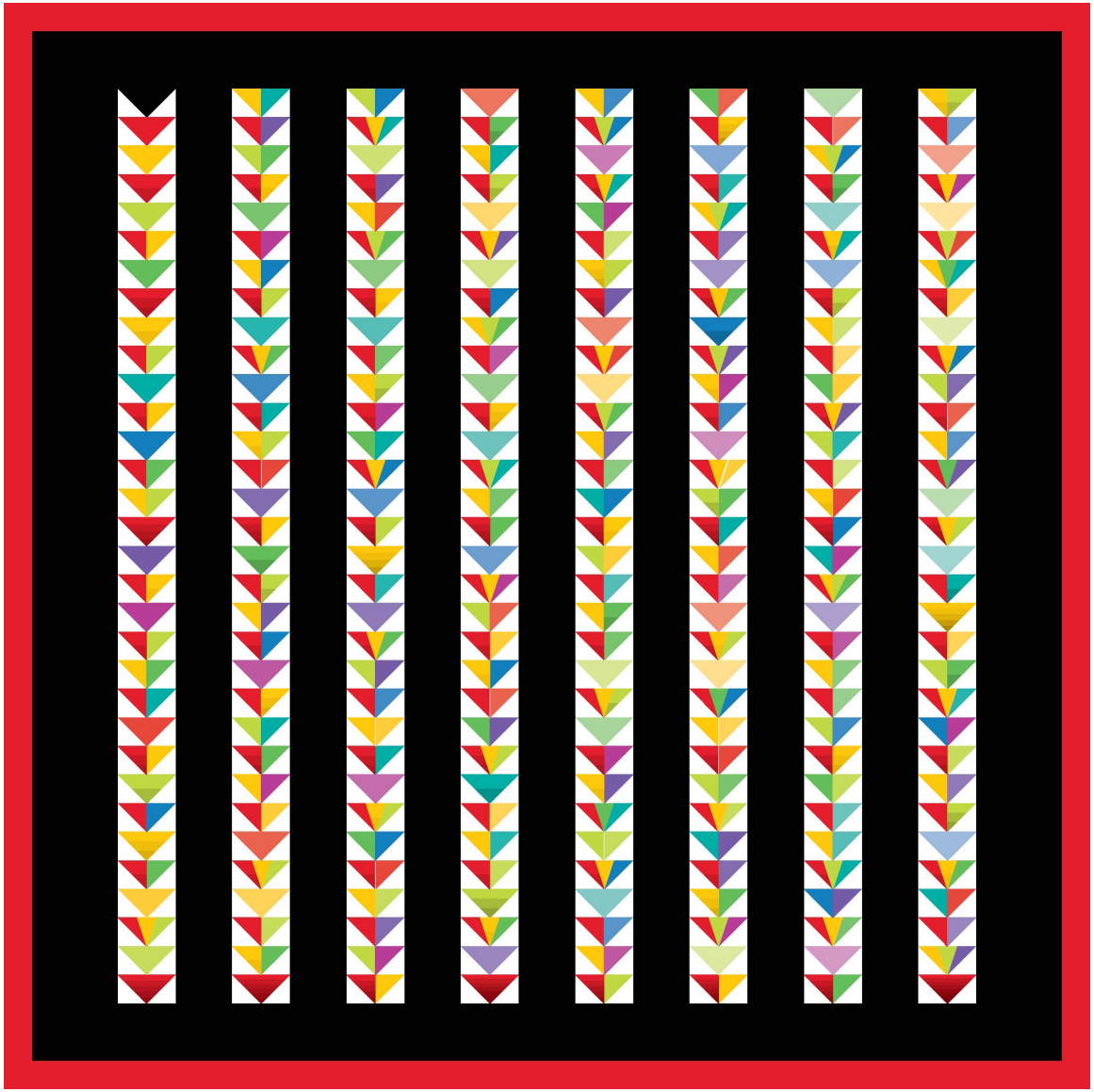
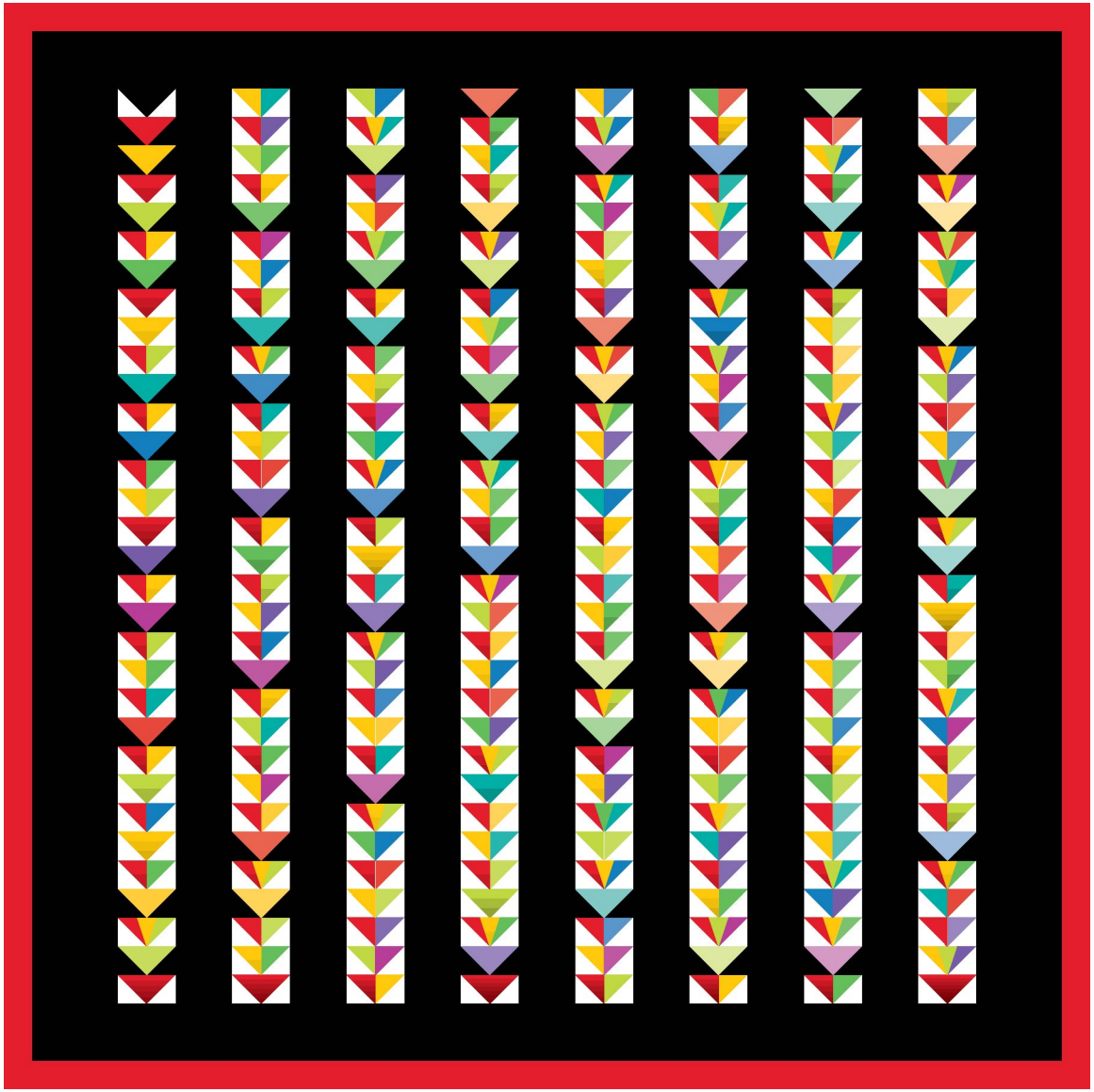
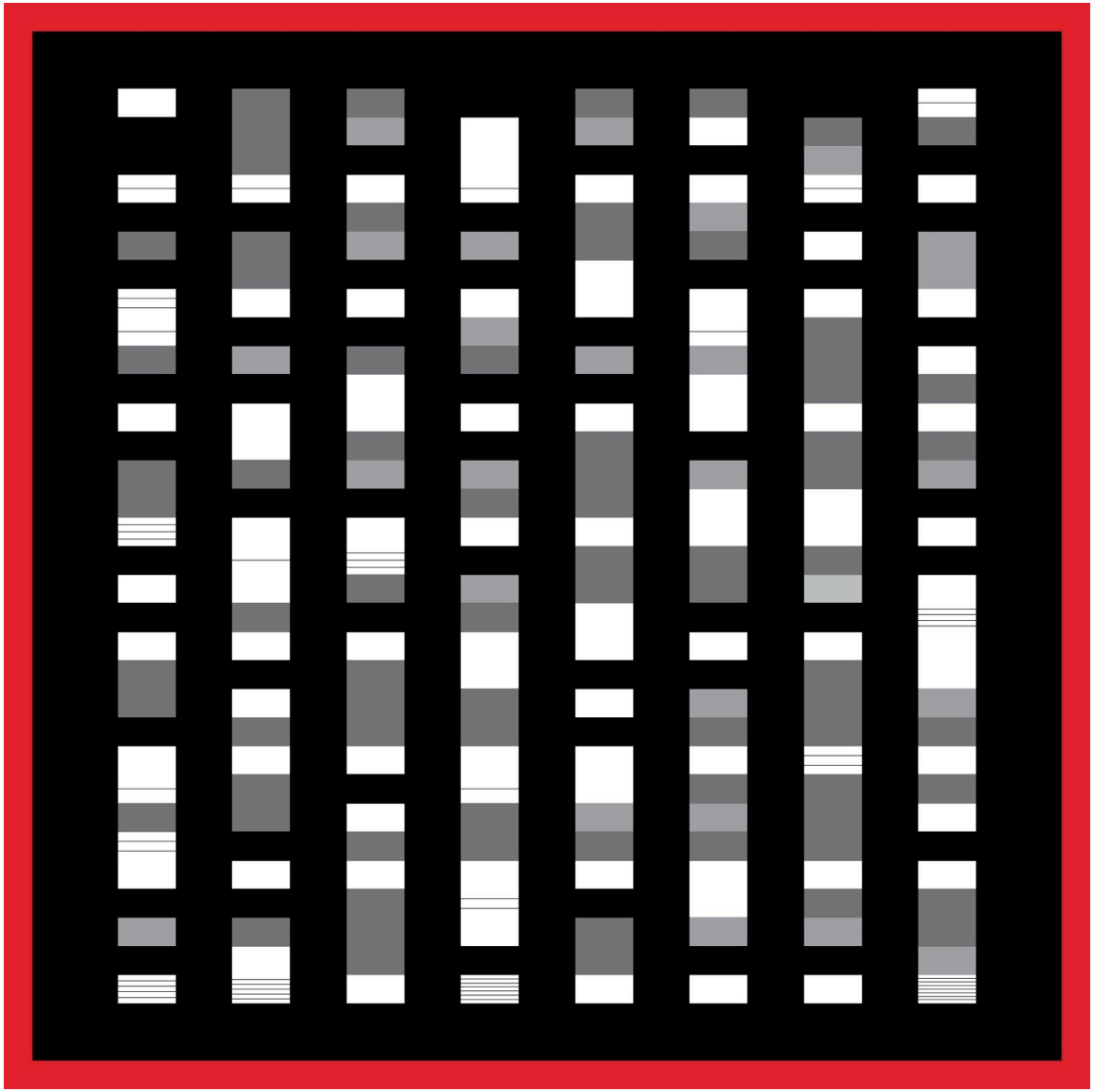
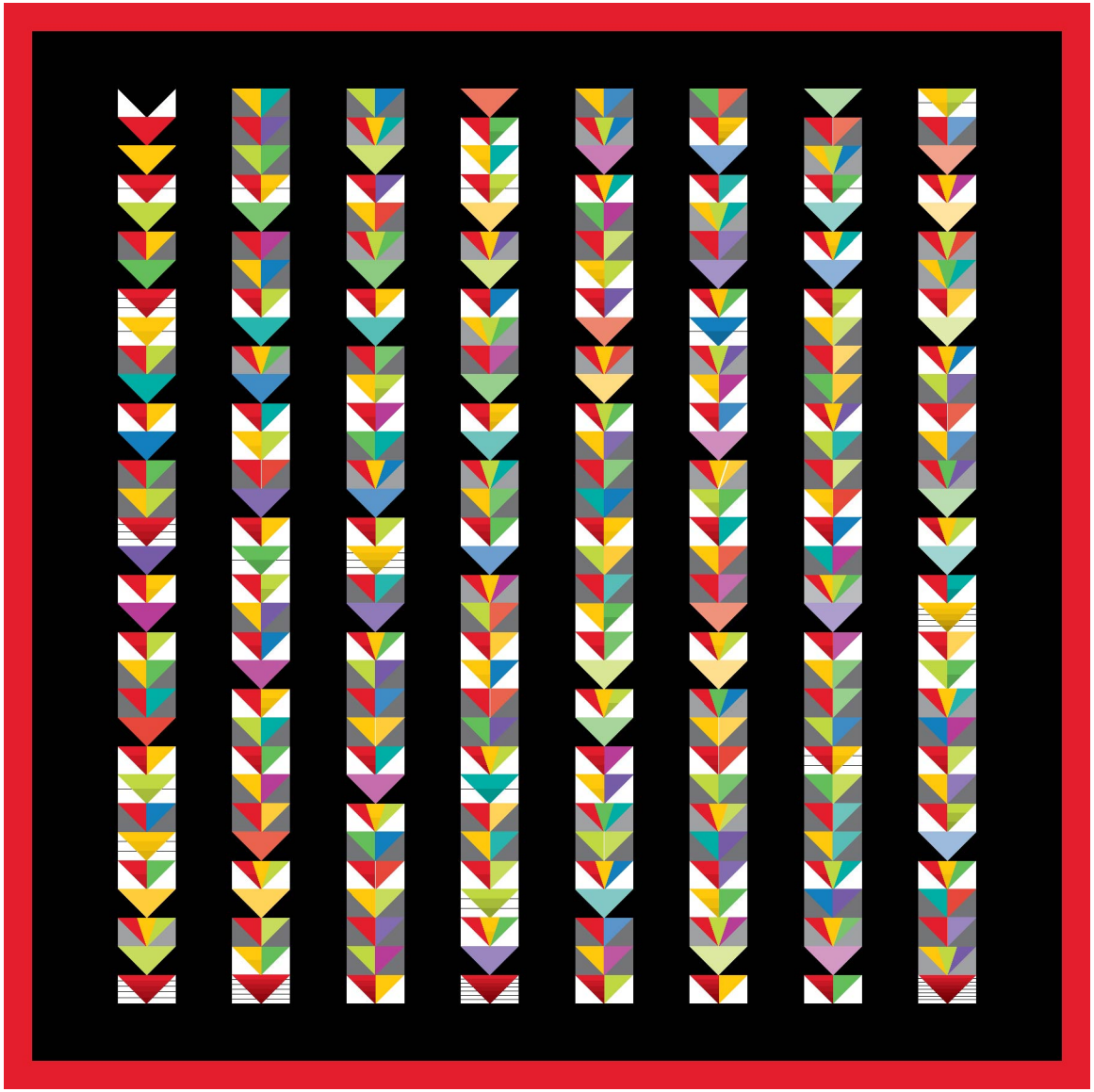
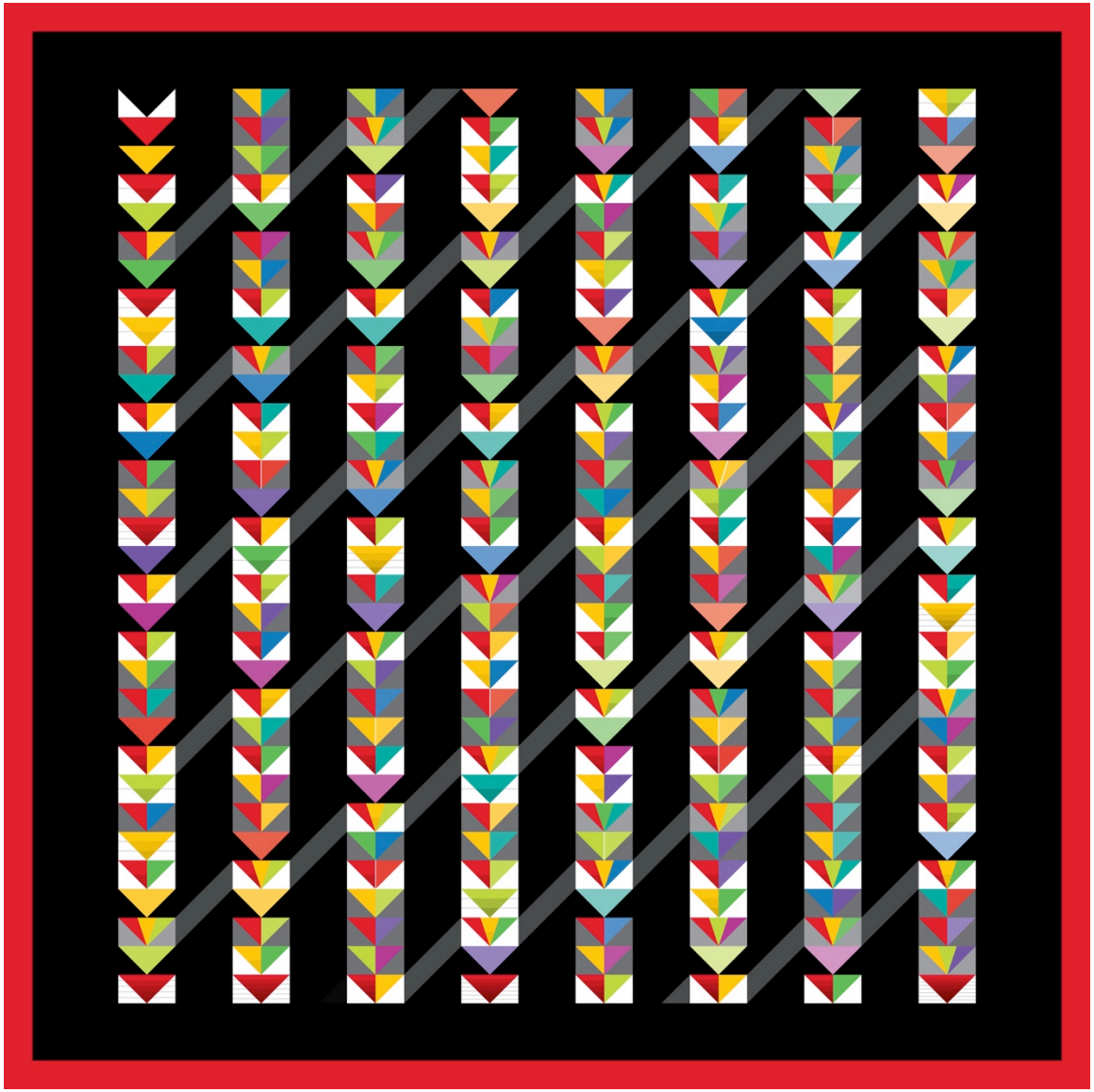
5	5	25	125	*625*					0 - 28.6
7	7	49	*343*						0 - 16.8
11	11	121	*1331*						0 - 18.5
13	13	169	*2197*						0 - 20.3











Prime Goose Chase

Partition Studies

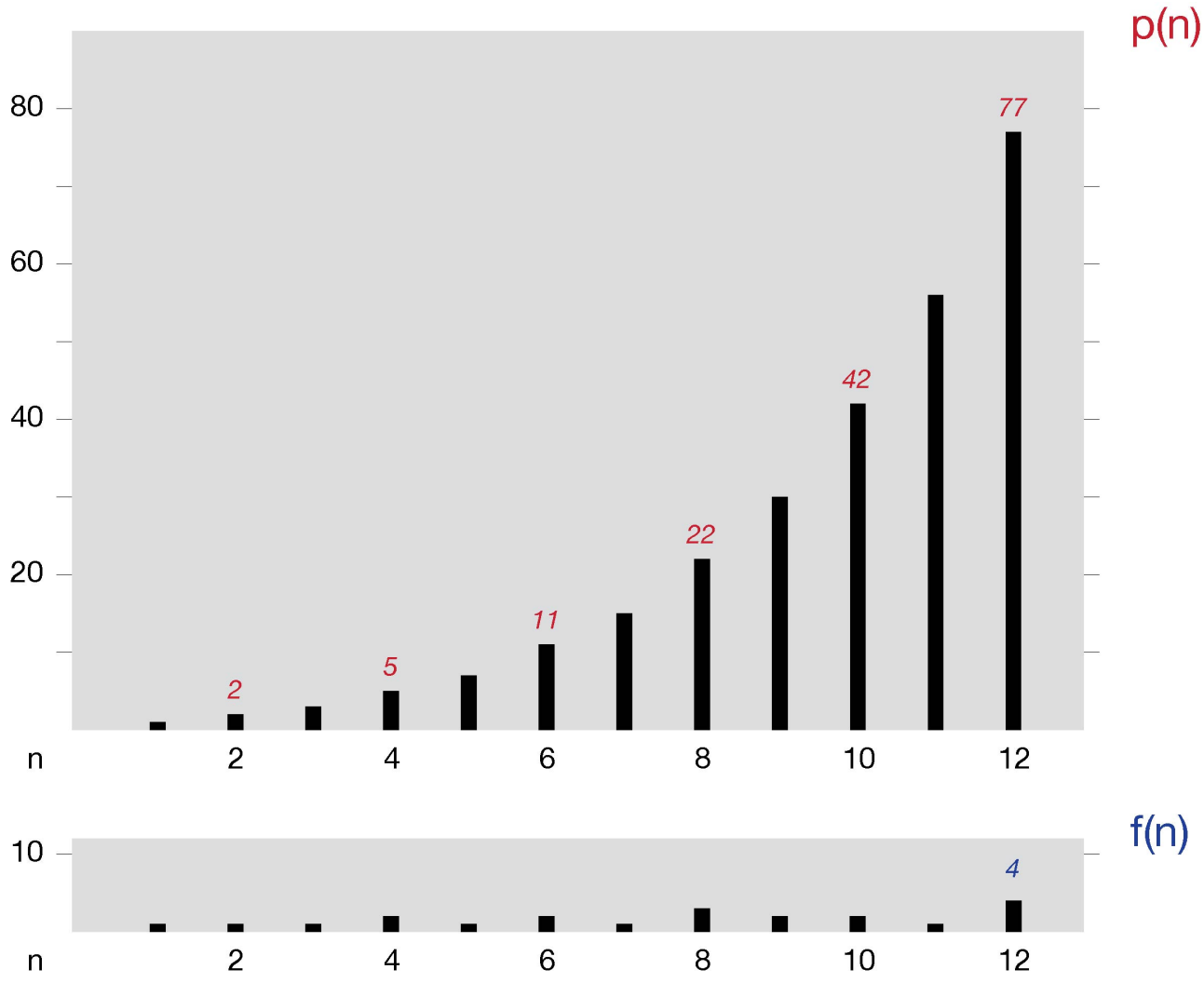
JMM 2015

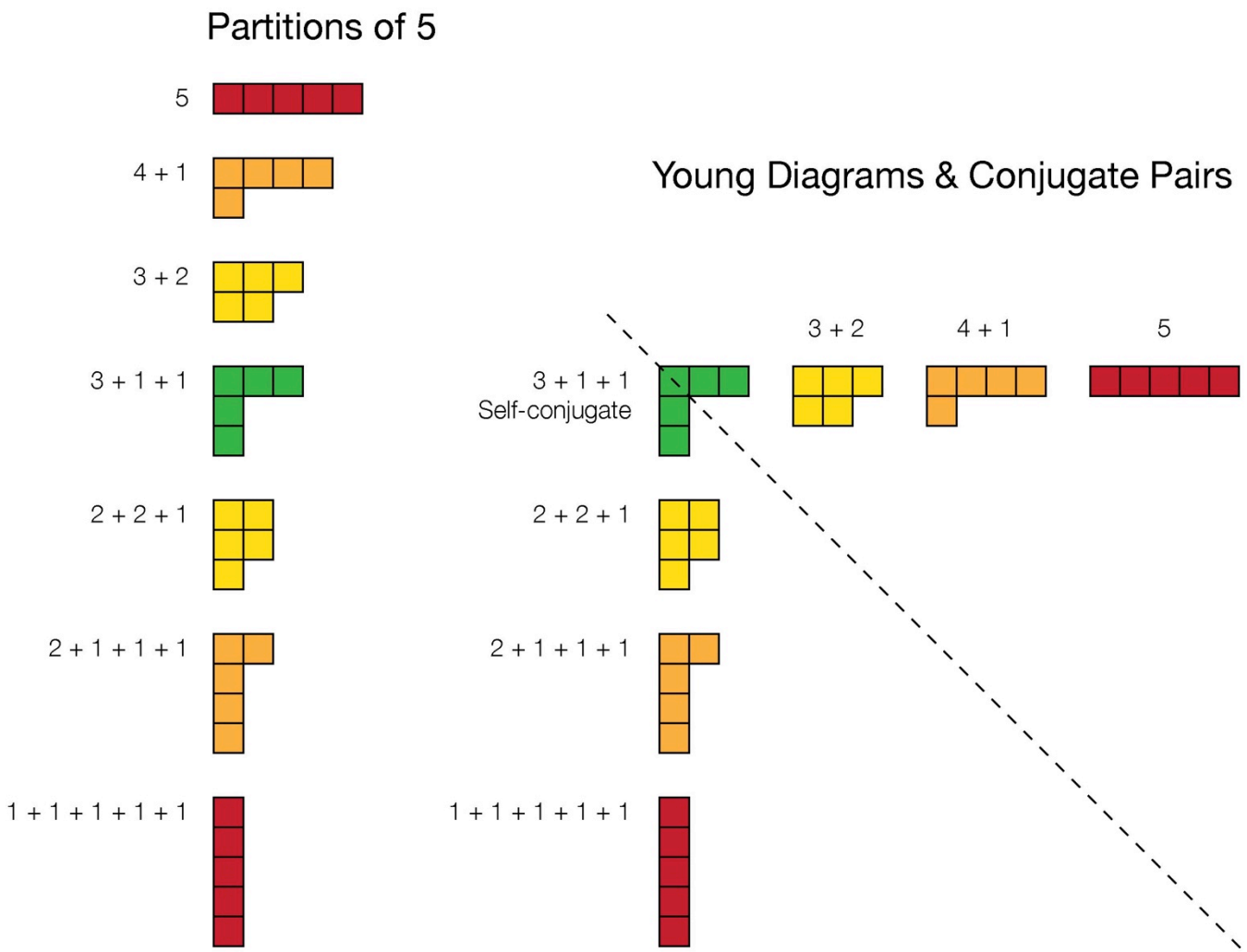
Examples of
Additive Partitions:

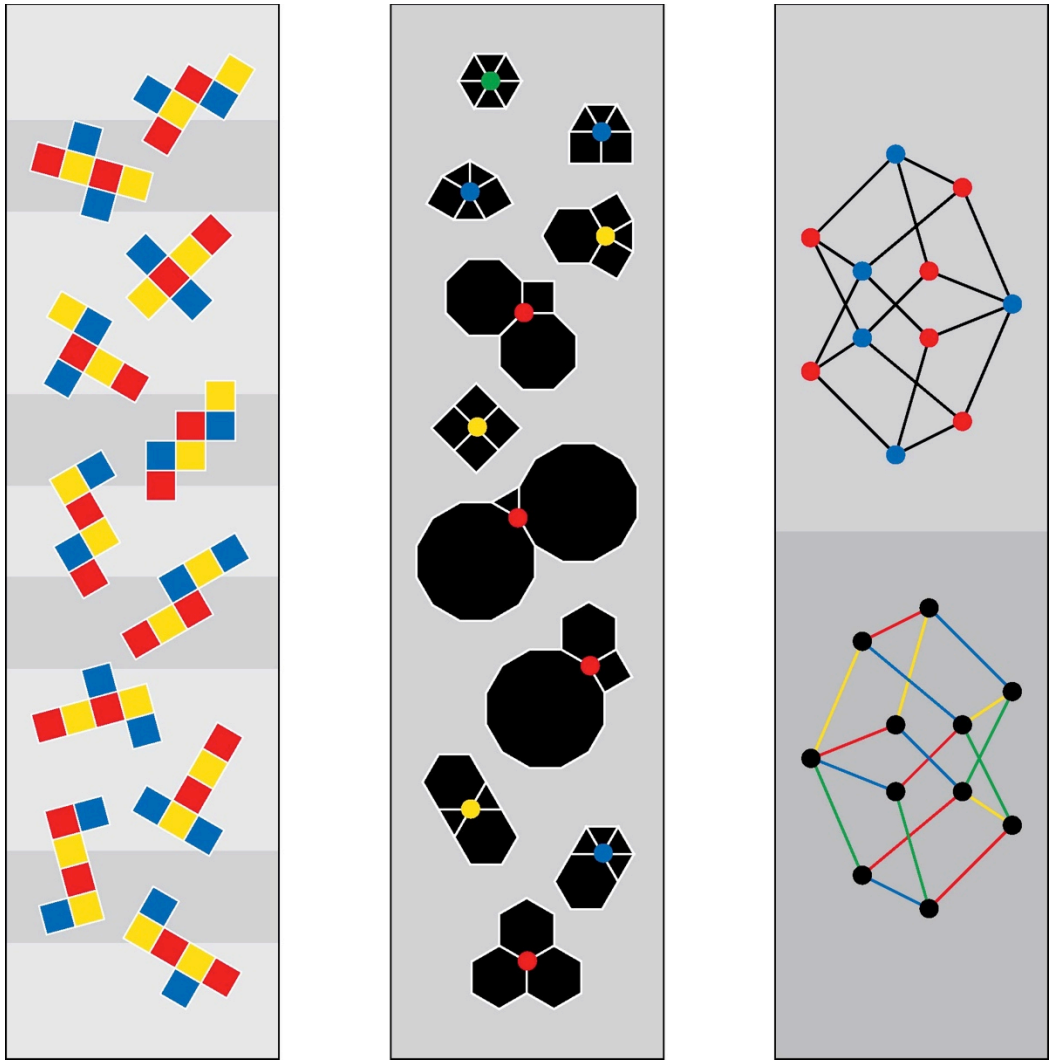
n		p(n)
1	= 1	1
2	= 2	2
	= 1 + 1	
3	= 3	3
	= 2 + 1 = 1 + 2	
	= 1 + 1 + 1	
4	= 4	5
	= 3 + 1	
	= 2 + 2	
	= 2 + 1 + 1	
	= 1 + 1 + 1 + 1	
5	= 5	7
	= 4 + 1	
	= 3 + 2	
	= 3 + 1 + 1	
	= 2 + 2 + 1	
	= 2 + 1 + 1 + 1	
	= 1 + 1 + 1 + 1 +1	

Examples of
Multiplicative Partitions:

n		f(n)
1	= 1	1
2	= 2	1
3	= 3	1
4	= 4 = 2 x 2	2
5	= 5	1
6	= 6 = 3 x 2 = 2 x 3	2
7	= 7	1
8	= 8 = 4 x 2 = 2 x 2 x 2	3
9	= 9 = 3 x 3	2


p(n)
80
60
40
20
n
2
4
6
8
10
12
2
5
11
22
42
77
f(n)
10
n
2
4
6
8
10
12
4


Partitions of 5
5
4 + 1
3 + 2
3 + 1 + 1
2 + 2 + 1
2 + 1 + 1 + 1
1 + 1 + 1 + 1 + 1
Young Diagrams & Conjugate Pairs
3 + 2
4 + 1
5
3 + 1 + 1
Self-conjugate
2 + 2 + 1
2 + 1 + 1 + 1
1 + 1 + 1 + 1 + 1



Ramanujan Congruences

$$p(5n + 4) \equiv 0 \pmod{5}$$

$$p(7n + 5) \equiv 0 \pmod{7}$$

$$p(11n + 6) \equiv 0 \pmod{11}$$

$$p(6) = 11$$

11 Partitions of 6

6

5 + 1

4 + 2

3 + 3

4 + 1 + 1

* 3 + 2 + 1

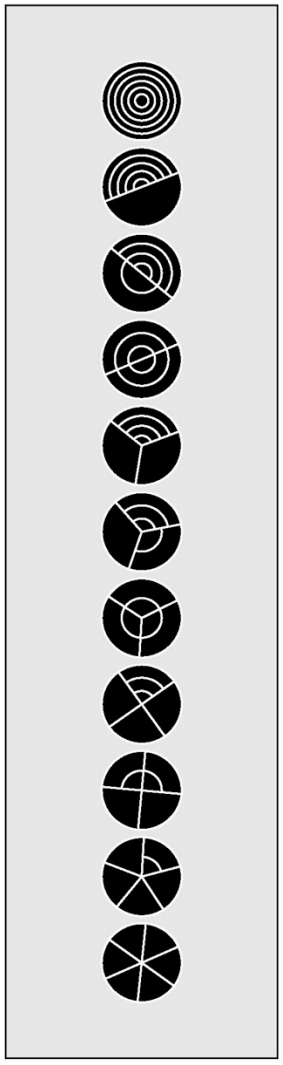
2 + 2 + 2

3 + 1 + 1 + 1

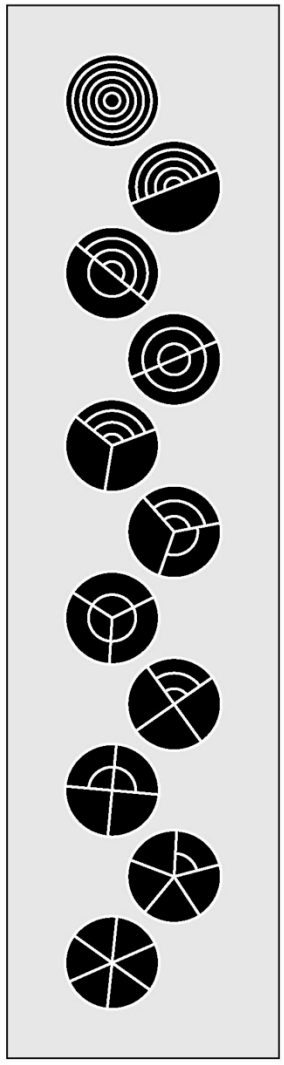
2 + 2 + 1 + 1

2 + 1 + 1 + 1 + 1

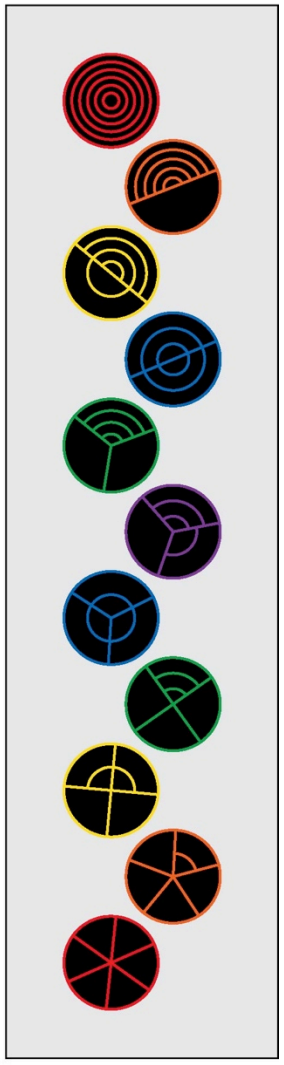
1 + 1 + 1 + 1+ 1+ 1



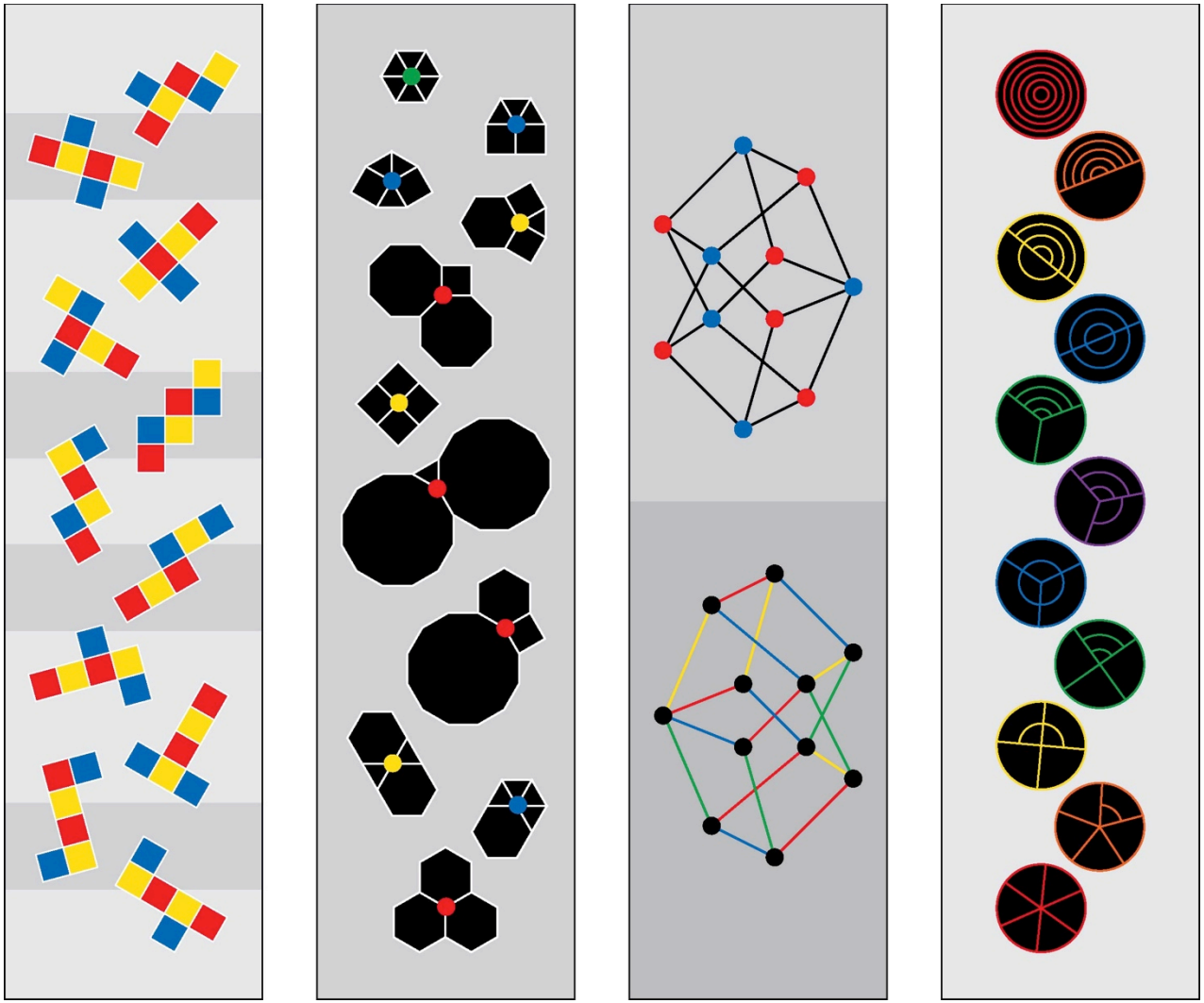
Invert & rotate



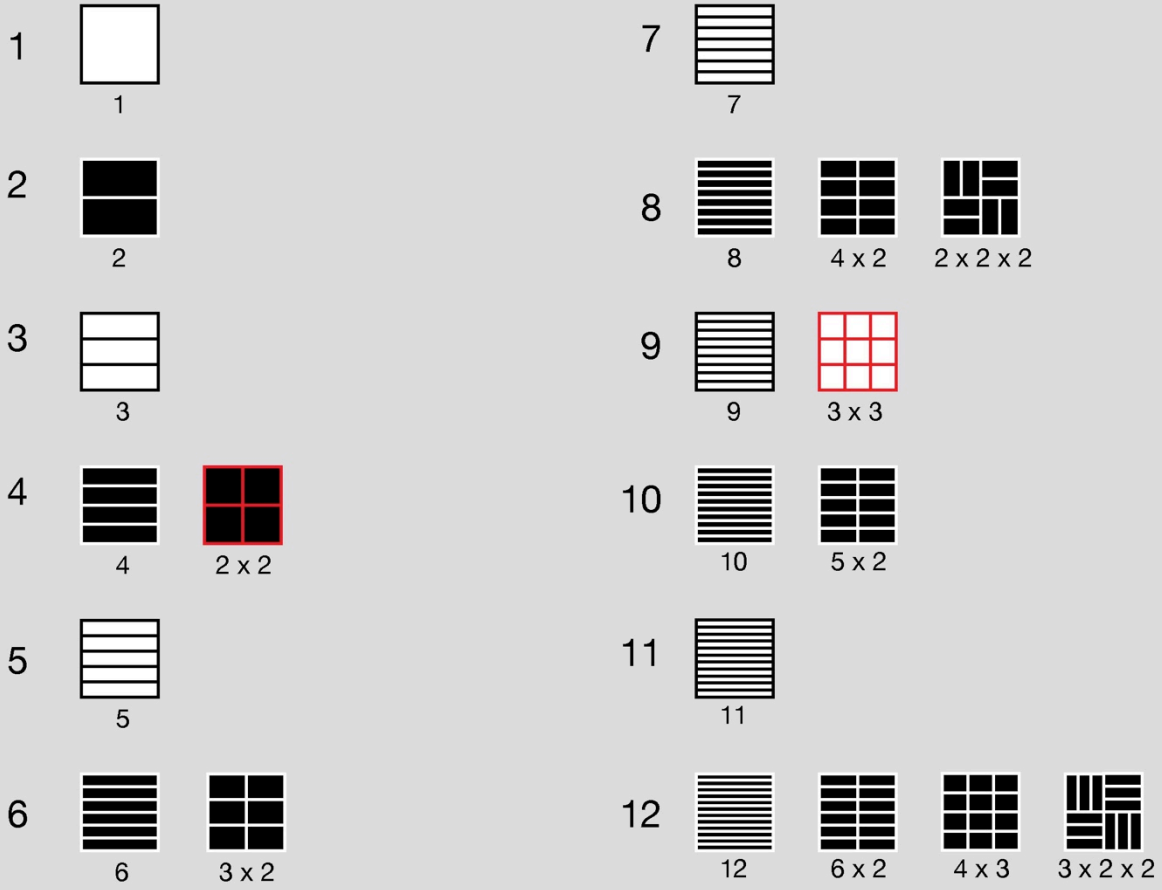
Scale & stagger



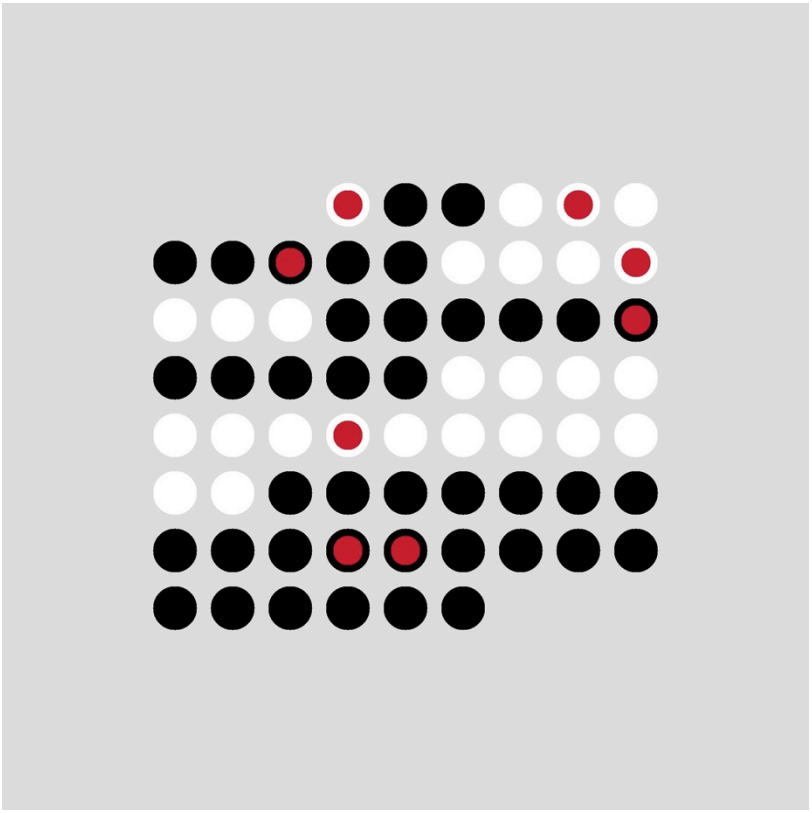
Color conjugates

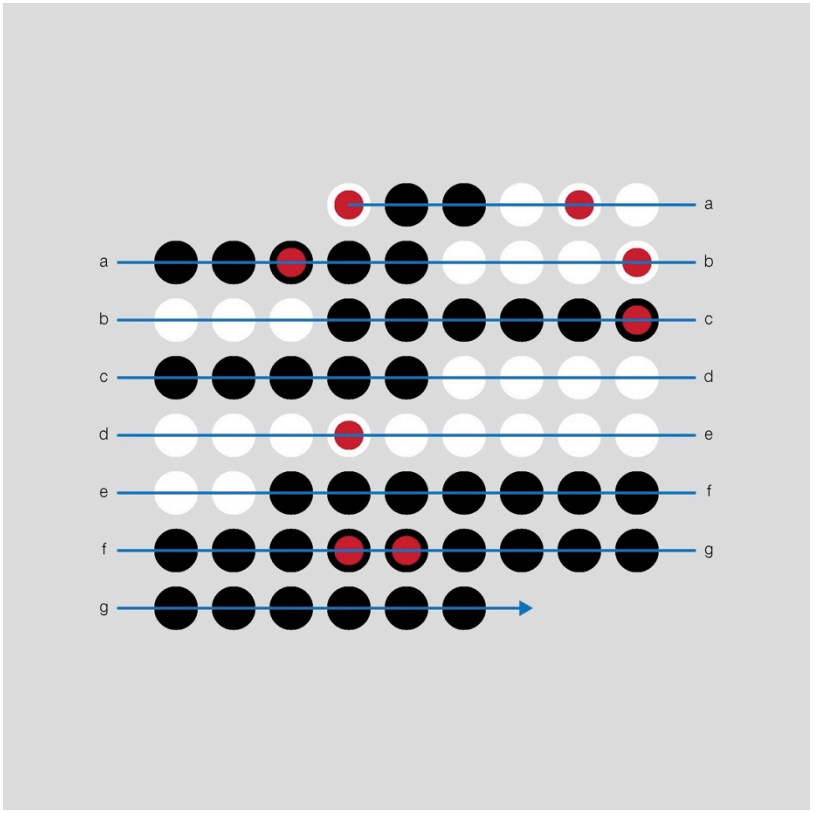


Factor Diagrams for 1 to 12

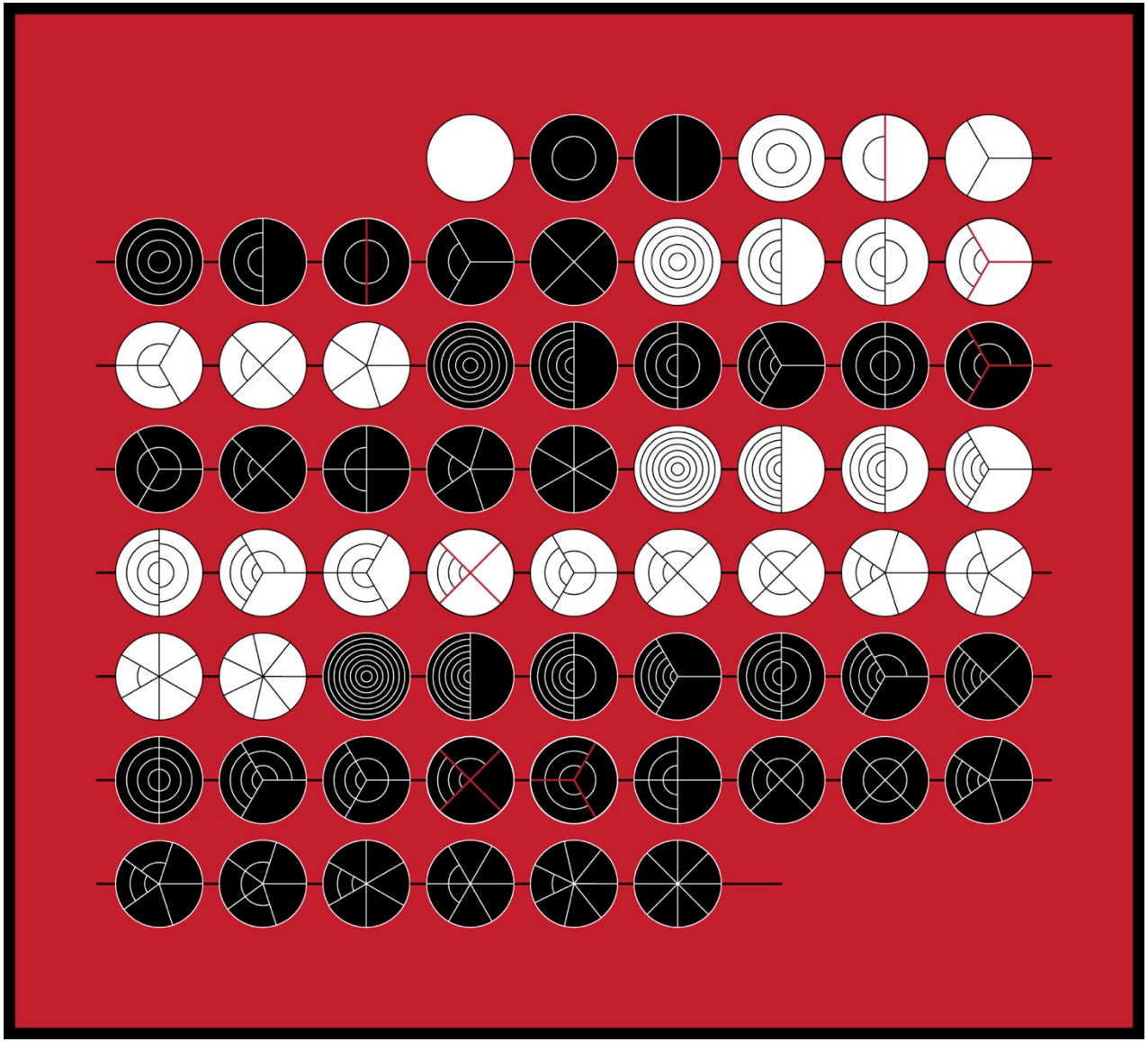


Additive Partitions				Multiplicative Partitions		
n	p(n)	Cum p(n)		n	f(n)	Cum f(n)
1	1	1		1	1	1
2	2	3		2	1	2
3	3	6		3	1	3
4	5	11		4	2	5
5	7	18		.	.	.
6	11	29		.	.	.
7	15	44		27	3	60
8	22	66	⟷	28	4	64
9	30	96		29	1	65
10	42	138		30	5	70

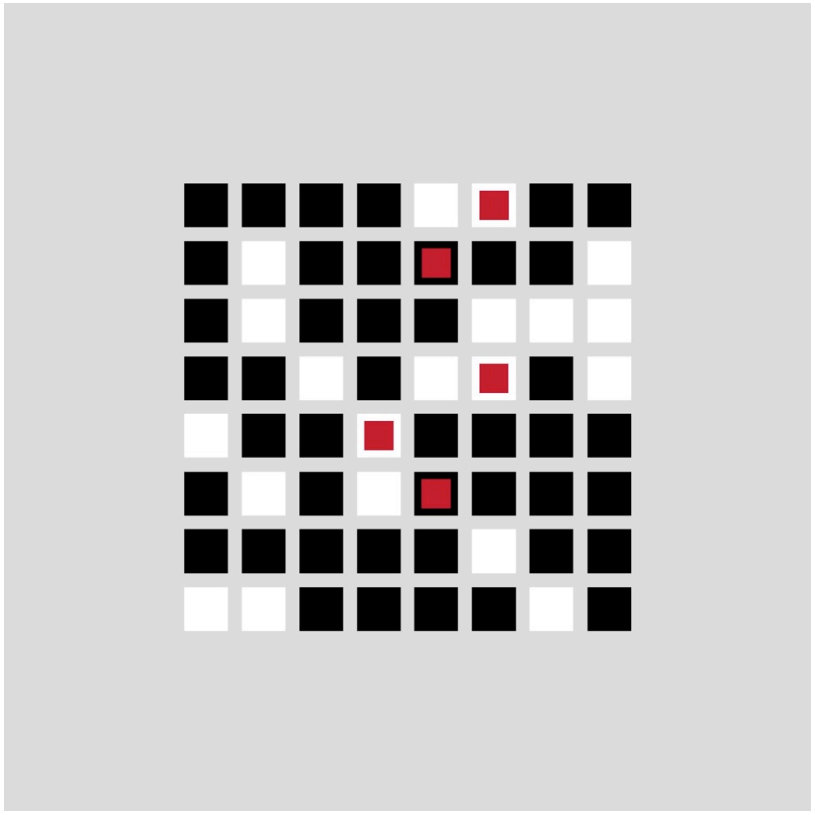


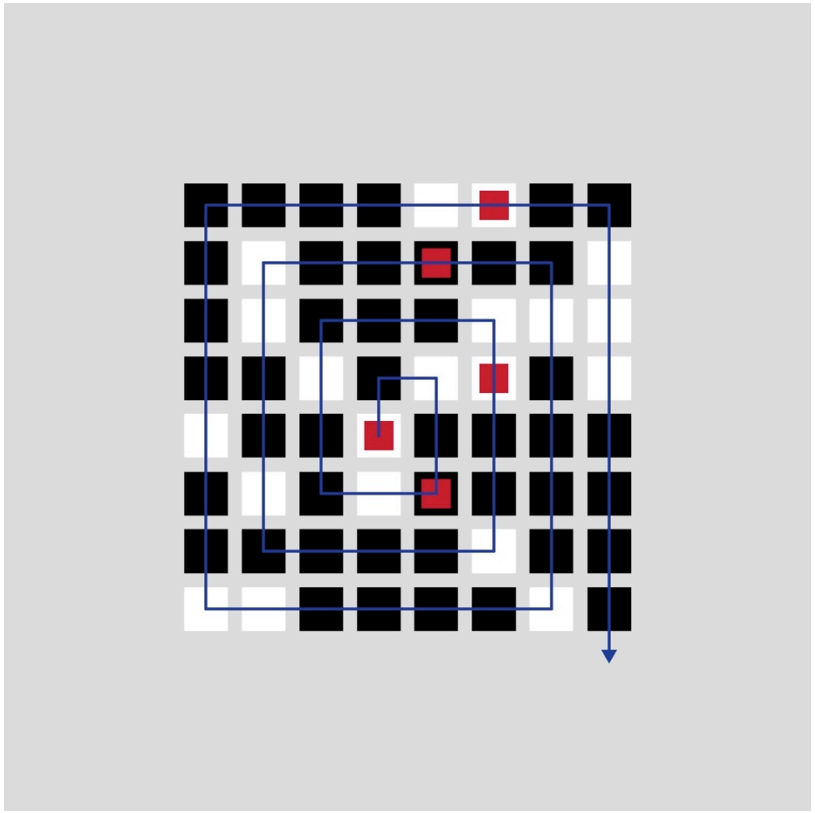


A Rectangular Layout for Additive Partitions

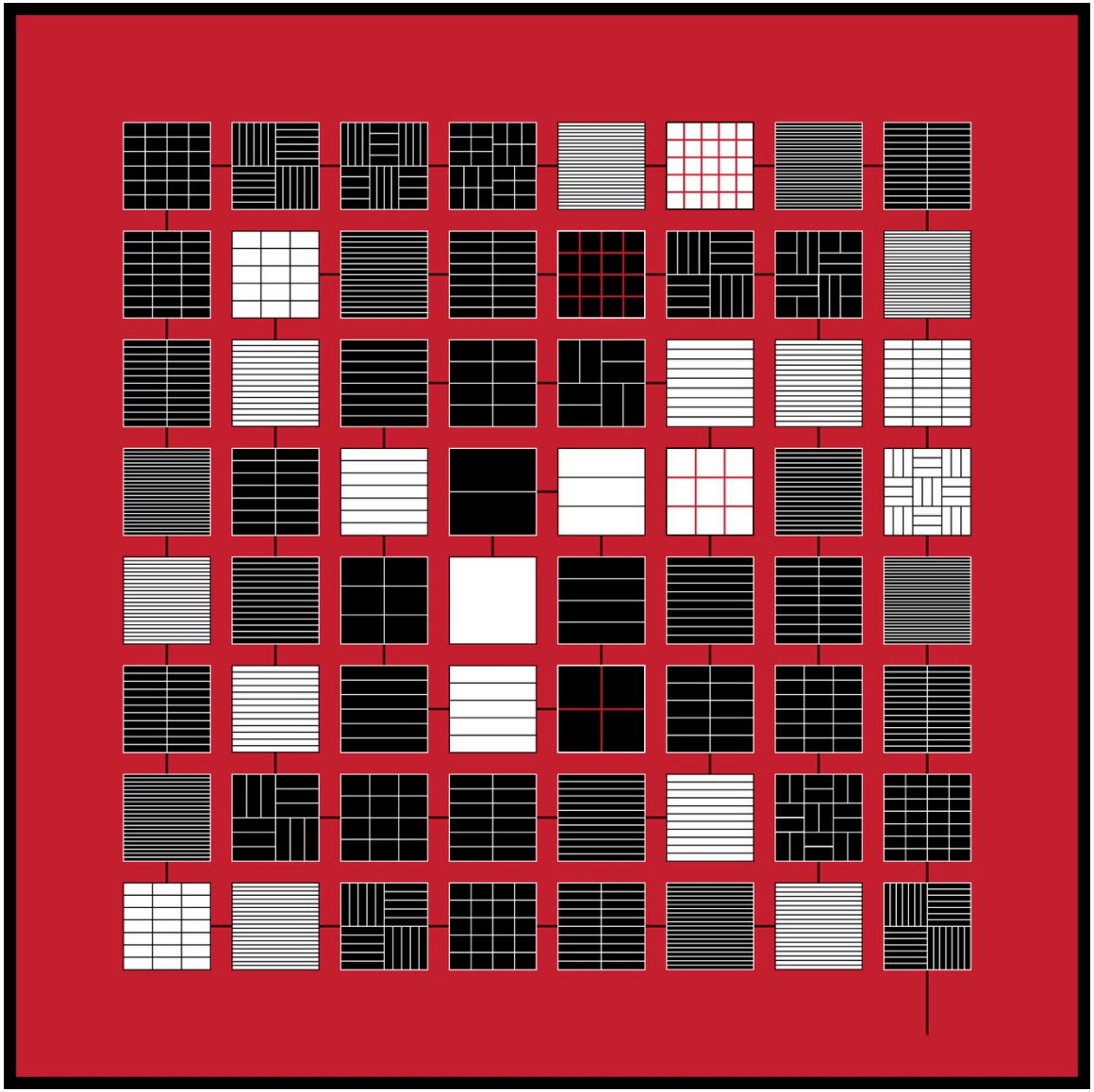


Partitions Study: Pieces of One to Eight

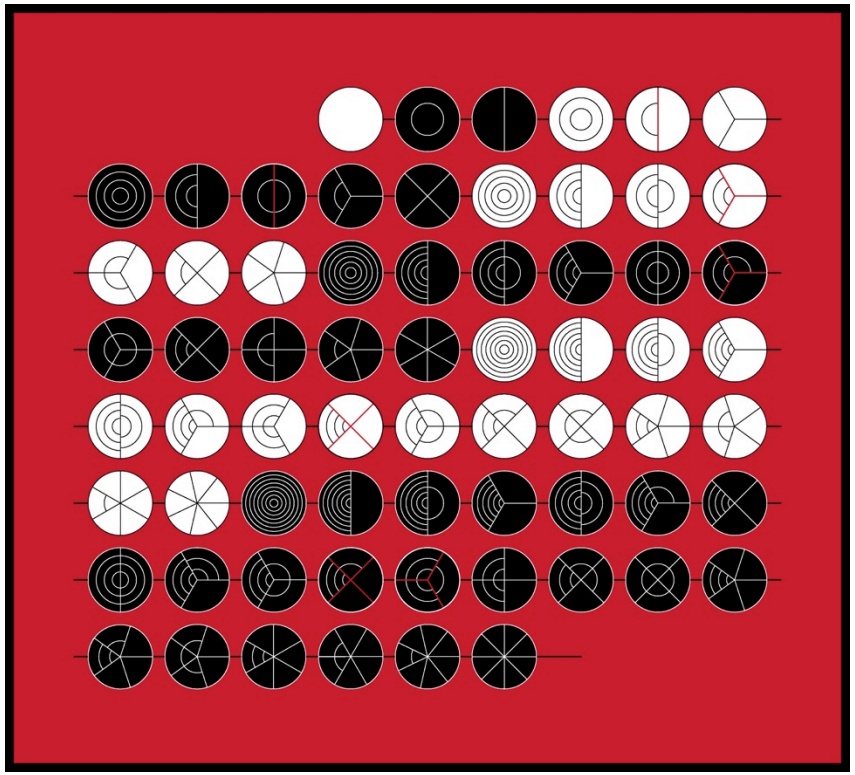


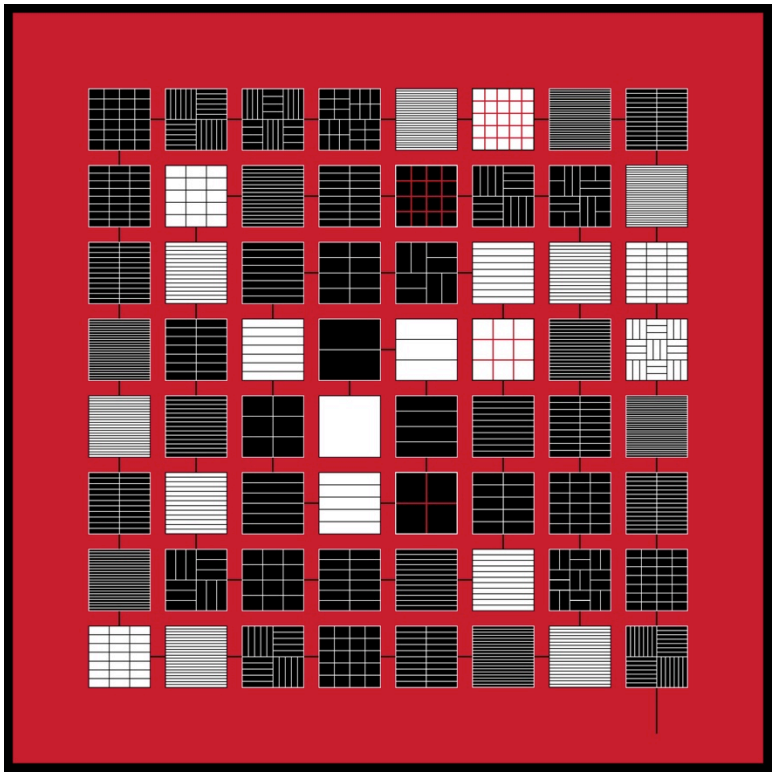


A Square Layout for Multiplicative Partitions



Partitions Study: On the Grid

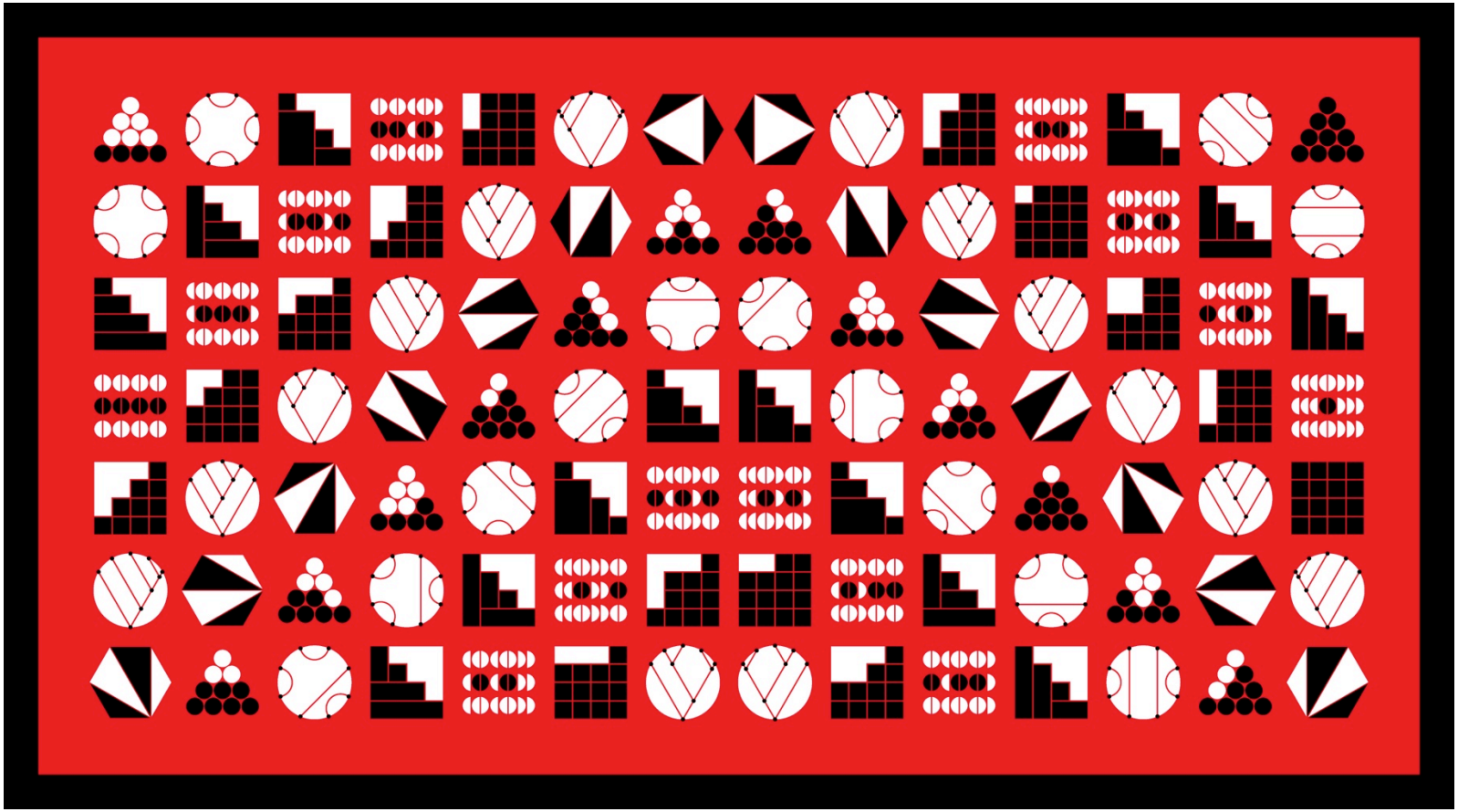




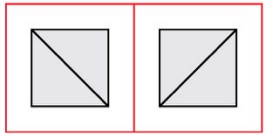
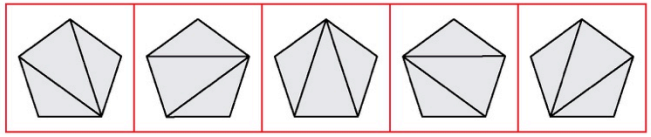
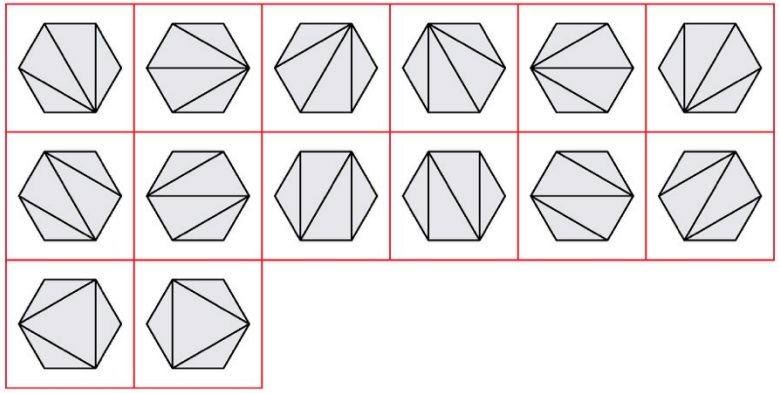
Catalan Connections

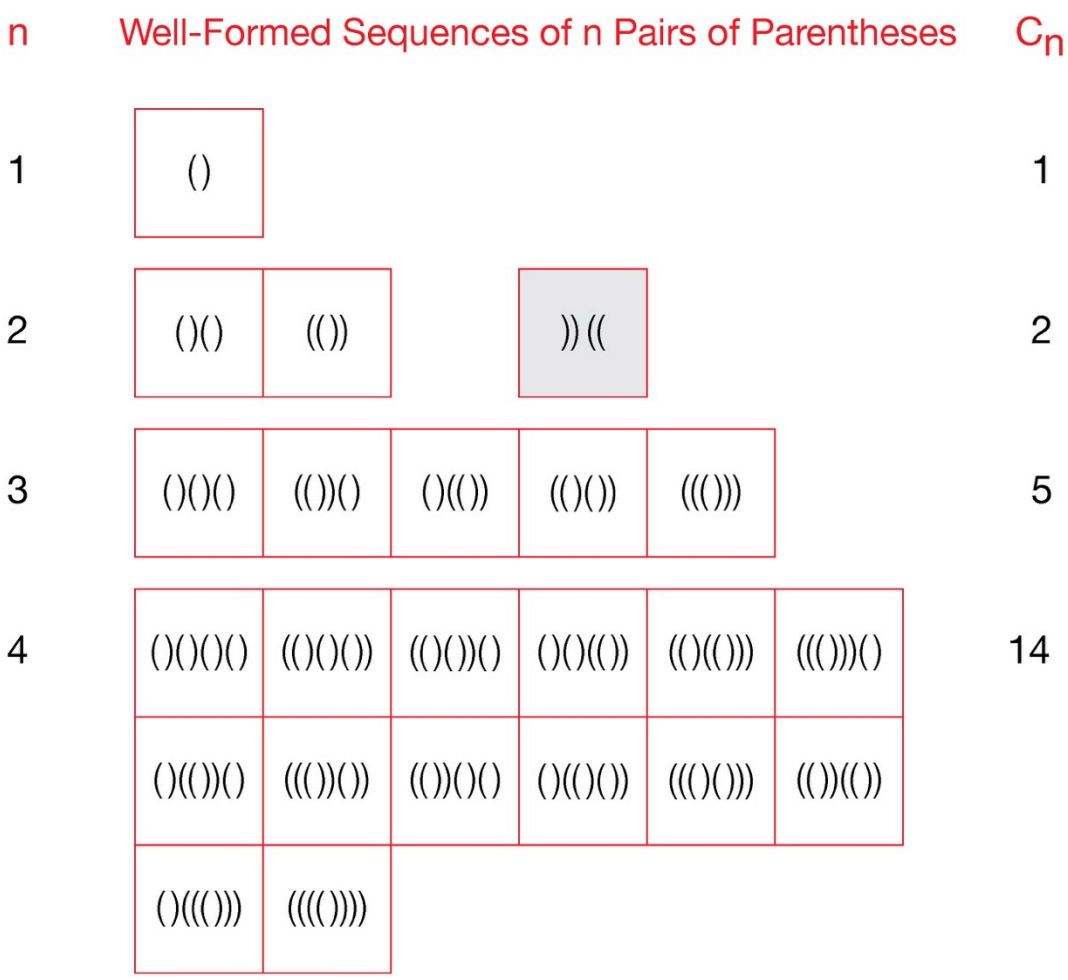
Bridges 2015

JMM 2016

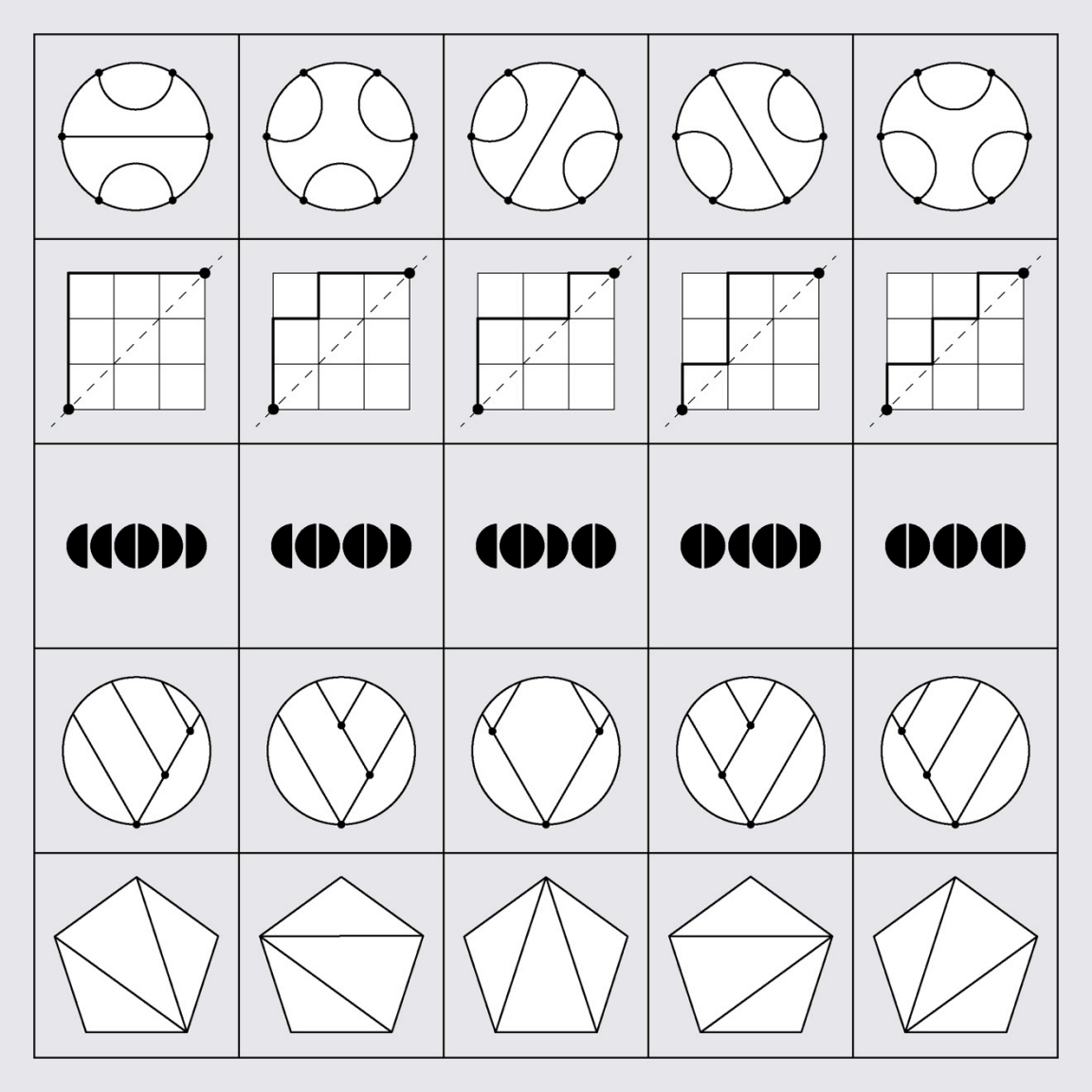


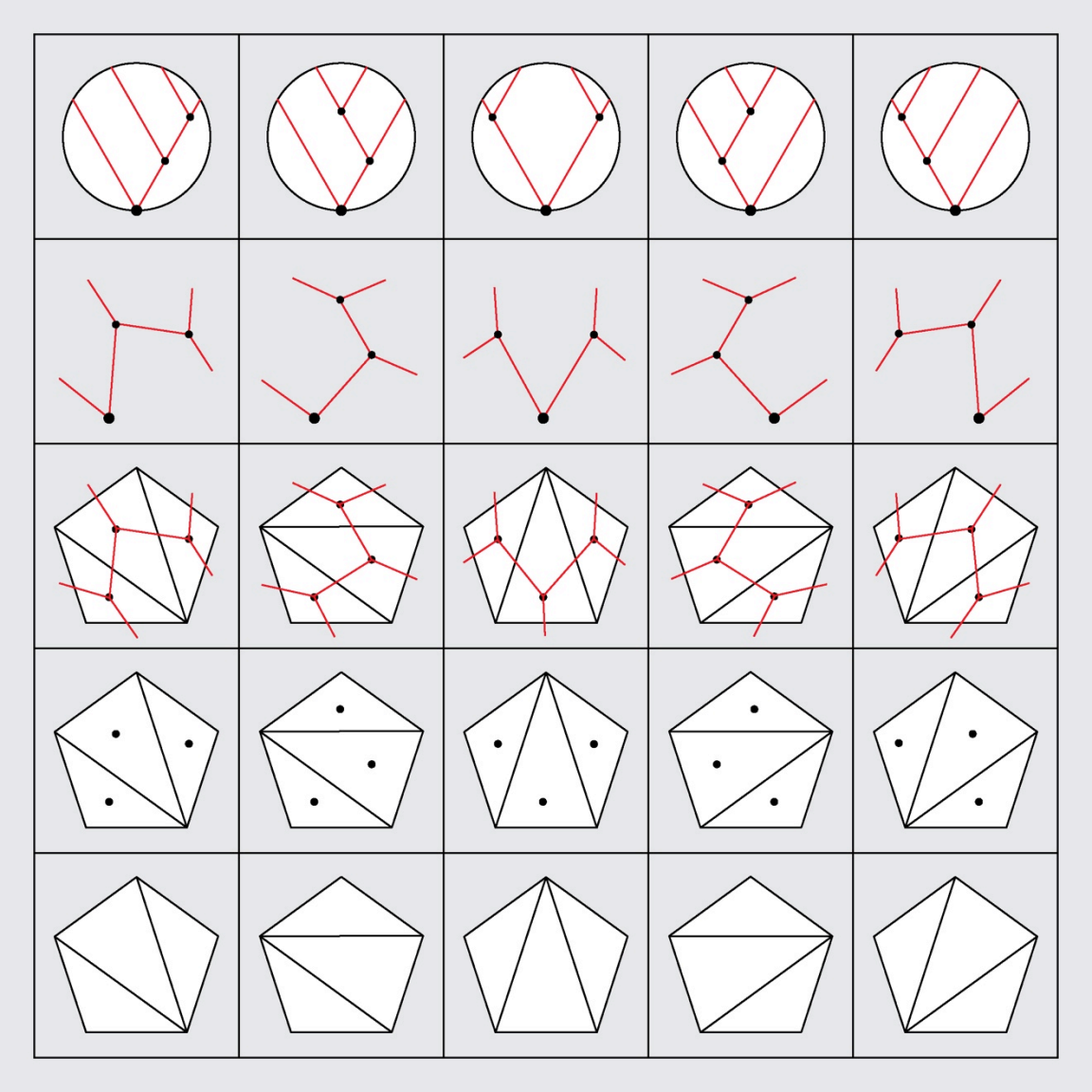
Catalan Connections: Level Four

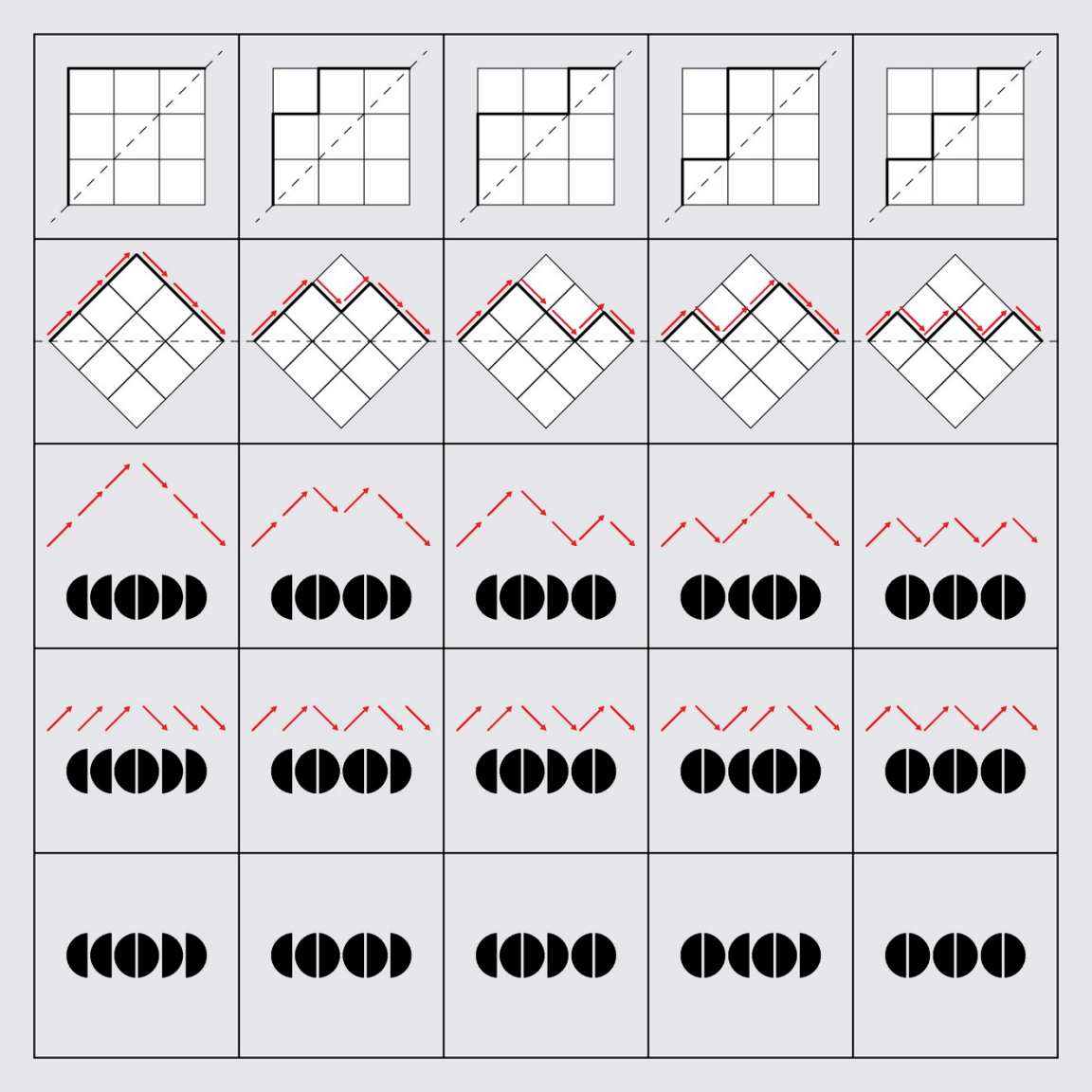
n	Triangulations of Convex Polygons with n+2 Sides	C_n
1		1
2		2
3		5
4		14

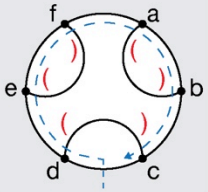
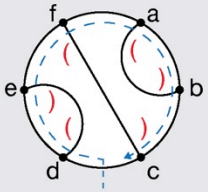
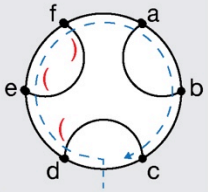
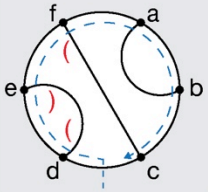
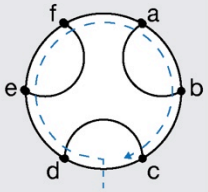
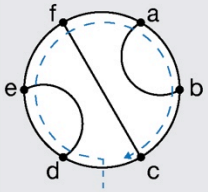
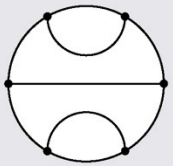
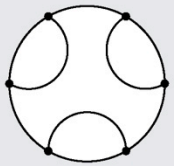
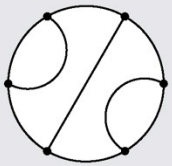
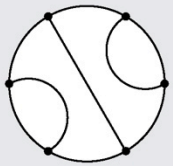
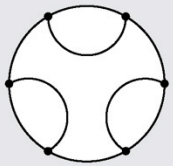


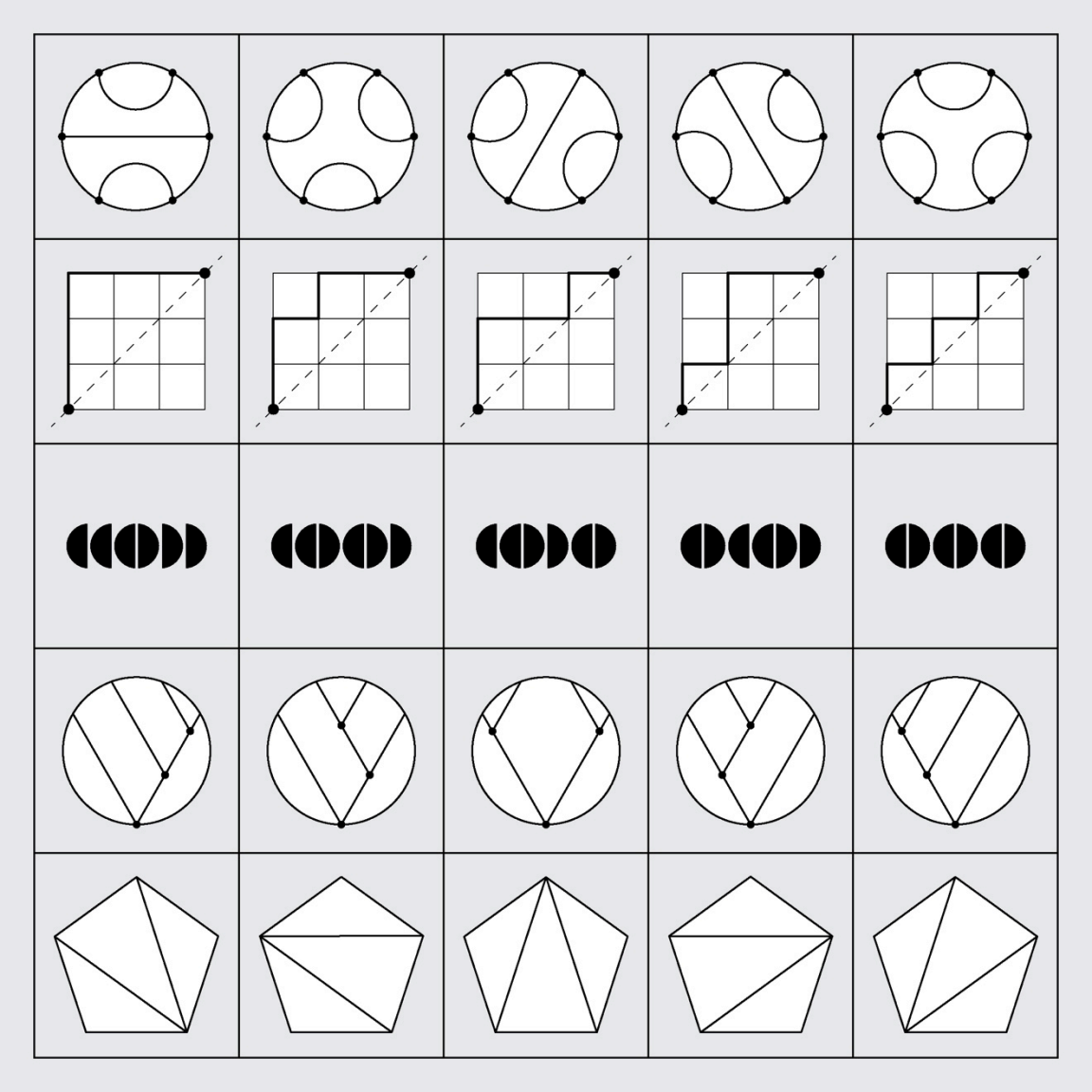
n	Well-Formed Sequences of n Pairs of Parentheses						C_n
1	()						1
2	()()	(())		)) ((			2
3	()()()	(())()	()(())	(()())	((()))		5
4	()()()()	(()()())	(()())()	()()(())	(()(()))	((()))()	14
	()(())()	((())())	(())()()	()(()())	((()()))	(())(())	
	()((()))	(((())))					

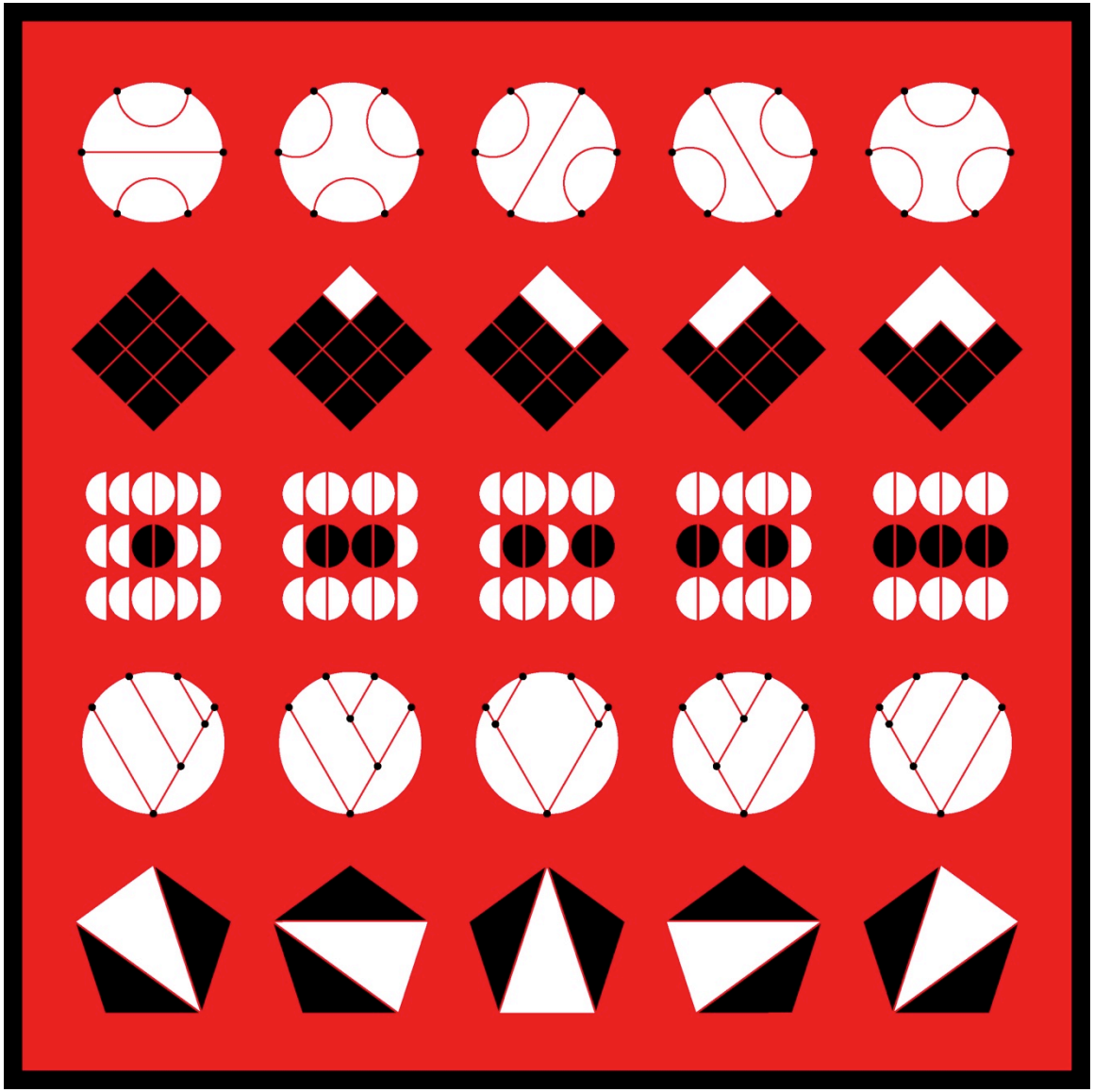


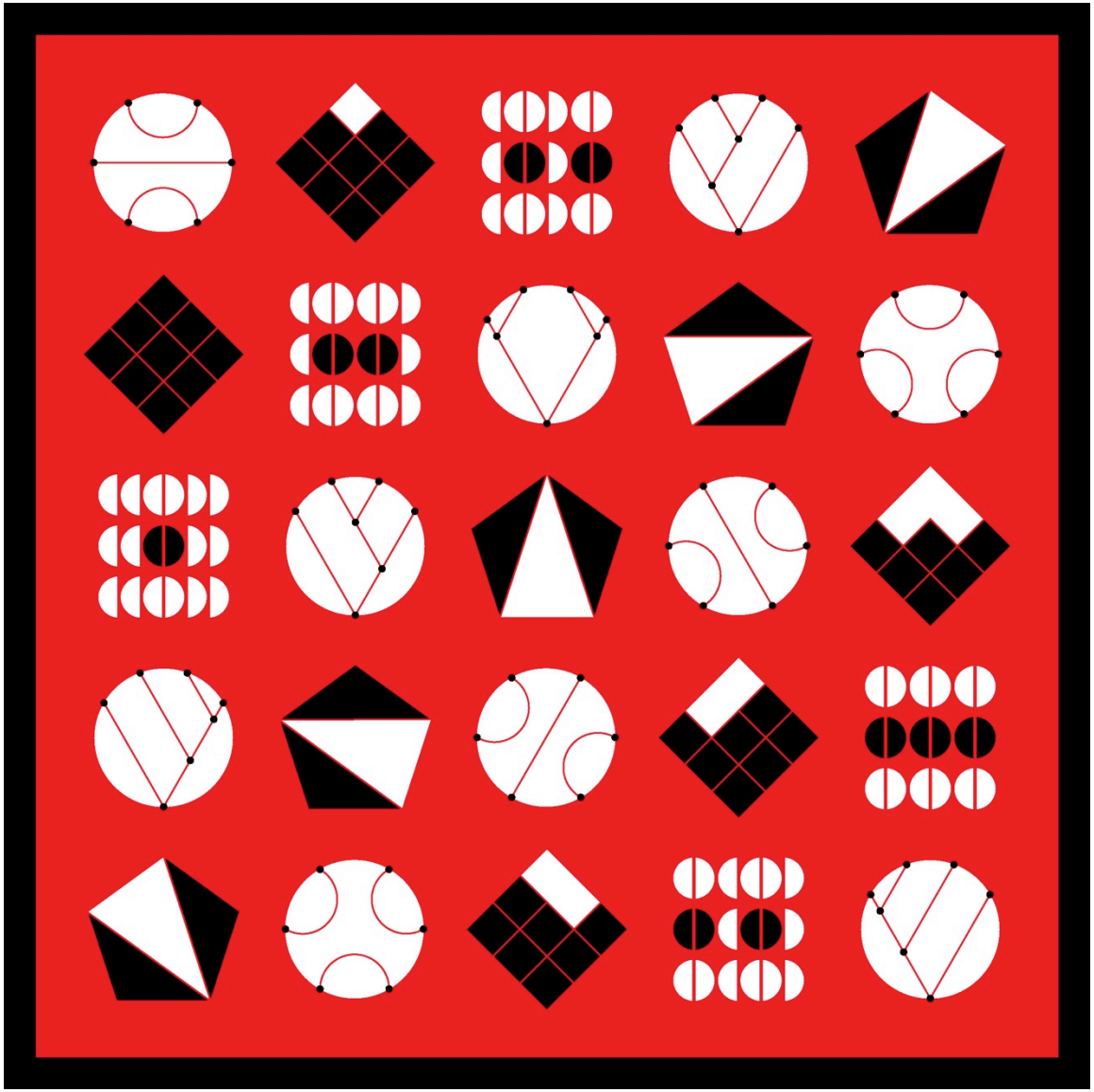




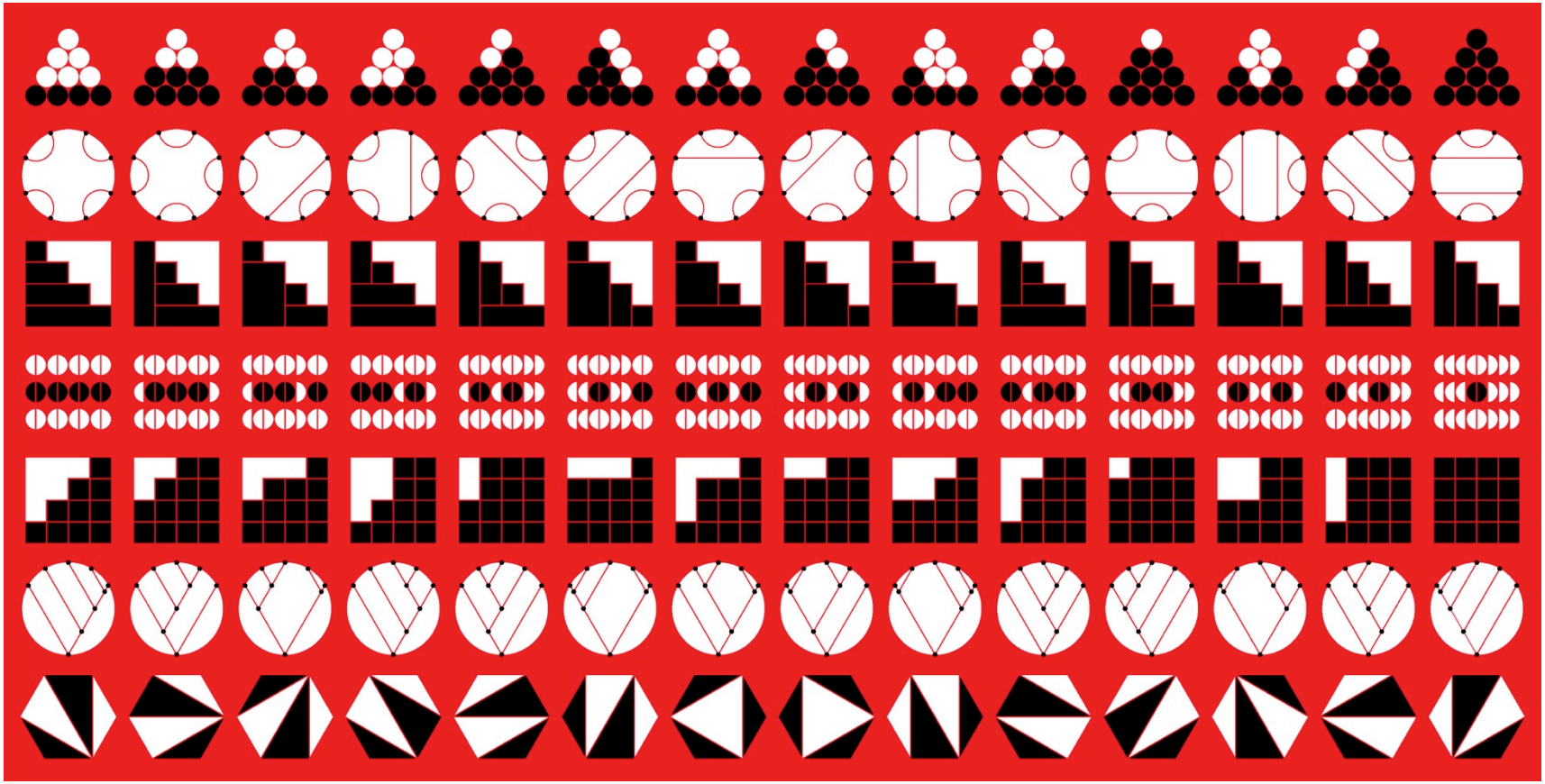
entire path		(( ) ( ))		( ) (( ))
left-half path		(( )		( ) (
				
				

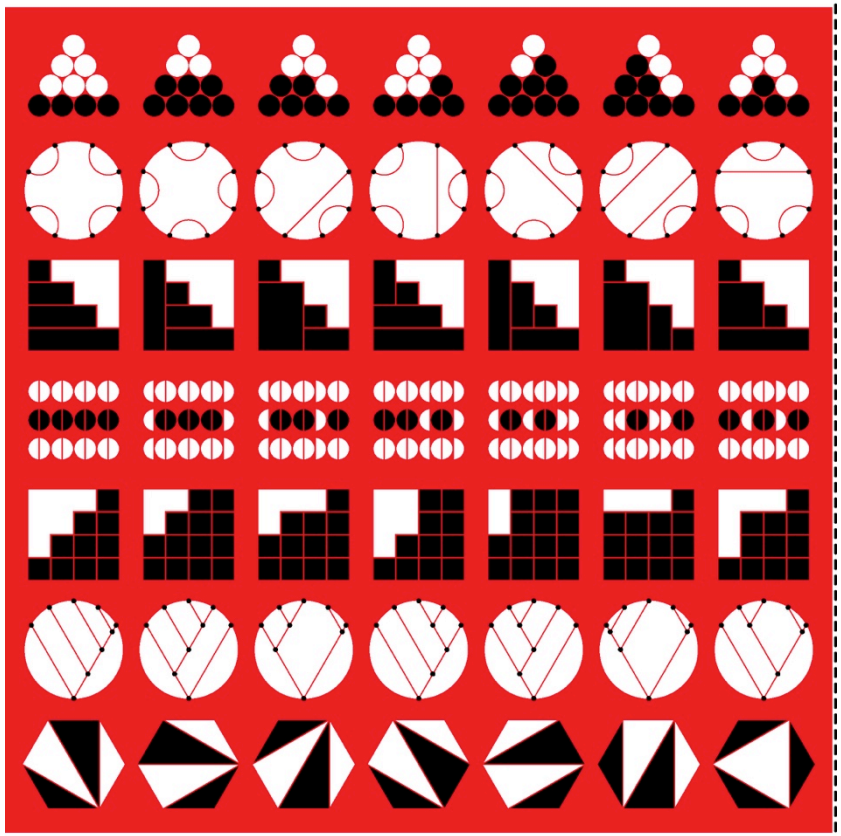






Catalan Connections Study [Bridges 2015]





Stacks of balls with a bottom row of 4 balls *

Handshake options for 8 people in a circle

Ways to tile a staircase with 4 rectangles *

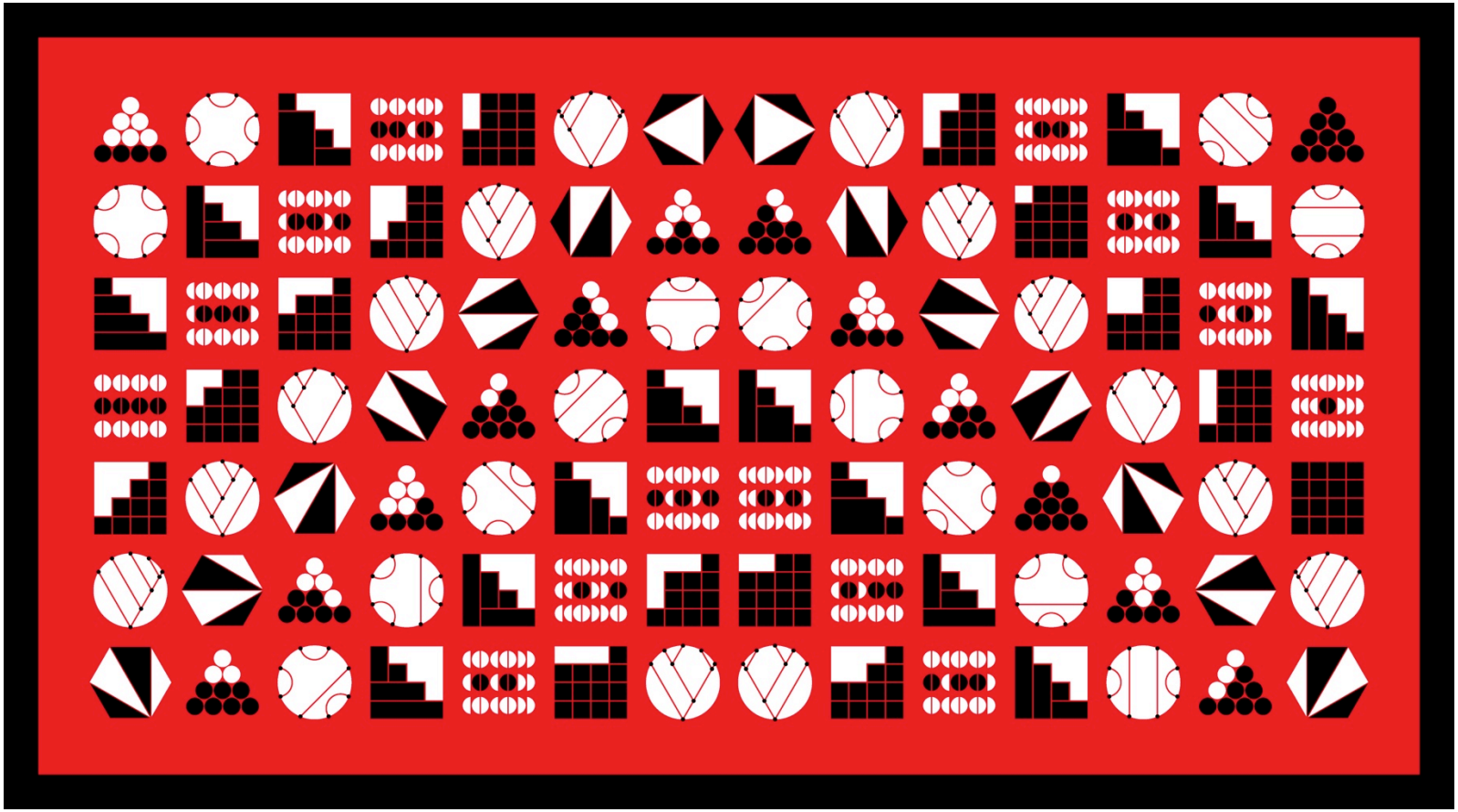
Well-formed sequences of 4 pairs of parentheses

Paths through a 4x4 grid (above the diagonal)

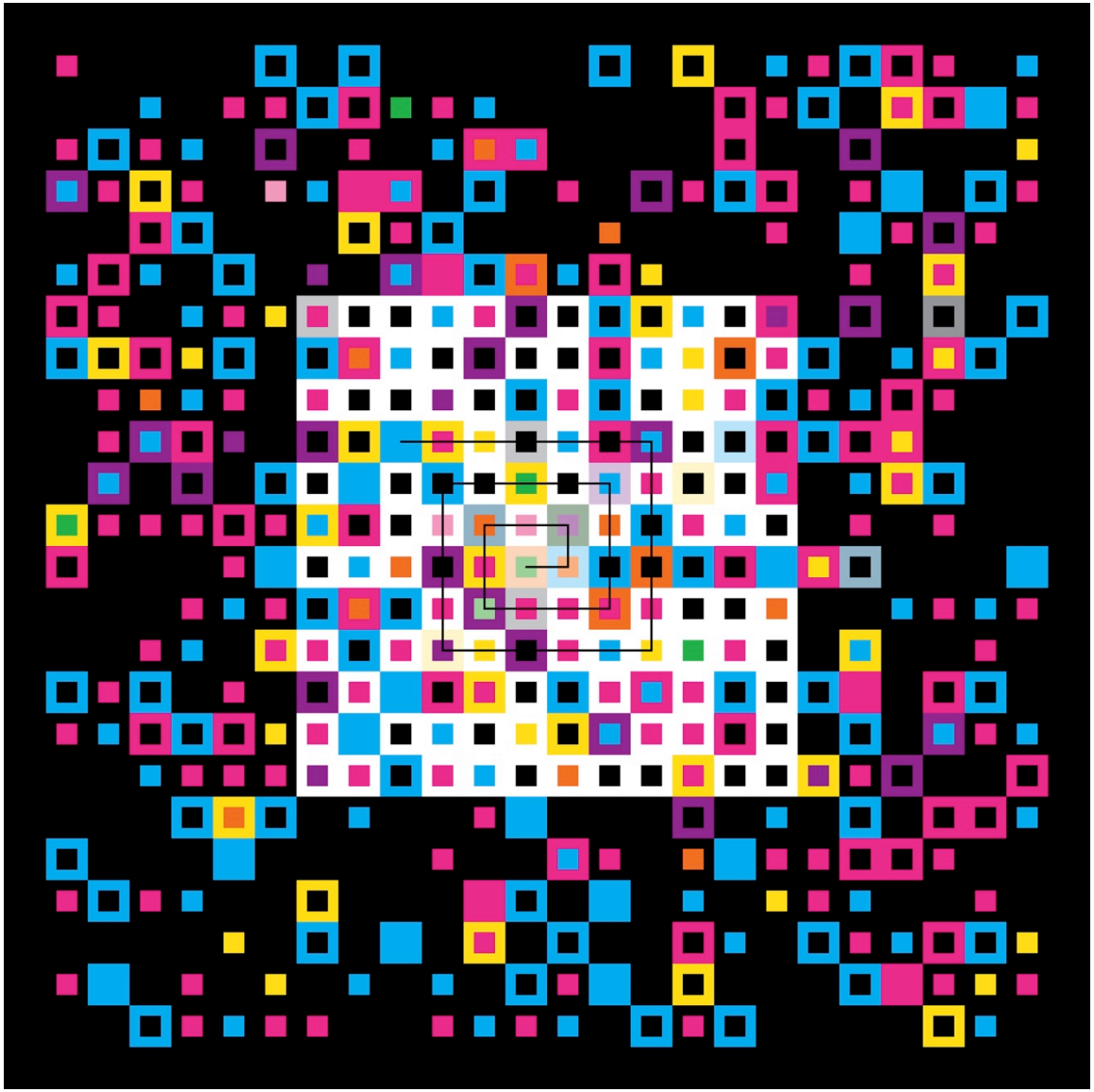
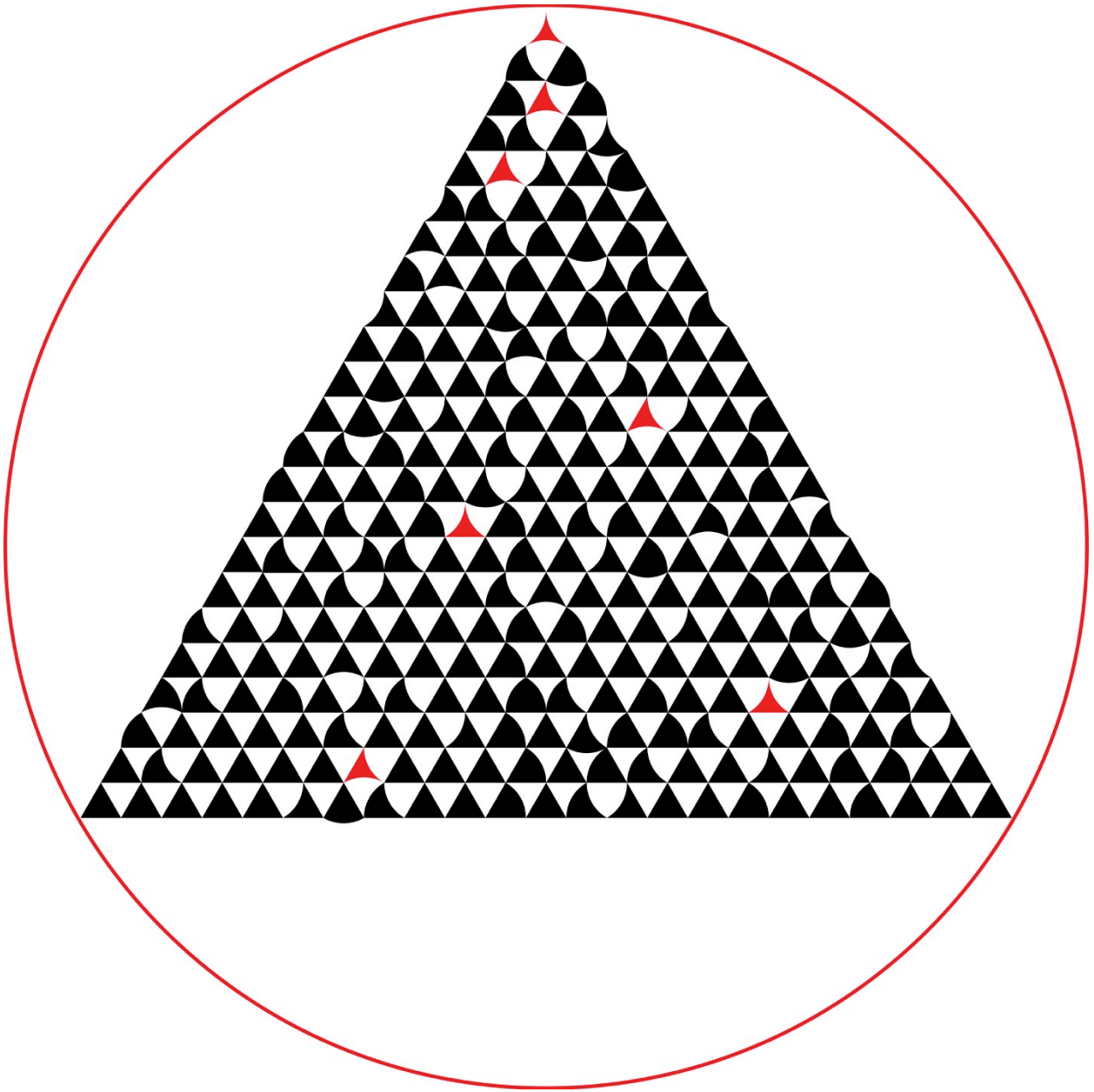
Binary trees with 4 internal branching nodes

Triangulations of a convex hexagon

Diagrams Associated with C4 (Incomplete)



Catalan Connections: Level Four [JMM 2016]

THANK YOU!

Margaret Kepner
mekvisysuals.yolasite.com
renpek1010@gmail.com