

# New Online Algorithms for Story Scheduling in Web Advertising

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## Online advertising

Worldwide online ad spending 2012/13: \$ 100 billion

Expected to surpass print ad spending soon

**Display advertising:** images, videos, animations

Content shown depending on **browsing history** of user

# Story boarding

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 Enter Symbol Look Up Thu, Jul 4, 2013, 4:49AM EDT - US Markets are closed

US	S&P 1,615.41 +1.33 +0.08%	Dow 14,988.55 +56.14 +0.38%	Nasdaq 3,443.67 +10.27 +0.30%	10-Year Bond 2.501 +0.032 +1.30%	Gold 1,255.10 11.70 +0.94%	Oil 101.34 1.74 +1.75%	EUR/USD 1.3001 -0.0008 -0.06%
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**STOCK WATCH** Bassett Sinks; Int'l Speedway Rises; Noodles & Co. Surges

### Market Turns Attention to Jobs

Breakout  
 Stocks were slightly lower Wednesday as news of a major shake-up in Portugal's government, turmoil in Egypt, and more softness in Chinese manufacturing was offset by encouraging jobs data on the homefront.

- U.S. private sector adds 188,000 jobs in June: ADP
- Jobs data upbeat, trade deficit widens
- Trade deficit widens sharply as imports rise

### The Company Steve Jobs Would've Bought: Wilcox

Breakout  
 From being named Motor Trend's Car of the Year to paying off a federal loan nine years early, things are clearly on the move at this company. Hint: Shares are up nearly 250% this year.

### Get Ready to Buy Ugly, Cheap Emerging Markets

Michael Santoli  
 If, as the Wall Street adage goes, the time to buy is when blood runs in the streets, then investors should prepare, sadly, to start picking up emerging-markets stocks.

### Service Sector Growth Slows to Three-Year Low in June: ISM

Reuters  
 The pace of growth in the services sector slowed in June to its weakest level in over

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## DAS SAMSUNG GALAXY S4 MIT ALLE-NETZE-FLATRATE

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49,99 €  
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## Story boarding

- Maintain ad position of a web site during browsing session of a user.
- Position depicts image sequences of advertisers.  
Advertiser pay unit shown.
- Depending on history/state user becomes interesting for advertisers.
- Maximize revenue of session.

## Model

- Session time is **slotted**

**Time  $t$ :** user continues surfing with **probability  $\beta$**   $0 < \beta < 1$

- $I = J_1, \dots, J_N$

$$J_i = (a_i, l_i, v_i)$$

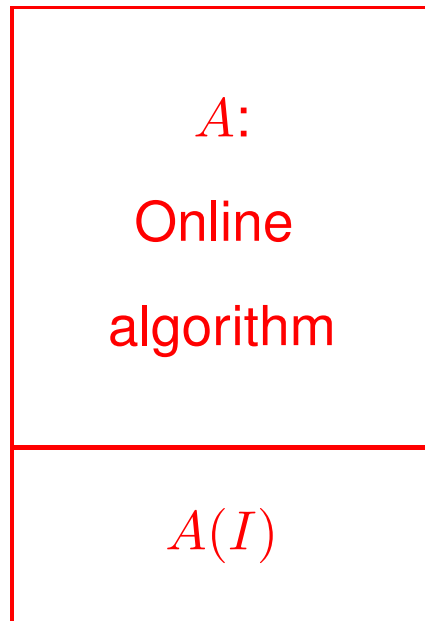
$a_i$  = arrival time       $l_i$  = length       $v_i$  = per-unit value

- **Preemption:** Job  $J_i$  may be scheduled for  $< l_i$  time units

- $\max \sum_{t \geq 0} \beta^t v(t)$

$v(t)$  = per-unit value of job scheduled at time  $t$

## Competitive analysis



$A$  is  $c$ -competitive if for all  $I$

$$A(I) \geq \frac{1}{c} \cdot OPT(I).$$

## Previous results

- Lower bound:  $c \geq 2$  for general  $\beta$        $c \geq \beta + \beta^2$
- Upper bound:  $c = 7$

ALG:

$\forall t$   $v_i$  of current  $J_i \iff$  loss in delaying  $J_k$  with  $v_k > v_i$  for 1 time unit

Dasgupta, Ghosh, Nazerzadeh, Raghavan SODA'09

## Previous results

- Lower bound:  $c \geq 2$  for general  $\beta$        $c \geq \beta + \beta^2$
- Upper bound:  $c = 7$
- Jobs to be scheduled **immediately** upon arrival  
 $c = \Omega(\sqrt{\log \mu / \log \log \mu})$        $\mu = \max\{l_{\max}/l_{\min}, v_{\max}/v_{\min}\}$
- Total value of  $v_i$  gained only if **entire**  $J_i$  is shown  
 $c = O(\log(v_{\max}/v_{\min}))$

Dasgupta, Ghosh, Nazerzadeh, Raghavan SODA'09



## Our contribution

- Upper bound:  $c = 4/(2 - \beta)$

- Upper bound:  $c = 1 + \Phi \approx 2.62$        $\Phi = \frac{1+\sqrt{5}}{2}$  Golden Ratio

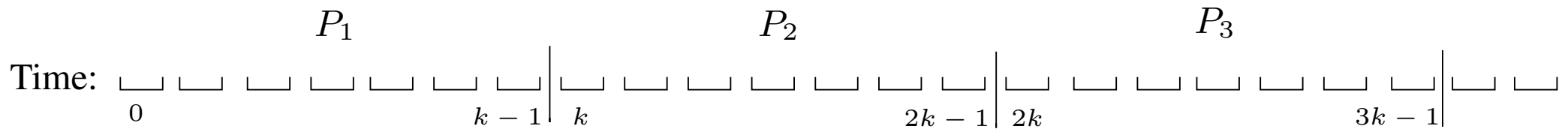
- **Problem extension:** Web page with  $m$  ad positions, where stories can be shown simultaneously; job migration not allowed

$$\max \sum_{t \geq 0} \sum_{j=1}^m \beta^t v(t, j)$$

$v(t, j)$  = per-unit value of job scheduled on ad position  $j$  at time  $t$

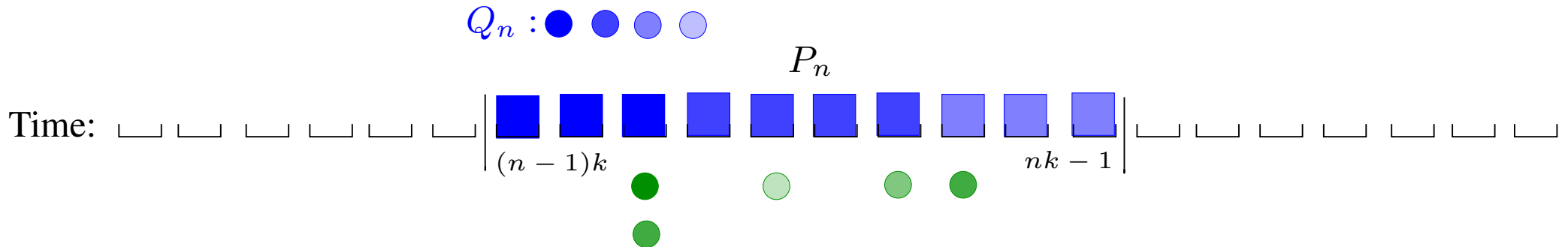
Upper bound:  $3.414 \approx 2/(2 - \sqrt{2}) \leq c \leq 1/(3 - 2\sqrt{2}) \approx 5.828$

## Algorithmic approach



- Phases  $P_1, P_2, P_3, \dots$  of  $k$  consecutive time steps.
- Scheduling decisions are made at the **beginning** of the phase.
- Jobs arriving during the phase are **ignored**.

## Simple algorithm



**ALG1:** Phase  $P_n$ :  $Q_n = \{\text{unscheduled jobs } J_i \text{ with } a_i \leq (n-1)k\}$

Schedule jobs of  $Q_n$  in order of **non-increasing** per-unit value

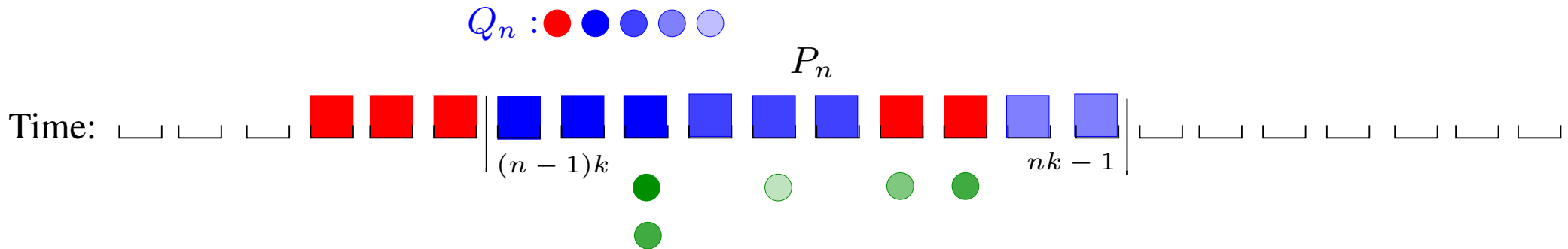
**Preempt** job at the end of  $P_n$

**Thm:**  $c = 1/(\beta^{k-1}(1 - \beta^k))$

$c = 4/(2 - \beta)$  for  $k = \lceil -\log_\beta 2 \rceil$

$c = 1/(1 - \beta)$  for  $k = 1$

## Refined algorithm



**ALG2:** Phase  $P_n$ :  $J_n = \text{remainder of last job in } P_{n-1}$

$$Q_n = \{J_n + \text{unscheduled jobs } J_i \text{ with } a_i \leq (n-1)k\}$$

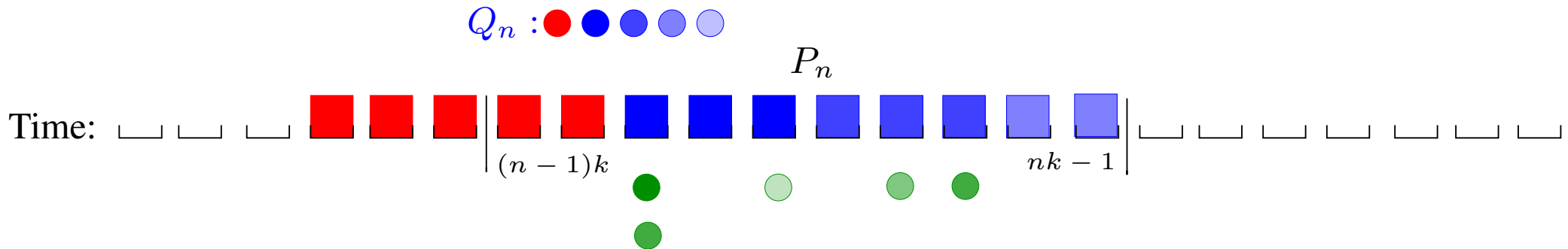
Schedule jobs of  $Q_n$  in order of **non-increasing** per-unit value

If  $J_n$  contained in schedule, move it to the front

**Thm:**  $c = 1/\beta^{k-1} \cdot \max\{1/\beta^{k-1}, 1/(1 - \beta^{2k}), \beta^{3k}/(1 - \beta^k)\}$

$c = 1 + \Phi \approx 2.618$ , where  $\Phi = (1 + \sqrt{5})/2$  for  $k = \lfloor -\frac{1}{2} \log_\beta(1 + \Phi) \rfloor + 1$

## Refined algorithm



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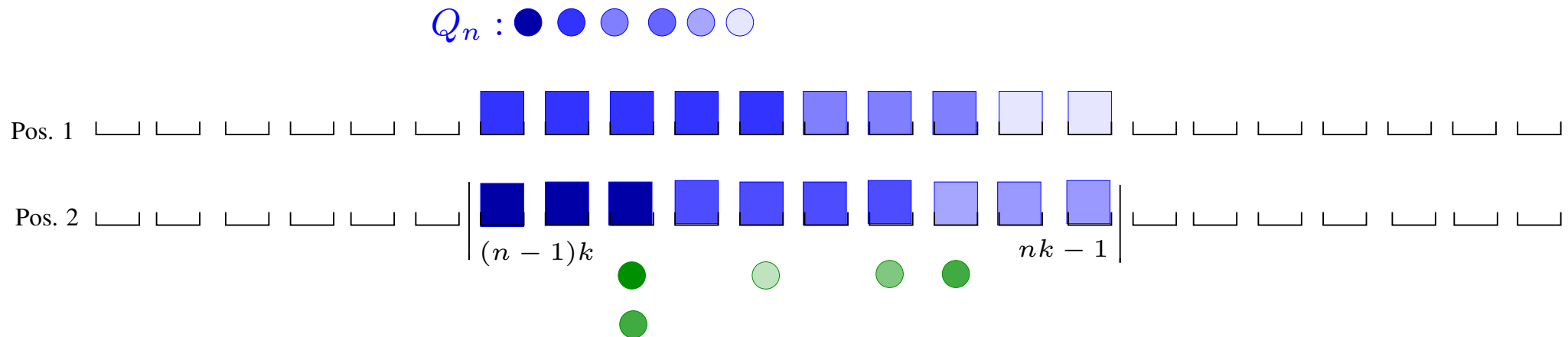
Schedule jobs of  $Q_n$  in order of **non-increasing** per-unit value

If  $J_n$  contained in schedule, move it to the front

**Thm:**  $c = 1/\beta^{k-1} \cdot \max\{1/\beta^{k-1}, 1/(1 - \beta^{2k}), \beta^{3k}/(1 - \beta^k)\}$

$c = 1 + \Phi \approx 2.618$ , where  $\Phi = (1 + \sqrt{5})/2$  for  $k = \lfloor -\frac{1}{2} \log_\beta(1 + \Phi) \rfloor + 1$

## Algorithm for $m$ ad positions



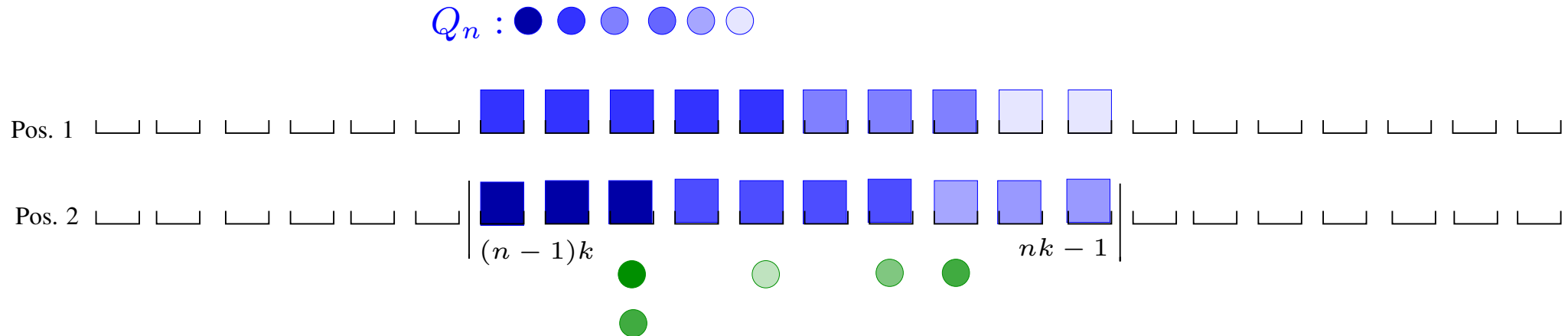
**ALG3:** Phase  $P_n$ :  $Q_n = \{\text{unscheduled jobs } J_i \text{ with } a_i \leq (n-1)k\}$

For  $t = (n-1)k, \dots, nk-1$ , schedule  $m$  jobs of highest per-unit value

Preempt jobs at the end of  $P_n$

Implementation: Units of a job are placed on same ad position

# Algorithm for $m$ ad positions



**ALG3:** Phase  $P_n$ :  $Q_n = \{\text{unscheduled jobs } J_i \text{ with } a_i \leq (n-1)k\}$

For  $t = (n-1)k, \dots, nk-1$ , schedule  $m$  jobs of highest per-unit value

Preempt jobs at the end of  $P_n$

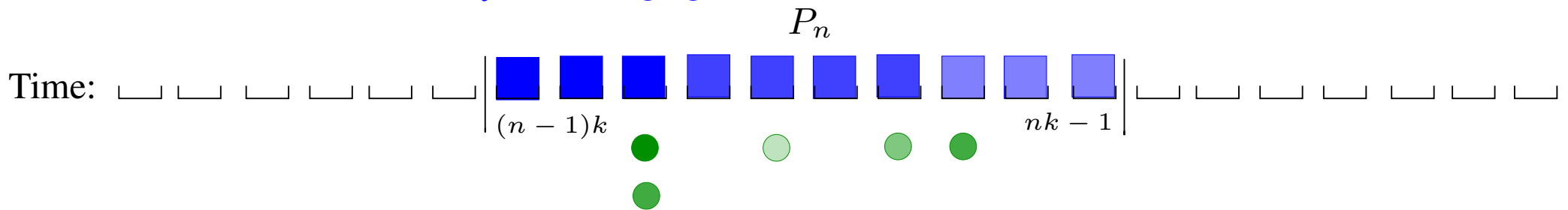
**Thm:**  $c = 1/\beta^{k-1} \cdot (1 + 1/(1 - \beta^k))$

$c = (1 + 1/(1 - \beta(2 - \sqrt{2}))) / (2 - \sqrt{2})$  for  $k = \lceil \log_\beta(2 - \sqrt{2}) \rceil$

$3.414 \approx 2/(2 - \sqrt{2}) \leq c \leq 1/(3 - 2\sqrt{2}) \approx 5.828$

## Simple algorithm

$Q_n$  : ● ● ● ●



**ALG1:** Phase  $P_n$ :  $Q_n = \{\text{unscheduled jobs } J_i \text{ with } a_i \leq (n-1)k\}$

Schedule jobs of  $Q_n$  in order of **non-increasing** per-unit value

Jobs of same value sorted in order of increasing arrival time

**Preempt** job at the end of  $P_n$

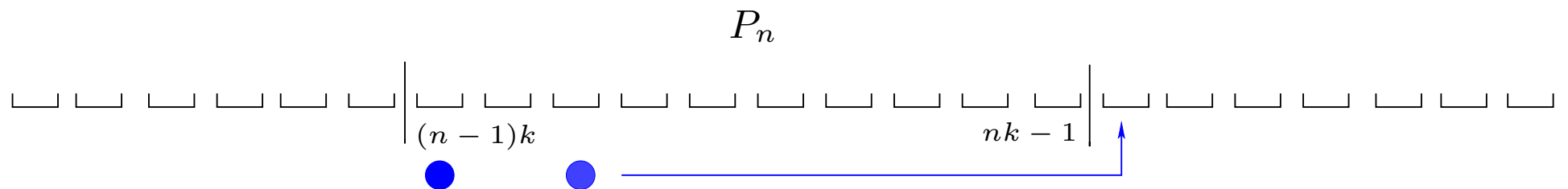


## Analysis simple algorithm

$$I = J_1, \dots, J_N \quad J_i = (a_i, l_i, v_i)$$

$k$ -quantized input

$$I_k = J'_1, \dots, J'_N \quad J'_i = (a'_i, l_i, v_i) \quad a'_i = k \lceil a_i/k \rceil$$



## Analysis simple algorithm

**Lemma:**  $OPT(I_k) \geq \beta^{k-1} \cdot OPT(I)$

**Proof:** Shift optimal schedule for  $I$  by  $k - 1$  time units to the right.

**Observation:**  $ALG1(I_k) = ALG1(I)$

## Relaxed offline algorithm

*CHOP*: Optimal algorithm that may resume preempted jobs

Always schedule job with highest per-unit value.  
Jobs with same per-unit value are scheduled  
in the **same order** as in ALG1.

**Observation:**  $CHOP(I_k) \geq OPT(I_k)$

## Timing property

$S$  = ALG1's schedule for  $I_k$

$t_S(i)$  = start time of  $J_i$  in  $S$

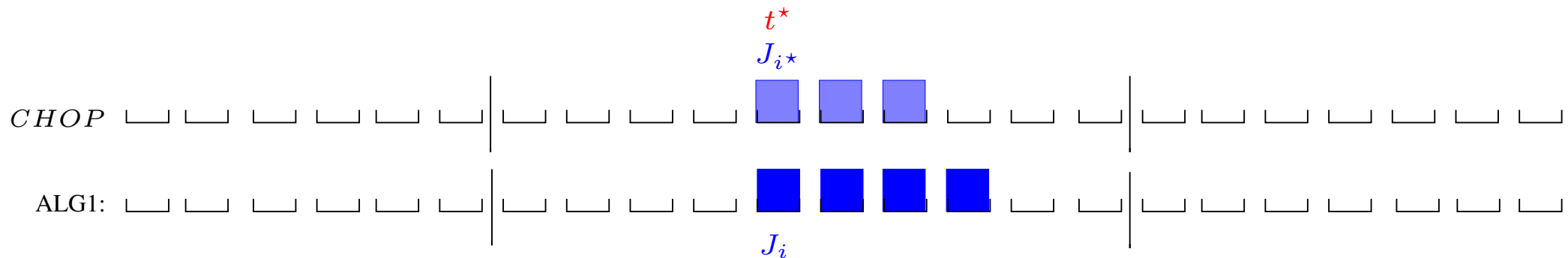
$S^*$  = CHOP's schedule for  $I_k$

$t_{S^*}(i)$  = start time of  $J_i$  in  $S^*$

**Lemma:**  $t_S(i) \leq t_{S^*}(i)$

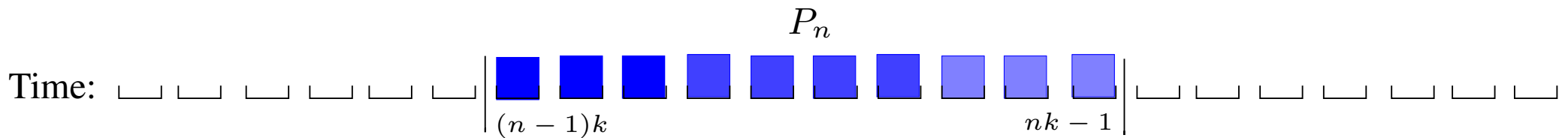
$$t_S(i) \leq t_{S^*}(i)$$

Let  $J_{i^*}$  be first job in  $S^*$  with  $t_S(i^*) > t_{S^*}(i^*) = t^*$



- $v_i \geq v_{i^*}$
- At time  $t^*$  *CHOP* has finished  $J_i$ 
  - $v_i > v_{i^*} : \checkmark$
  - $v_i = v_{i^*} : CHOP$  schedules jobs of value  $v_i$  in the same order as *ALG1*
- $t_{S^*}(i) \leq t^* - l_i \quad t_S(i) \geq t^* - l_i + 1$

## Phase analysis



$\mathcal{J}_n = \{\text{jobs scheduled by ALG1 in } P_n\}$

$\text{ALG1}(P_n) = \text{value achieved for } \mathcal{J}_n$

$$\text{ALG1}(I_k) = \sum_n \text{ALG1}(P_n)$$

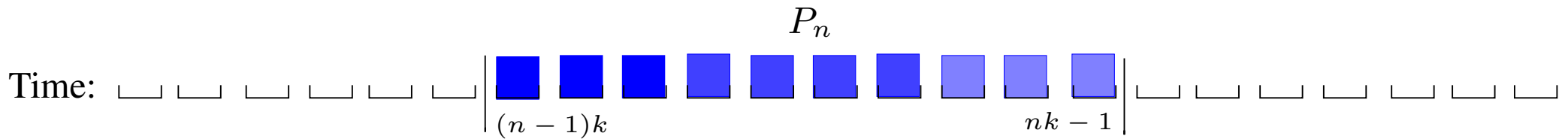
$\text{CHOP}(P_n) = \text{value achieved for } \mathcal{J}_n$

**Lemma:**  $\text{CHOP}(I_k) = \sum_n \text{CHOP}(P_n)$

**Proof:** For any  $J_i$  scheduled by  $\text{CHOP}$ ,  $t_S(i) \leq t_{S^*}(i)$

$$\{\text{jobs scheduled by } \text{CHOP}\} \subseteq \bigcup_n \mathcal{J}_n$$

## Phase analysis



**Lemma:**  $CHOP(P_n) \leq ALG1(P_n)/(1 - \beta^k)$

**Proof:**  $J$  = last job of  $P_n$  with value  $v$

$CHOP(P_n) \leq ALG1(P_n) + \text{extra value}$  in scheduling preempted portion of  $J$

$$ALG1(P_n) = \sum_{t=(n-1)k}^{nk-1} \beta^t v = (\beta^{(n-1)k} - \beta^{nk}) / (1 - \beta) \cdot v$$

$$\text{extra value} = \sum_{t \geq nk} \beta^t v = \beta^{nk} / (1 - \beta) \cdot v$$

$$CHOP(P_n) / ALG1(P_n) \leq 1 + \beta^{nk} / (\beta^{(n-1)k} - \beta^{nk}) = 1 / (1 - \beta^k)$$

## Wrapping up

$$CHOP(I_k) \leq \text{ALG1}(I_k)/(1 - \beta^k)$$

$$\beta^{k-1} \cdot \text{OPT}(I) \leq CHOP(I_k)$$

$$\text{OPT}(I) \leq \text{ALG1}(I)/(\beta^{k-1}(1 - \beta^k))$$



## Analysis refined algorithm

- Take care of **delays** if last job of **previous phase** is continued.
- Loss of **preempted** jobs have to amortized over **several phases**; segments of up to three phases.

## Analysis $m$ ad positions

- $\exists S^*$  value is at least as high as that of optimal schedule
- $S^*$  schedules up to  $2m$  jobs at any time
- $\forall t$ : each  $J_i$  scheduled in  $S^*$  but not in  $S$  can be mapped
  - (a) to a unit  $v_{i'} \geq v_i$  scheduled in  $S$  or
  - (b) to a job preempted by  $S$  at time  $t' < t$

## Open problems

- Tight bounds for deterministic algorithms
- Design randomized algorithms