Vexillary signed permutations and essential sets

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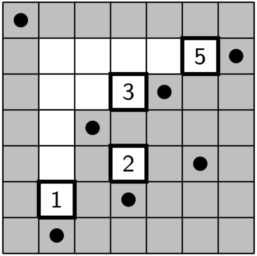
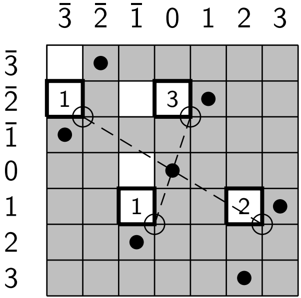
Diagrams and essential sets



$\ell(v) = 12$
$\lambda(v) = (5, 3, 2, 1, 1)$

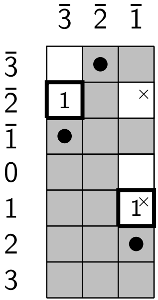
Diagrams and essential sets

$v = \bar{1}\ \bar{3}\ 2\ 0\ \bar{2}\ 3\ 1$



Diagrams and essential sets

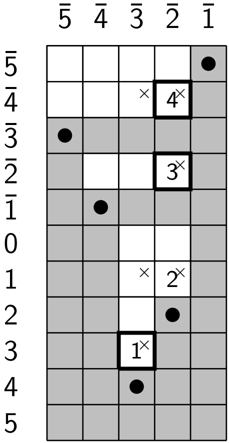
$w = \bar{2}\,3\,1$



$\ell(w) = 3$

$\mathscr{E}ss(w) = \{\, (1, 3, \bar{1})\,,\, (1, 1, 2) \,\}$

Diagrams and essential sets



$w = 5\ \bar{2}\ \bar{4}\ 1\ 3$

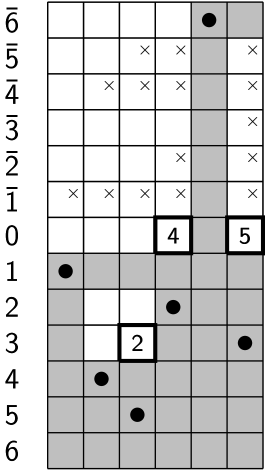
$\ell(w) = 11$

$\mathscr{E}ss(w) = \{\, (1, 3, 4)\,,\ (3, 2, \bar{1})\,,\ (4, 2, \bar{3})\,\}$

vexillary: (fr. Latin *vexillum*, pl. *vexilla*, flag).

Vexillary signed permutations

$\boldsymbol{\tau} = (\ 2\ 4\ 5,\ 4\ 3\ 1,\ 4\ 1\ 1\ )$



$\lambda(\boldsymbol{\tau}) = (8, 7, 4, 3, 1)$

$w(\boldsymbol{\tau}) = \bar{3}\ 6\ \bar{2}\ \bar{5}\ \bar{4}\ \bar{1}$

$\ell(w) = 23$

$\mathscr{E}ss(w) = \{\ (2, 4, 5)\ ,\ (4, 3, 1)\ ,\ (5, 1, 1)\ \}$

Formulas for orthogonal flag varieties / degeneracy loci

Theorem (A.-Fulton)

For a vexillary signed permutation $w = w(\boldsymbol{\tau})$ *and strict partition* $\lambda = \lambda(\boldsymbol{\tau})$ *we have*

$$[\Omega_w] = \frac{1}{2^{k_s}} \mathrm{Pfaff}(M),$$

where M *is the skew-symmetric matrix having* (k, ℓ)*-entry*

$$c(k)_{\lambda_k} c(\ell)_{\lambda_\ell} + 2 \sum_{j=1}^{\lambda_\ell} (-1)^j c(k)_{\lambda_k + j} c(\ell)_{\lambda_\ell - j},$$

with $c(k_i) = c(V - E_{p_i} - F_{q_i})$ *and* $c(k) = c(k_i)$ *for* $k_{i-1} < k \leq k_i$.