# Two manifestations of rigidity phenomena in random point sets : forbidden regions and maximal degeneracy

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- Namely, given a domain $D$, how does the point configuration outside of $D$ impact the distribution of the points inside $D$ ?
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- This has implications in the study of stochastic geometry on these point processes, notably in the use of Burton and Keane type arguments, or the "finite energy" property.

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- Rigidity of particle numbers was also established for the zeros of the planar Gaussian analytic function [G. - Peres]

$$f(z) = \sum_{k=0}^{\infty} \xi_k \frac{z^k}{\sqrt{k!}}.$$

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- Projection kernel in the above is necessary ! [G.-Krishnapur]

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- Natural to ask about rigidity of more general functionals of a point process (other than the particle count), particularly higher moments of the points in $D$.

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- These are the only rigid observables !
- For the standard planar case ($\alpha = 1$), this implies that the total mass as well as the centre of mass of the points in a bounded domain are determined by the outside point configuration.
- In particular, if there happens to be only one point in a bounded domain (an event of positive probability), then the exact location of that point is completely determined by the outside configuration.

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- Rigidity of general observables connected with suppressed fluctuation of other linear statistics.
- Rigidity is also connected with faster decay of hole probabilities and singularity of Palm measures

- (Moment-matching) [G.] Consider a point process $\Pi$ having precisely the first $m$ moments rigid, and two collections of points $\underline{\zeta} = (\zeta_1, \cdots, \zeta_k)$ and $\underline{\eta} = (\eta_1, \cdots, \eta_l)$. Then Palm measures $[\Pi]_{\underline{\zeta}}$ and $[\Pi]_{\underline{\eta}}$ are mutually absolutely continuous iff the first $m$ moments of $\zeta$ and $\eta$ match,

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- However, very few rigorous theorems establishing general implications like the above between these concepts.

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- In other words, what causes a large hole (a rare event) to occur ?
- The most interesting scale to look at this question turns out to be the scale when the "hole" is rescaled to have size 1.

# Conditioning on a large hole: the Ginibre ensemble

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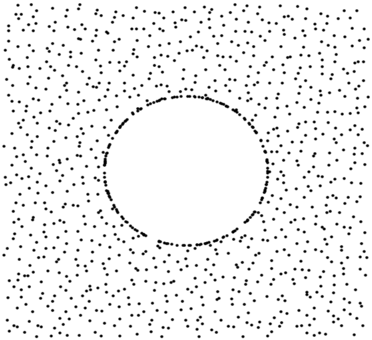

Ginibre Ensemble

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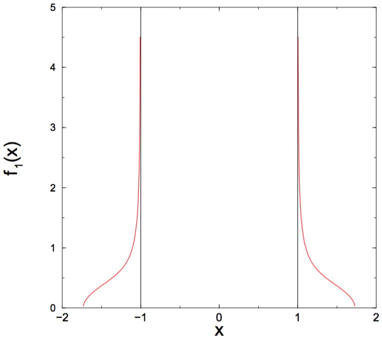


GUE

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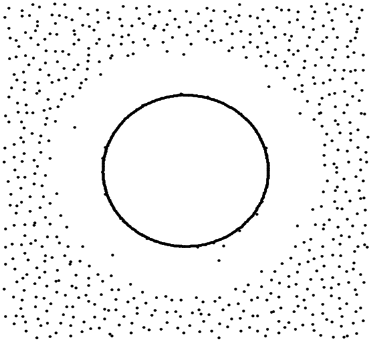

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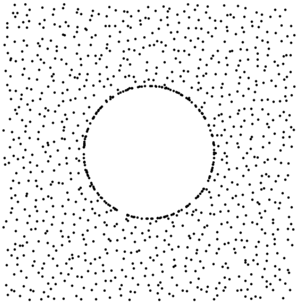

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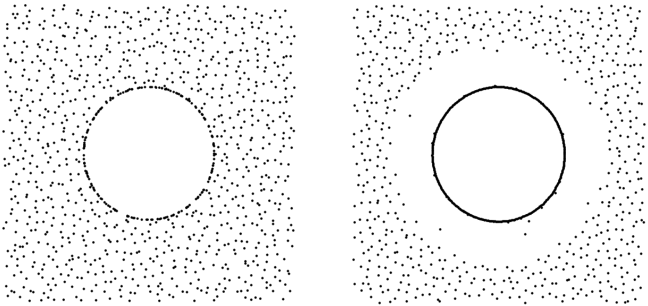

# Heuristics

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- No zeros in the hole $D$ is the same as $\underline{Z}(D) = 0$.
- To find the "most likely configuration" given that there is hole is roughly the same as minimizing the rate functional $I$ over the space of probability measures (on $\mathbb{C}$) under the constraint that there is zero mass on $D$.

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- Heuristics made rigorous by obtaining "effective" versions of large deviation estimates and approximating the analytic function zeros by those of the polynomials.

# Stealthy random fields

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- Stealthy particle systems conjectured to have deterministically bounded holes [Zhang-Stillinger-Torquato].

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Special case : Guassian process with a gap (or fast decay) in the spectrum

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# Heuristics

- The existence of a gap / fast decay in the spectrum can be exploited to construct linear functionals of the process which have low variance.
- A linear functional with a low variance is approximately constant, so this gives an approximate linear constraint
- Sufficiently rich class of such constraints can be exploited to deduce degenerate behaviour.

Thank you !!