Eisenstein Series on Covers of Odd Orthogonal Groups

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Metaplectic Groups

Fix

- An integer $n \geq 1$.
- A reductive algebraic group G.
- A number field F containing the full group μ_N of N-th roots of unity, where we take (for convenience) $N = 2n$.

Then there is an *n-fold metaplectic cover* $\widetilde{G}$ of $G(\mathbb{A}_F)$. Key aspects:

- The elements of $\widetilde{G}$ consist of ordered pairs (g, ζ) with $g \in G(\mathbb{A}_F)$, $\zeta \in \mu_n$.
- There is a two-cocycle $\sigma : G(\mathbb{A}_F) \times G(\mathbb{A}_F) \to \mu_n$ such that the multplication

$$(g_1, \zeta_1) \cdot (g_2, \zeta_2) = (g_1\, g_2\,,\, \zeta_1\, \zeta_2\, \sigma(g_1, g_2))$$

  makes $\widetilde{G}$ into a group.
- The two-cocycle σ is computed arithmetically, that is using local Hilbert symbols on completions of F.

The Big Picture

Let

- U by a maximal unipotent subgroup of G.
- ψ be a character of $U(\mathbb{A}_F)$ that is trivial on $U(F)$ and is *generic*: its restriction to the subgroup corresponding to each simple root is non-trivial.

Given an automorphic form f on $\widetilde{G}$, its *Whittaker coefficient* is

$$W(g) = \int_{U(F)\backslash U(\mathbb{A}_F)} f((u,1)\,g)\,\psi(u)\,du. \qquad g \in \widetilde{G}$$

Conjecture/Theorem

Suppose that f is an Eisenstein series on $\widetilde{G}$. Then its Whittaker coefficients may be expressed in terms of crystal graphs and inducing data.

What's Known

- Proved for $G = GL_{r+1}$ for the Borel (i.e. minimal parabolic) Eisenstein series by Brubaker, Bump and Friedberg (Annals, 2011); follows earlier work that is joint with Hoffstein. (In this case there is no nontrivial inducing data and the answer is given fully in terms of crystal graphs.)
- Proved for more general Eisenstein series on GL_{r+1} by Brubaker and Friedberg (2013).
- Conjectures for Borel Eisenstein series on other types of classical groups by Beineke, Brubaker and Frechette (odd covers of odd orthogonal groups), Brubaker, Bump, Chinta, Gunnells (even covers of symplectic groups), Chinta and Gunnells (even orthogonal groups); Lie-theoretic conjecture for all groups for $n \gg 0$ (Brubaker, Bump, Friedberg).
- Proved using Casselman-Shalika formulae in lowest degree cover cases.
- Work of Chinta-Gunnells, Chinta-Offen, McNamara is related.

Our Result

Theorem (Friedberg/Zhang)

Let f be a Borel Eisenstein series on the n-fold cover of a split special odd orthogonal group. Then its Whittaker coefficients may be expressed in terms of crystal graphs of type C.

- This proves the Conjecture of Beineke, Brubaker and Frechette for odd degree covers.
- This proves the Conjecture of Brubaker, Bump and Friedberg for covers with $n \gg 0$.
- This gives a new recipe for even degree covers.
- Proof is quite involved, and in particular uses subtle aspects of the type A theory, established by Brubaker, Bump and Friedberg (Annals of Math Studies 175, 2011).