Polar Coding

Part 2 - Construction and Performance

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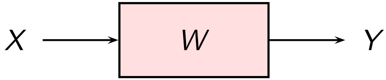
The channel

Let $W : X \to Y$ be a binary-input discrete memoryless channel



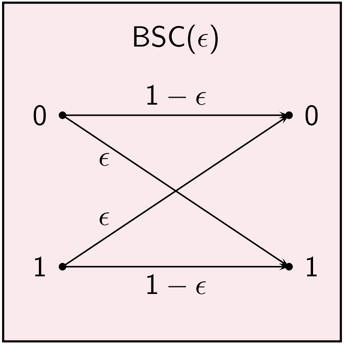
- input alphabet: $\mathcal{X} = \{0, 1\}$,
- output alphabet: $\mathcal{Y}$,
- transition probabilities:

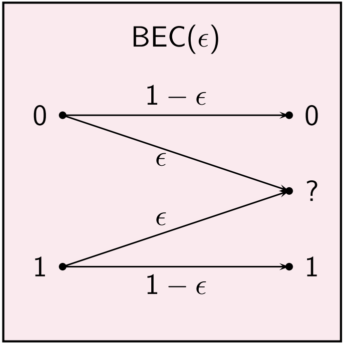
$$W(y|x), \quad x \in \mathcal{X}, y \in \mathcal{Y}$$

Symmetry assumption

Assume that the channel has "input-output symmetry."

Examples:





Capacity

For channels with input-output symmetry, the capacity is given by

$$C(W) \stackrel{\Delta}{=} I(X;Y), \quad \text{with } X \sim \text{unif. } \{0,1\}$$

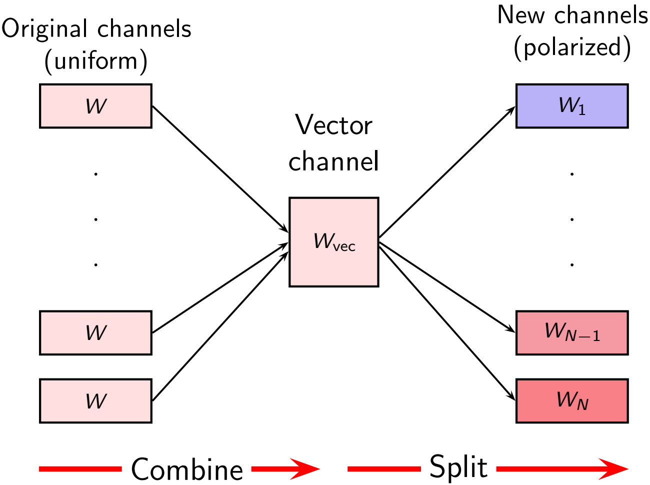
Use base-2 logarithms:

$$0 \leq C(W) \leq 1$$

The main idea

- Channel coding problem trivial for two types of channels
  - Perfect: $C(W) = 1$
  - Useless: $C(W) = 0$
- Transform ordinary W into such extreme channels

The method: aggregate and redistribute capacity



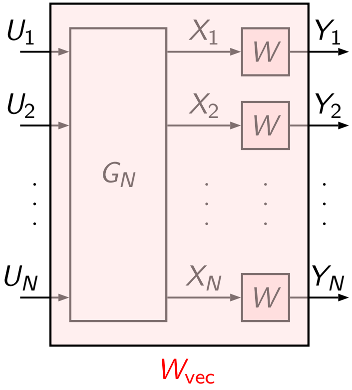
Combining

- Begin with N copies of W,
- use a 1-1 mapping

$$G_N : \{0,1\}^N \to \{0,1\}^N$$

- to create a vector channel

$$W_{\text{vec}} : U^N \to Y^N$$

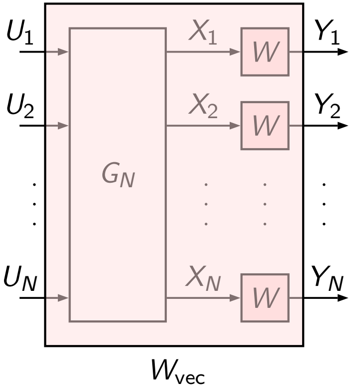


Conservation of capacity

Combining operation is lossless:

- Take $U_1, \ldots, U_N$ i.i.d. unif. $\{0, 1\}$
- then, $X_1, \ldots, X_N$ i.i.d. unif. $\{0, 1\}$
- and

$$\begin{aligned} C(W_{\text{vec}}) &= I(U^N; Y^N) \\ &= I(X^N; Y^N) \\ &= NC(W) \end{aligned}$$

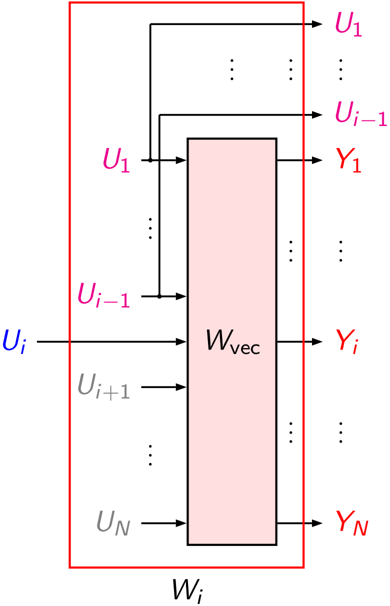


Splitting

$$C(W_{\text{vec}}) = I(U^N; Y^N)$$
$$= \sum_{i=1}^{N} I(U_i; Y^N, U^{i-1})$$
$$= \sum_{i=1}^{N} C(W_i)$$

Define bit-channels

$$W_i : U_i \to (Y^N, U^{i-1})$$

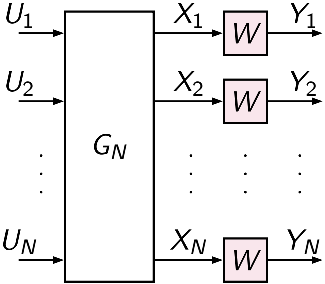


Polarization is commonplace

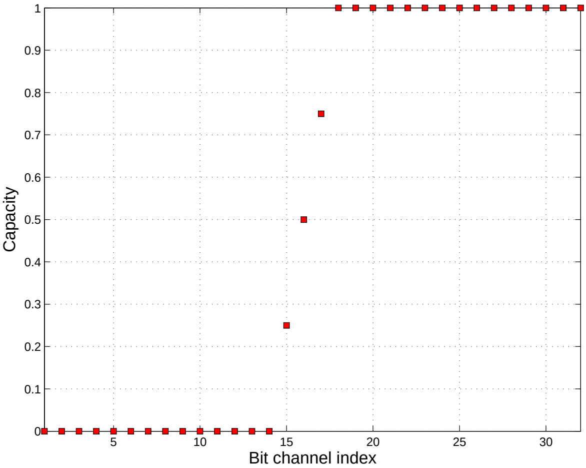
- Polarization is the rule not the exception
- A random permutation

$$G_N : \{0,1\}^N \to \{0,1\}^N$$

  is a good polarizer with high probability
- Equivalent to Shannon's random coding approach



Random polarizers: stepwise, isotropic



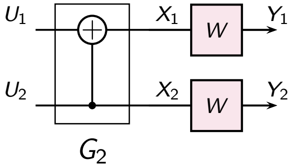
Isotropy: any redistribution order is as good as any other.

The complexity issue

- Random polarizers lack structure, too complex to implement
- Need a low-complexity polarizer
- May sacrifice stepwise, isotropic properties of random polarizers in return for less complexity

Basic module for a low-complexity scheme

Combine two copies of W

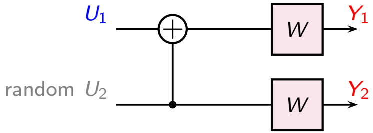


and split to create two bit-channels

$$W_1 : U_1 \to (Y_1, Y_2)$$
$$W_2 : U_2 \to (Y_1, Y_2, U_1)$$

The first bit-channel W_1

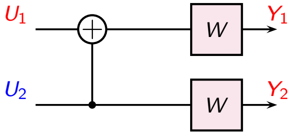
$$W_1 : U_1 \to (Y_1, Y_2)$$



$$C(W_1) = I(U_1; Y_1, Y_2)$$

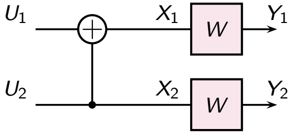
The second bit-channel W_2

$$W_2 : U_2 \to (Y_1, Y_2, U_1)$$



$$C(W_2) = I(U_2; Y_1, Y_2, U_1)$$

Capacity conserved but redistributed unevenly



- Conservation:

$$C(W_1) + C(W_2) = 2C(W)$$

- Extremization:

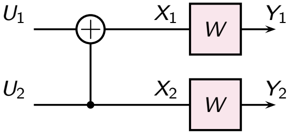
$$C(W_1) \leq C(W) \leq C(W_2)$$

with equality iff $C(W)$ equals 0 or 1.

Extremality of BEC

$$H(U_1|Y_1 Y_2) \leq H(X_1|Y_1) + H(X_2|Y_2) - H(X_1|Y_1)H(X_2|Y_2)$$

with equality iff W is a BEC.

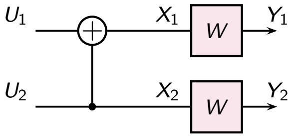


Extremality of BSC (Mrs. Gerber's lemma)

Let $\mathcal{H}^{-1} : [0, \frac{1}{2}] \to [0, \frac{1}{2}]$ be the inverse of the binary entropy function $\mathcal{H}(p) = -p\log(p) - (1-p)\log(1-p)$, $0 \le p \le \frac{1}{2}$.

$$H(U_1|Y_1Y_2) \ge \mathcal{H}\big(\mathcal{H}^{-1}(H(X_1|Y_1)) * \mathcal{H}^{-1}(H(X_2|Y_2))\big)$$

with equality iff W is a BSC.



Notation

The two channels created by the basic transform

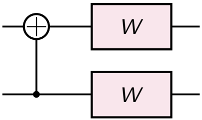
$$(W, W) \to (W_1, W_2)$$

will be denoted also as

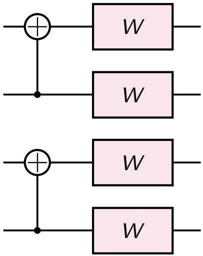
$$W^- = W_1 \quad \text{and} \quad W^+ = W_2$$

Likewise, we write W^{--}, W^{-+} for descendants of W^-; and W^{+-}, W^{++} for descendants of W^+.

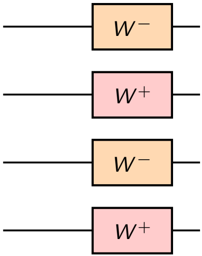
For the size-4 construction



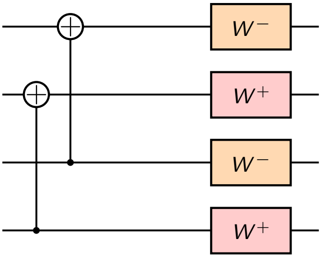
… duplicate the basic transform



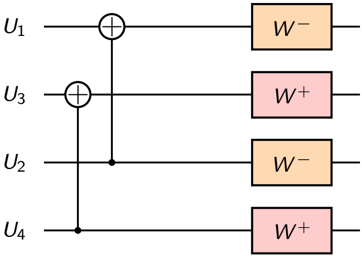
… obtain a pair of W^- and W^+ each



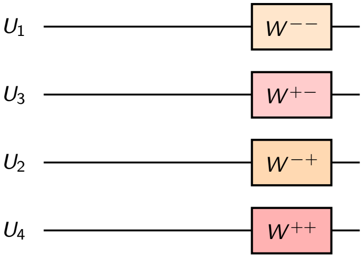
… apply basic transform on each pair



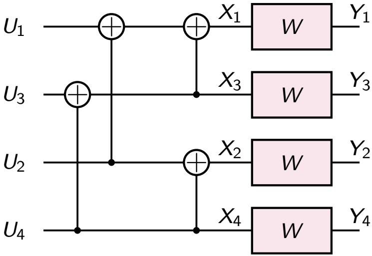
… decode in the indicated order



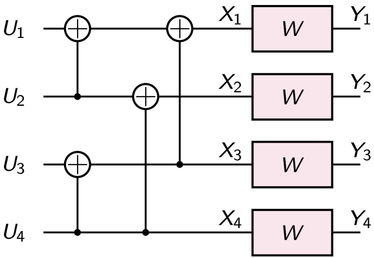
... obtain the four new bit-channels



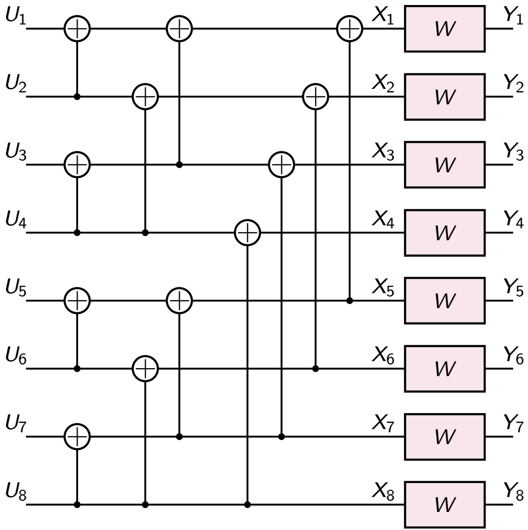
Overall size-4 construction



"Rewire" for standard-form size-4 construction



Size 8 construction



Polarization of a BEC W

Polarization is easy to analyze when W is a BEC.

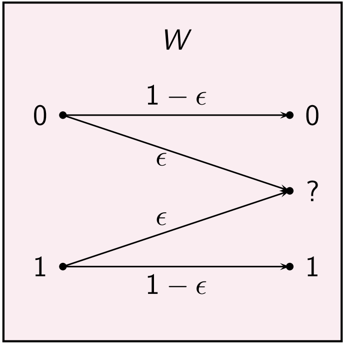
If W is a BEC(ϵ), then so are W^- and W^+, with erasure probabilities

$$\epsilon^- \triangleq 2\epsilon - \epsilon^2$$

and

$$\epsilon^+ \triangleq \epsilon^2$$

respectively.



The first bit channel W^-

The first bit channel W^- is a BEC.

If W is a BEC(ϵ), then so are W^- and W^+, with erasure probabilities

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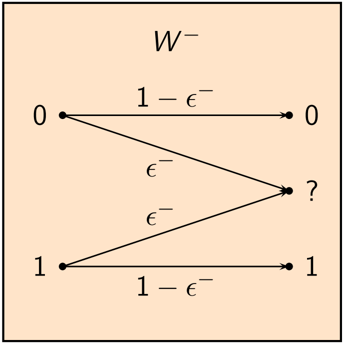


The second bit channel W^+

The second bit channel W^+ is a BEC.

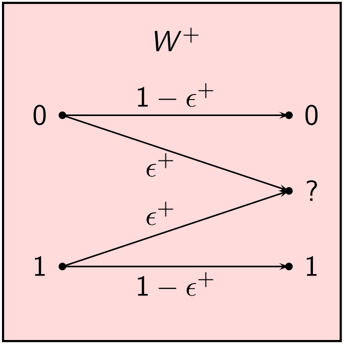
If W is a BEC(ϵ), then so are W^- and W^+, with erasure probabilities

$$\epsilon^- \stackrel{\Delta}{=} 2\epsilon - \epsilon^2$$

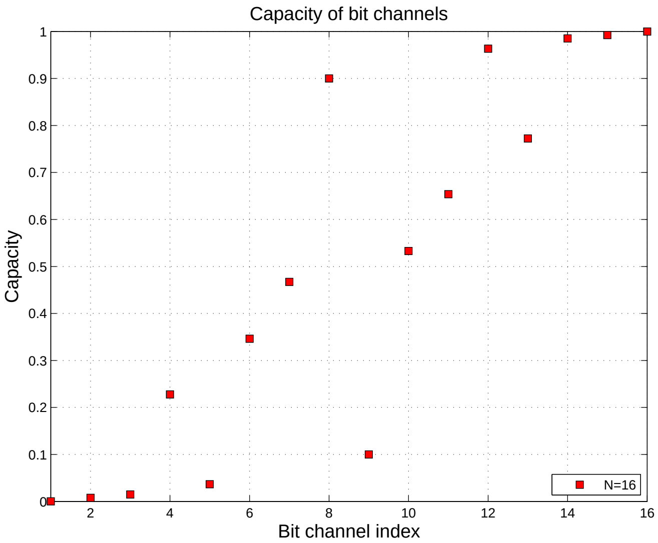
and

$$\epsilon^+ \stackrel{\Delta}{=} \epsilon^2$$

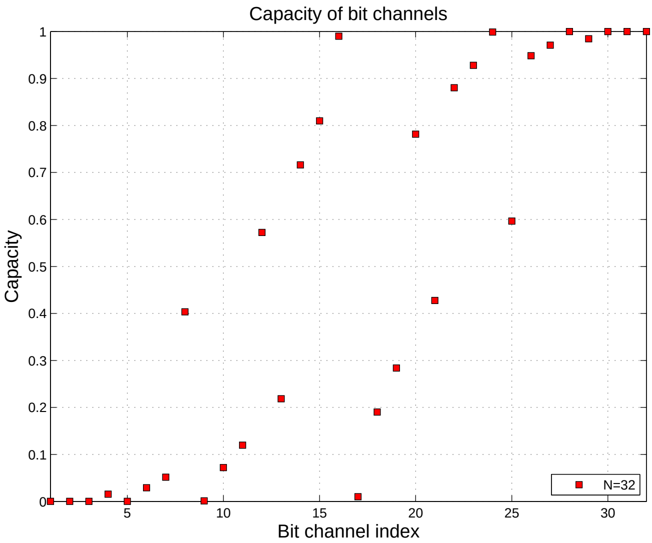
respectively.



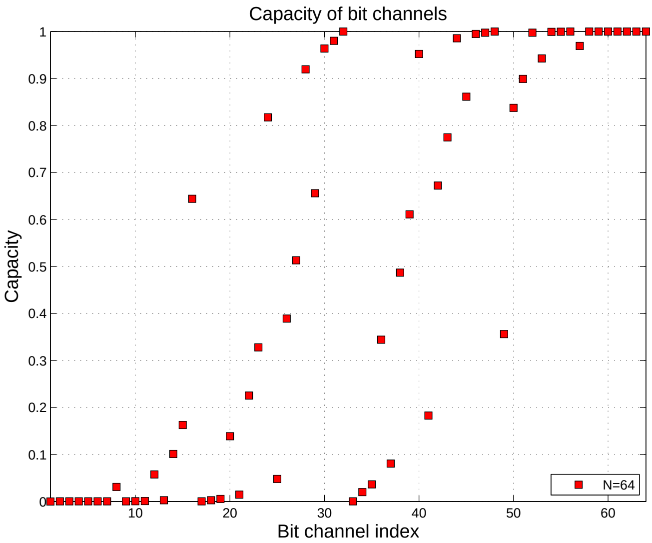
Polarization for BEC($\frac{1}{2}$): $N = 16$



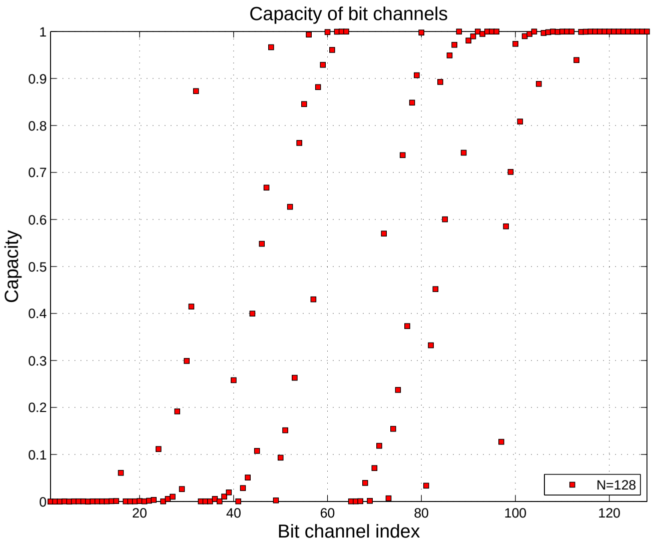
Polarization for BEC($\frac{1}{2}$): $N = 32$



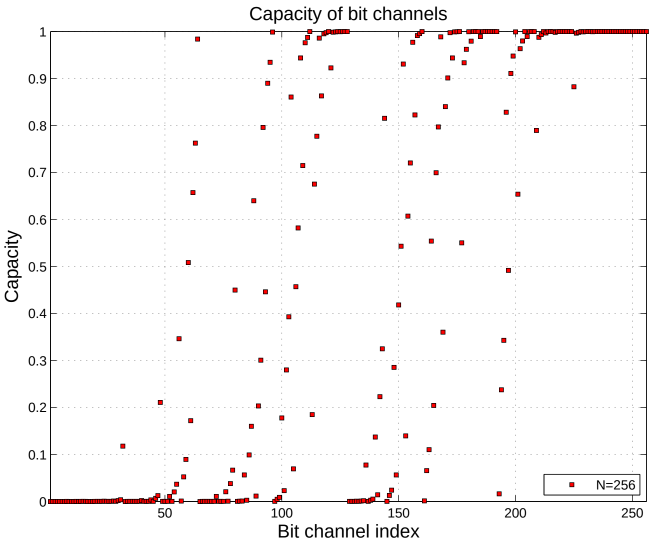
Polarization for BEC($\frac{1}{2}$): $N = 64$



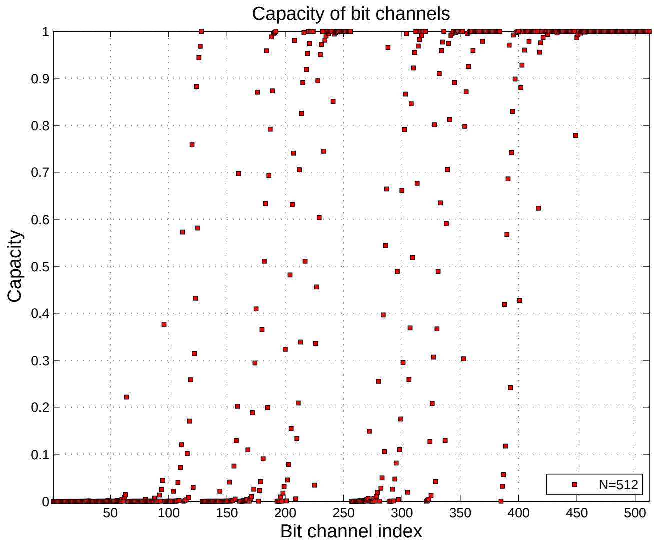
Polarization for BEC($\frac{1}{2}$): $N = 128$



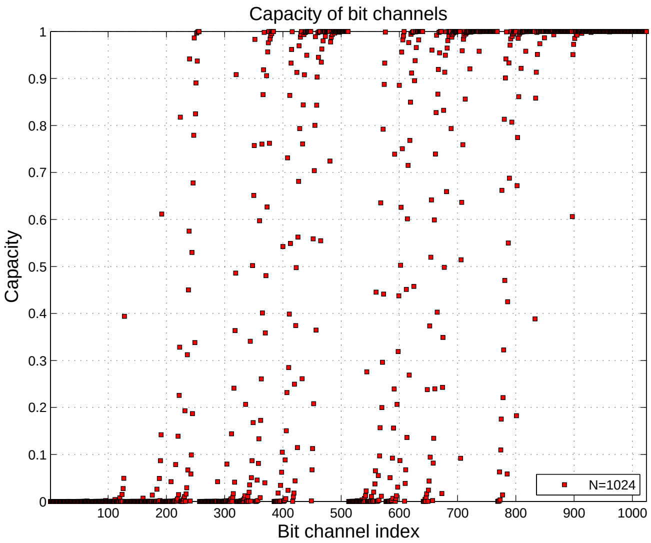
Polarization for BEC($\frac{1}{2}$): $N = 256$



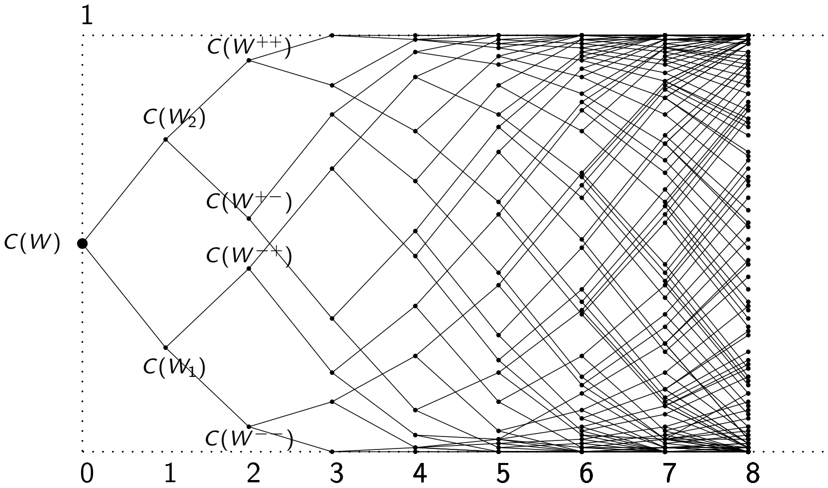
Polarization for BEC($\frac{1}{2}$): $N = 512$



Polarization for BEC($\frac{1}{2}$): $N = 1024$



Polarization martingale



Theorem (Polarization, A. 2007)

The bit-channel capacities $\{C(W_i)\}$ *polarize: for any* $\delta \in (0,1)$*, as the construction size N grows*

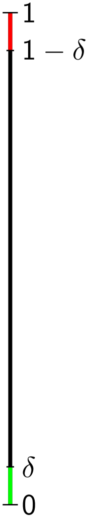
$$\left[\frac{\text{no. channels with } C(W_i) > 1-\delta}{N}\right] \longrightarrow C(W)$$

and

$$\left[\frac{\text{no. channels with } C(W_i) < \delta}{N}\right] \longrightarrow 1 - C(W)$$

Theorem (Rate of polarization, A. and Telatar (2008))

Above theorem holds with $\delta \approx 2^{-\sqrt{N}}$.



Channel polarization

Polar coding

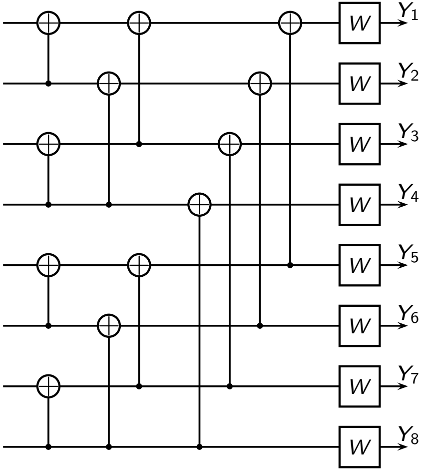
Polar codes for selected applications

Polar coding

- Code construction
- Encoding
- Decoding
- Performance

Polar code example: $W = \text{BEC}(\frac{1}{2})$, $N = 8$, rate $1/2$

$I(W_i)$	Rank	
0.0039	8	frozen U_1
0.1211	7	frozen U_2
0.1914	6	frozen U_3
0.6836	4	data U_4
0.3164	5	frozen U_5
0.8086	3	data U_6
0.8789	2	data U_7
0.9961	1	data U_8



Polar code example: $W = \mathrm{BEC}(\frac{1}{2})$, $N = 8$, rate $1/2$

$I(W_i)$	Rank
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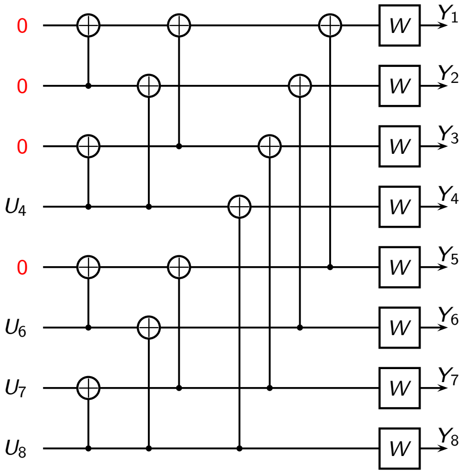


Encoding complexity

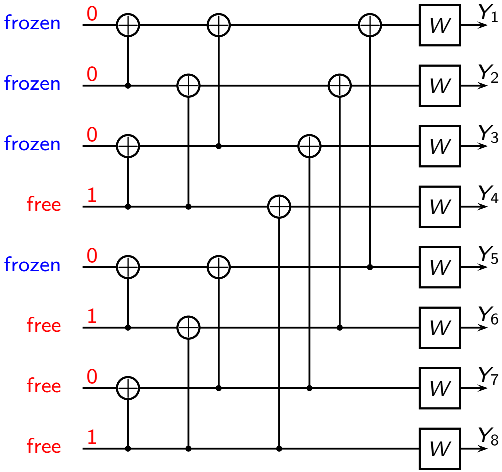
Theorem

Encoding complexity for polar coding is $\mathcal{O}(N \log N)$.

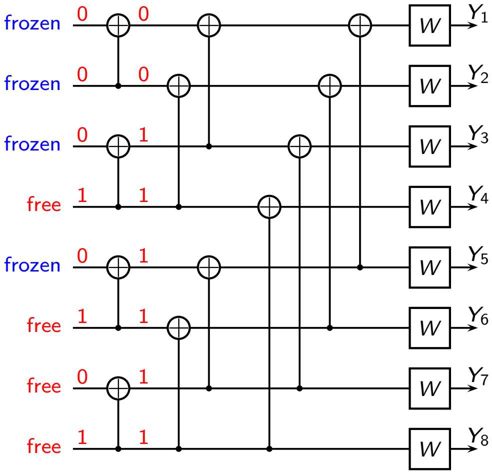
Proof:

- Polar coding transform can be represented as a graph with $N[1 + \log(N)]$ variables.
- The graph has $(1 + \log(N))$ levels with N variables at each level.
- Computation begins at the source level and can be carried out level by level.
- Space complexity $O(N)$, time complexity $O(N \log N)$.

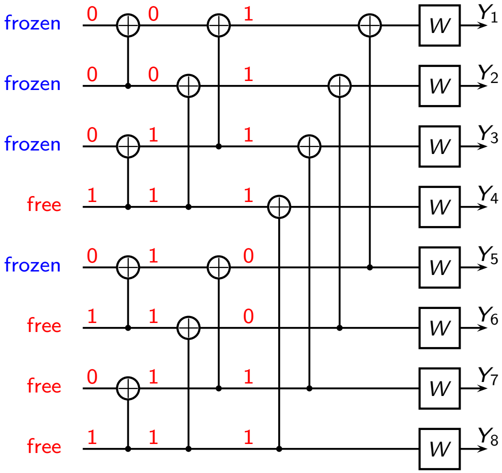
Encoding: an example



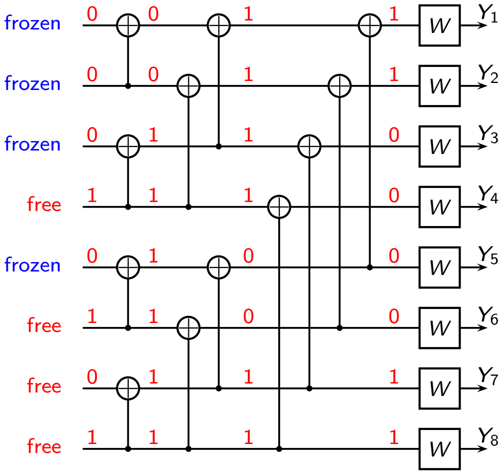
Encoding: an example



Encoding: an example



Encoding: an example



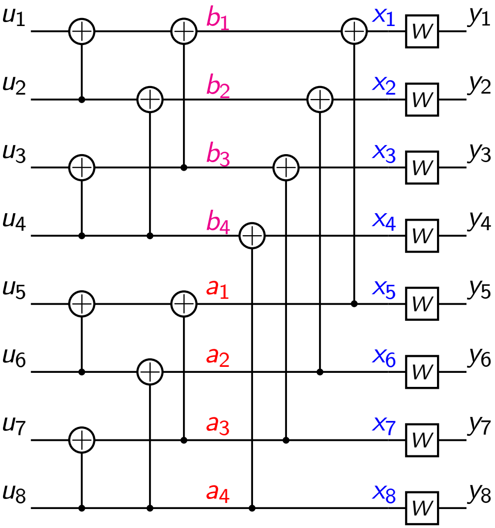
Successive Cancellation Decoding (SCD)

Theorem

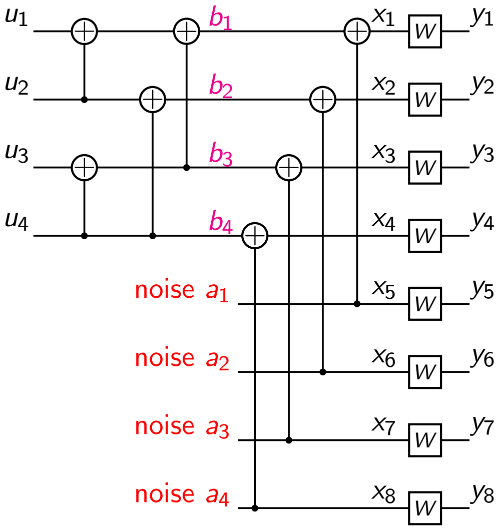
The complexity of successive cancellation decoding for polar codes is $\mathcal{O}(N \log N)$.

Proof: Given below.

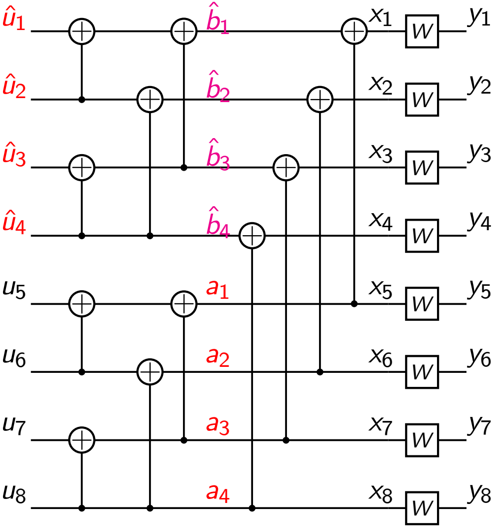
SCD: Exploit the $\mathbf{x} = |\mathbf{a}|\mathbf{a}+\mathbf{b}|$ structure



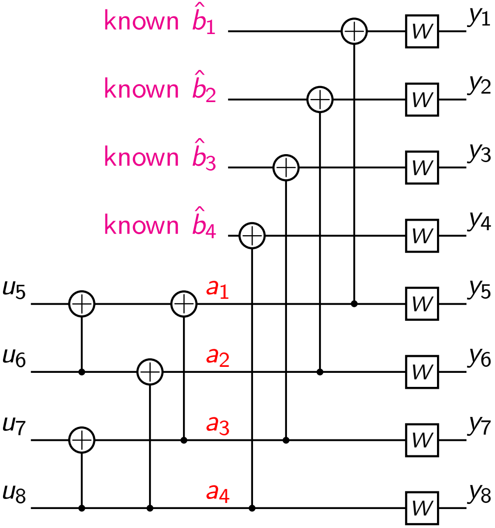
First phase: treat **a** as noise, decode (u_1, u_2, u_3, u_4)



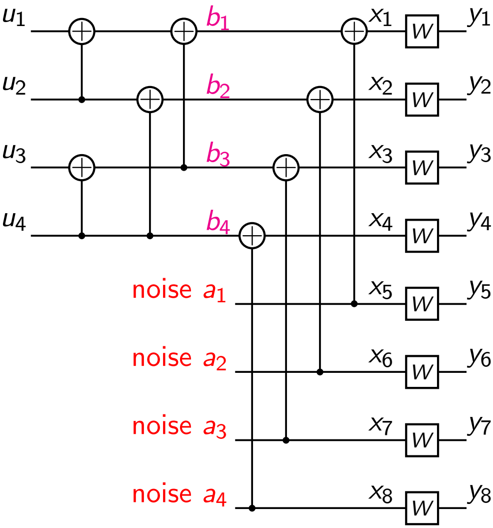
End of first phase



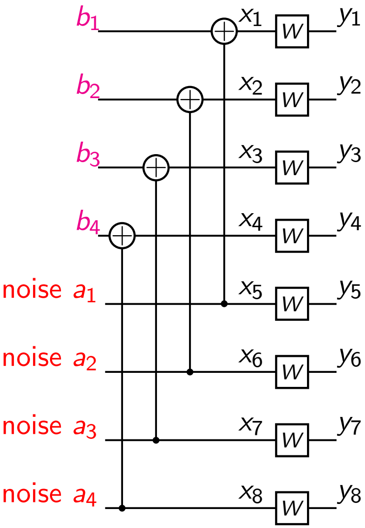
Second phase: Treat $\hat{\mathbf{b}}$ as known, decode (u_5, u_6, u_7, u_8)



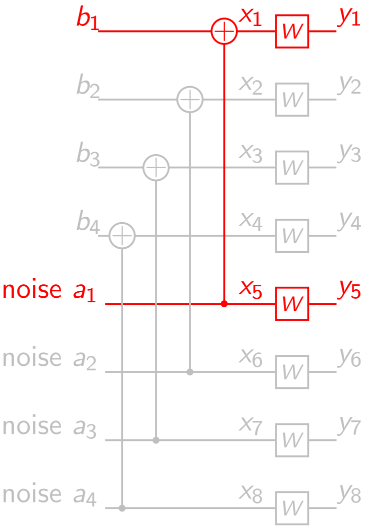
First phase in detail



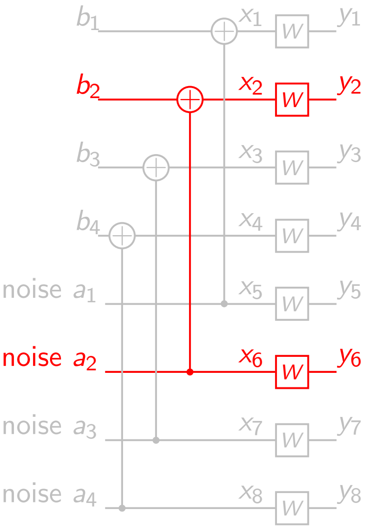
Equivalent channel model



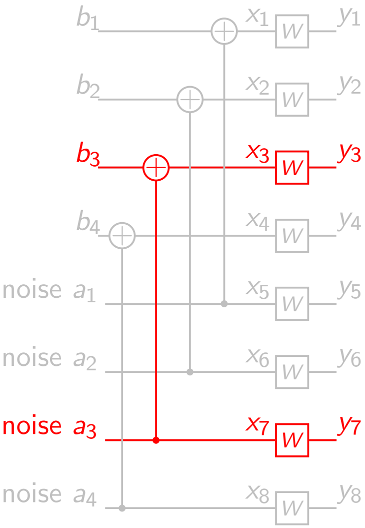
First copy of W^-



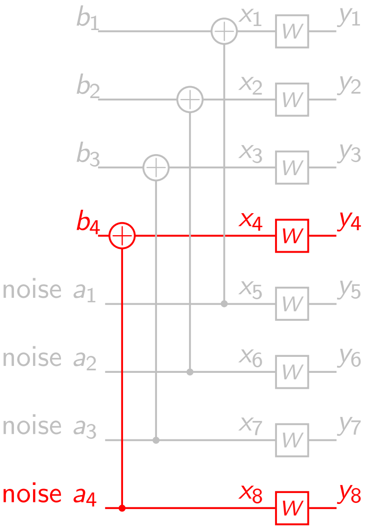
Second copy of W^-



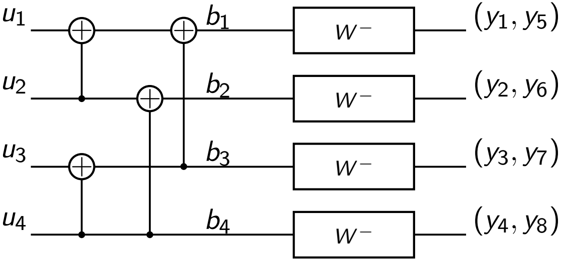
Third copy of W^-



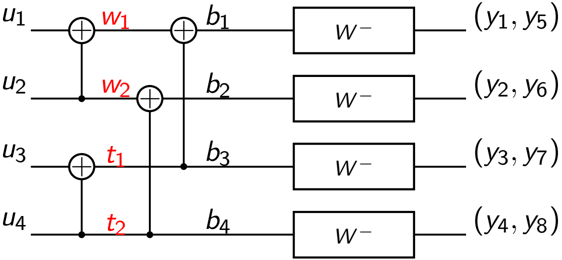
Fourth copy of W^-



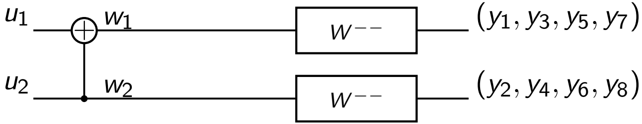
Decoding on W^-



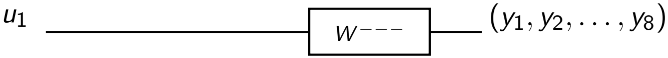
$\mathbf{b} = |\mathbf{t}|\mathbf{t} + \mathbf{w}|$



Decoding on W^{--}



Decoding on W^{---}



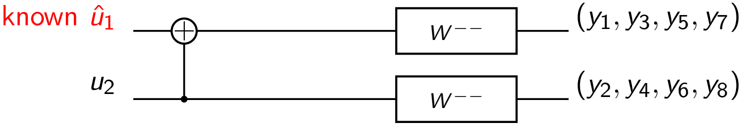
Compute

$$L^{---} \triangleq \frac{W^{---}(y_1, \ldots, y_8 \mid u_1 = 0)}{W^{---}(y_1, \ldots, y_8 \mid u_1 = 1)}$$

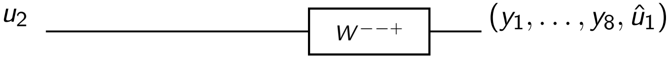
and set

$$\hat{u}_1 = \begin{cases} u_1 & \text{if } u_1 \text{ is frozen} \\ 0 & \text{else if } L^{---} > 0 \\ 1 & \text{else} \end{cases}$$

Decoding on W^{--+}



Decoding on W^{--+}



Compute

$$L^{--+} \triangleq \frac{W^{--+}(y_1, \ldots, y_8, \hat{u}_1 \mid u_2 = 0)}{W^{--+}(y_1, \ldots, y_8, \hat{u}_1 \mid u_2 = 1)}$$

and set

$$\hat{u}_2 = \begin{cases} u_2 & \text{if } u_2 \text{ is frozen} \\ 0 & \text{else if } L^{--+} > 0 \\ 1 & \text{else} \end{cases}$$

Complexity for successive cancelation decoding

- Let C_N be the complexity of decoding a code of length N
- Decoding problem of size N for W reduced to two decoding problems of size $N/2$ for W^- and W^+
- So
$$C_N = 2C_{N/2} + kN$$
for some constant k
- This gives $C_N = \mathcal{O}(N \log N)$

Performance of polar codes

Probability of Error (A. and Telatar (2008)

For any binary-input symmetric channel W, the probability of frame error for polar coding at rate $R < C(W)$ and using codes of length N is bounded as

$$P_e(N, R) \leq 2^{-N^{0.49}}$$

for sufficiently large N.

A more refined versions of this result has been given given by S. H. Hassani, R. Mori, T. Tanaka, and R. L. Urbanke (2011).

Construction complexity

Construction Complexity

Polar codes can be constructed in time $\mathcal{O}(N\text{poly}(log(N)))$.

This result has been developed in a sequence of papers by

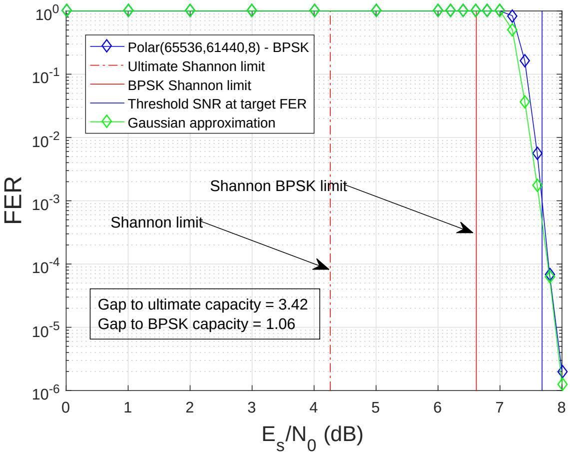
- R. Mori and T. Tanaka (2009)
- I. Tal and A. Vardy (2011)
- R. Pedarsani, S. H. Hassani, I. Tal, and E. Telatar (2011)

Gaussian approximation

- Trifonov (2011) introduced a Gaussian approximation technique for constructing polar codes
- Dai *et al.* (2015) studied various refinements of Gaussian approximation for polar code construction
- These methods work extremely well although a satisfactory explanation of why they work is still missing

Example of Gaussian approximation

Polar code construction and performance estimation by Gaussian approximation



Polar coding summary

Summary

Given W, $N = 2^n$, and $R < I(W)$, a polar code can be constructed such that it has

- construction complexity $\mathcal{O}(N\text{poly}(log(N)))$,
- encoding complexity $\approx N \log N$,
- successive-cancellation decoding complexity $\approx N \log N$,
- frame error probability $P_e(N, R) = o\left(2^{-\sqrt{N}+o(\sqrt{N})}\right)$.

Performance improvement for polar codes

- Concatenation to improve minimum distance
- List decoding to improve SC decoder performance

Concatenation

Method	Ref
Block turbo coding with polar constituents	AKMOP (2009)
Generalized concatenated coding with polar inner	AM (2009)
Reed-Solomon outer, polar inner	BJE (2010)
Polar outer, block inner	SH (2010)
Polar outer, LDPC inner	EP (ISIT'2011)

AKMOP: A., Kim, Markarian, Özgür, Poyraz

GCC: A., Markarian

BJE: Bakshi, Jaggi, and Effros

SH: Seidl and Huber

EP: Eslami and Pishro-Nik

Overview of decoders for polar codes

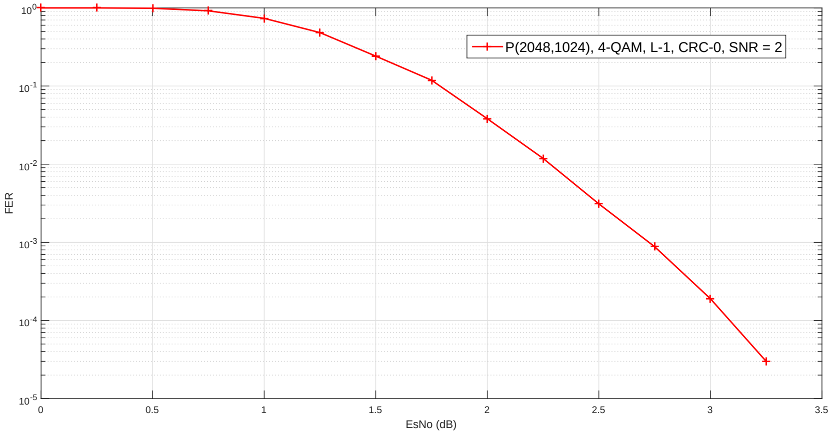
- Successive cancellation decoding: A depth-first search method with complexity roughly $N \log N$
  - Sufficient to prove that polar codes achieve capacity
  - Equivalent to an earlier algorithm by Schnabl and Bossert (1995) for RM codes
  - Simple but not powerful enough to challenge LDPC and turbo codes in short to moderate lengths
- List decoding: A breadth-first search algorithm with limited branching (known as "beam search" in AI).
  - First proposed by Tal and Vardy (2011) for polar codes.
  - List decoding was used earlier by Dumer and Shabunov (2006) for RM codes
  - Complexity grows as $O(LN \log N)$ for a list size L. But hardware implementation becomes problematic as L grows due to sorting and memory management.
- Sphere-decoding ("British Museum" search with branch and bound, starts decoding from the opposite side).

List decoder for polar codes

- First produce L candidate decisions
- Pick the most likely word from the list
- Complexity $\mathcal{O}(LN \log N)$

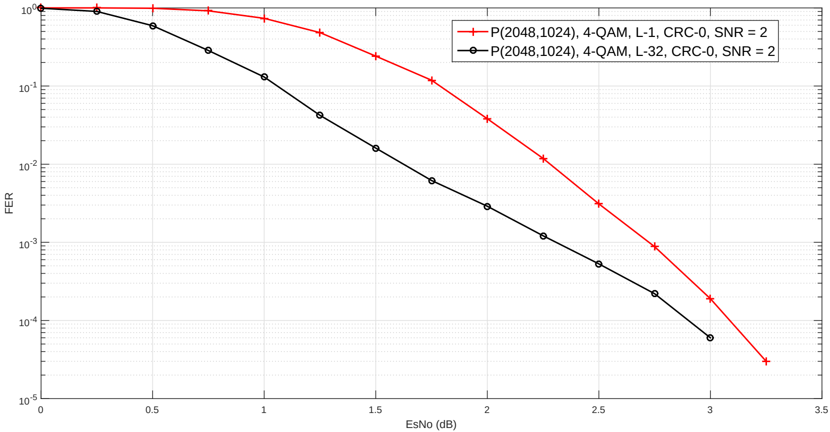
Polar code performance

Successive cancellation decoder



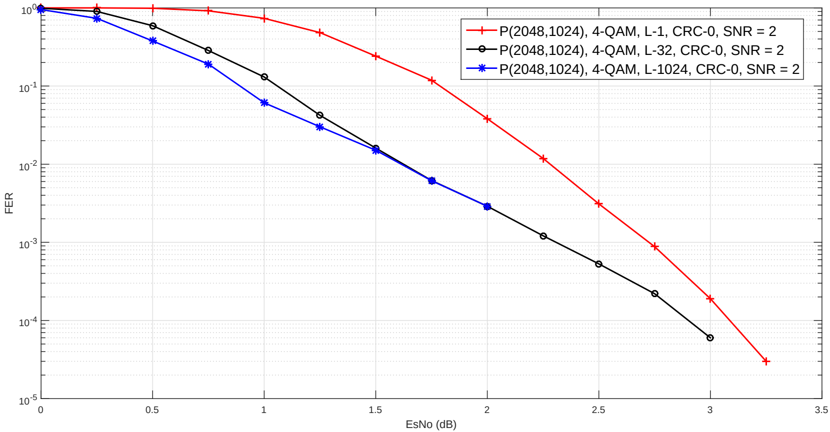
Polar code performance

Improvement by list-decoding: List-32



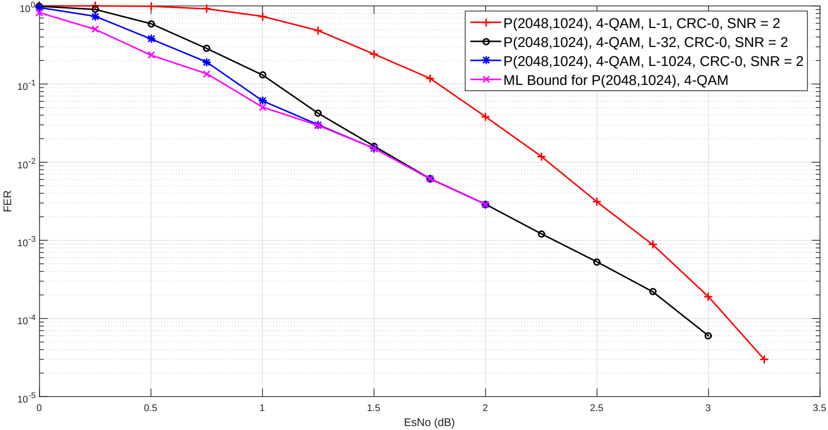
Polar code performance

Improvement by list-decoding: List-1024



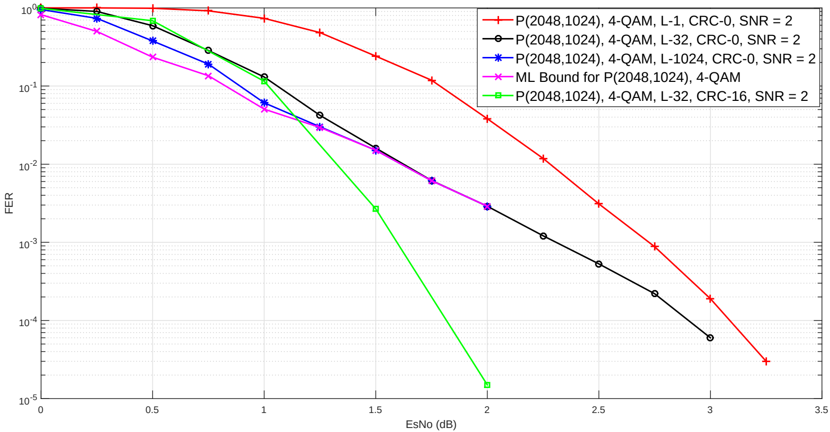
Polar code performance

Comparison with ML bound



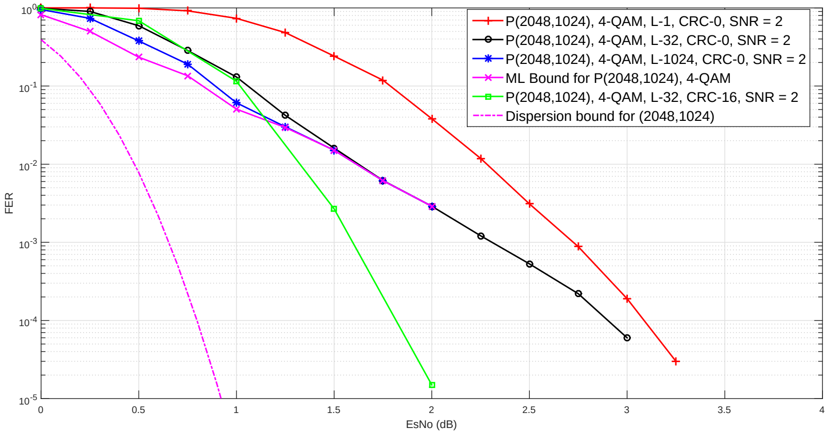
Polar code performance

Introducing CRC improves performance at high SNR



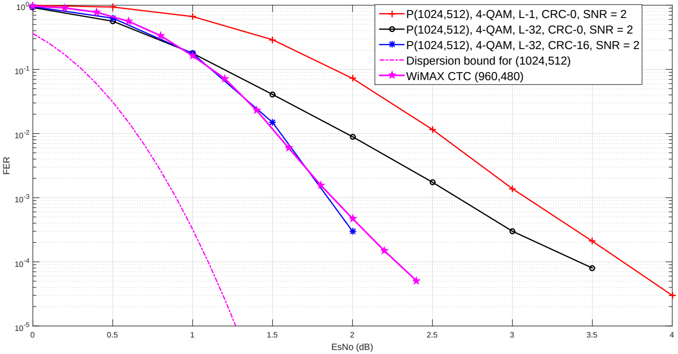
Polar code performance

Comparison with dispersion bound



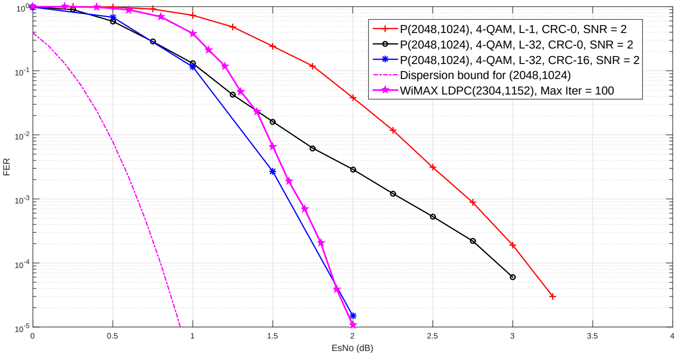
Polar codes vs WiMAX Turbo Codes

Comparable performance obtained with List-32 + CRC



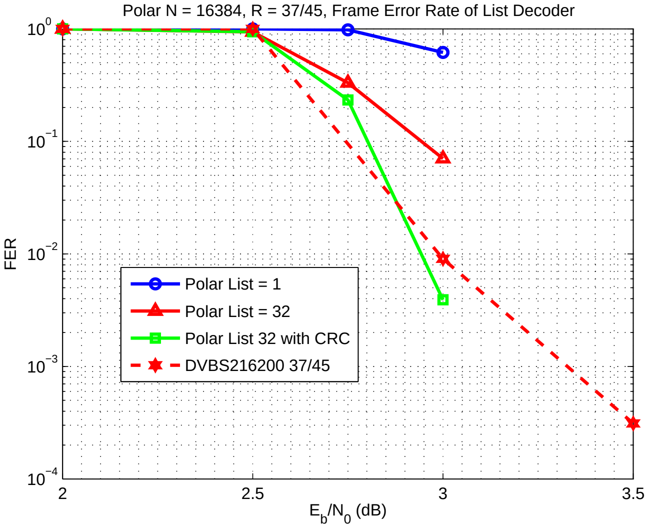
Polar codes vs WiMAX LDPC Codes

Better performance obtained with List-32 + CRC



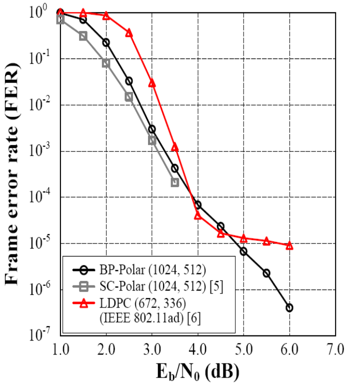
Polar Codes vs DVB-S2 LDPC Codes

LDPC (16200,13320), Polar (16384,13421). Rates = 0.82. BPSK-AWGN channel.



Polar codes vs IEEE 802.11ad LDPC codes

Park (2014) gives the following performance comparison.



(Park's result on LDPC conflicts with reference IEEE 802.11-10/0432r2. Whether there exists an error floor as shown needs to be confirmed independently.)

Source: Youn Sung Park, "Energy-Effcient Decoders of Near-Capacity Channel Codes," PhD Dissertation, The University of Michigan, 2014.

Summary of performance comparisons

- Successive cancellation decoder is simplest but inherently sequential which limits throughput
- BP decoder improves throughput and with careful design performance
- List decoder but significantly improves performance at low SNR
- Adding CRC to list decoding improves performance significantly at high SNR with little extra complexity
- Overall, polar codes under list-32 decoding with CRC offer performance comparable to codes used in present wireless standards

Implementation performance metrics

Implementation performance is measured by

- Chip area (mm2)
- Throughput (Mbits/sec)
- Energy efficiency (nJ/bit)
- Hardware efficiency (Mb/s/mm2)

Successive cancellation decoder comparisons

	[1]	[2][1]	[3][2]	
Decoder Type	SC	SC	BP	
Block Length	1024	1024	1024	
Technology	90 nm	65 nm	65 nm	
Area [mm^2]	3.213	0.68	1.476	
Voltage [V]	1.0	1.2	1.0	0.475
Frequency [MHz]	2.79	1010	300	50
Power [mW]	32.75	-	477.5	18.6
Throughput [Mb/s]	2860	497	4676	779.3
Engy.-per-bit [pJ/b]	11.45	-	102.1	23.8
Hard. Eff. [$Mb/s/mm^2$]	890	730	3168	528

[1] O. Dizdar and E. Arıkan, arXiv:1412.3829, 2014.

[2] Y. Fan and C.-Y. Tsui, "An efficient partial-sum network architecture for semi-parallel polar codes decoder implementation," Signal Processing, IEEE Transactions on, vol. 62, no. 12, pp. 3165-3179, June 2014.

[3] C. Zhang, B. Yuan, and K. K. Parhi, "Reduced-latency SC polar decoder architectures," arxiv.org, 2011.

[1] Throughput 730 Mb/s calculated by technology conversion metrics

[2] Performance at 4 dB SNR with average no of iterations 6.57

BP decoder comparisons

Property	Unit	[1]	[2]	[3]	[3]	[4]	[4]
Decoding type and Scheduling		SCD with folded HPPSN	Specialized SC	BP Circular Unidirectional	BP Circular Unidirectional	BP All-ON, Fully Parallel	BP Circular Unidirectional, Reduced Complexity
Block length		1024	16384	1024	1024	1024	1024
Rate			0.9	0.5	0.5	0.5	0.5
Technology		CMOS	Altera Stratix 4	CMOS	CMOS	CMOS	CMOS
Process	nm	65	40	65	65	45	45
Core area	mm^2	0.068		1.48	1.48	12.46	1.65
Supply	V	1.2	1.35	1	0.475	1	1
Frequency	MHz	1010	106	300	50	606	555
Power	mW			477.5	18.6	2056.5	328.4
Iterations		1	1	15	15	15	15
Throughput*	Mb/s	497	1091	1024	171	2068	1960
Energy efficiency	pJ/b			102.1	23.8	110.5	19.3
Energy eff. per iter.	pJ/b/iter			15.54	3.63	7.36	1.28
Area efficiency	$Mb/s/mm^2$	7306.78		693.77	99.80	166.01	1187.71
Normalized to 45 nm according to ITRS roadmap							
Throughput*	Mb/s	613.4		1263.8	210.6	2068	1960
Energy efficiency	pJ/b			149.6	34.9	110.5	19.3
Area efficiency	$Mb/s/mm^2$	18036.5		1250.21	179.85	166.01	1187.71

∗ Throughput obtained by disabling the BP early-stopping rules for fair comparison.

[1] Y.-Z. Fan and C.-Y. Tsui, "An efficient partial-sum network architecture for semi-parallel polar codes decoder implementation," *IEEE Transactions on Signal Processing*, vol. 62, no. 12, pp. 3165–3179, June 2014.

[2] G. Sarkis, P. Giard, A. Vardy, C. Thibeault, and W. J. Gross, "Fast polar decoders: Algorithm and implementation," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 5, pp. 946–957, May 2014.

[3] Y. S. Park, "Energy-efficient decoders of near-capacity channel codes," in *http://deepblue.lib.umich.edu/handle/2027.42/108731*, 23 October 2014 PhD.

[4] A. D. G. Biroli, G. Masera, E. Arıkan, "High-throughput belief propagation decoder architectures for polar codes," submitted 2015.

Channel polarization

Polar coding

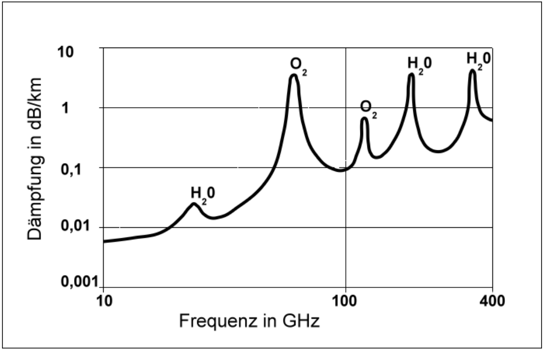
Polar codes for selected applications

Polar codes for selected applications

- 60 GHz wireless
- Optical access networks
- 5G

Millimeter Wave 60 GHz Communications

- 7 GHz of bandwidth available (57-64 GHz allocated in the US)
- Free-space path loss $(4\pi d/\lambda)^2$ is high at $\lambda = 5$ mm but compensated by large antenna arrays.
- Propagation range limited severely by O_2 absorption. Cells confined to rooms.

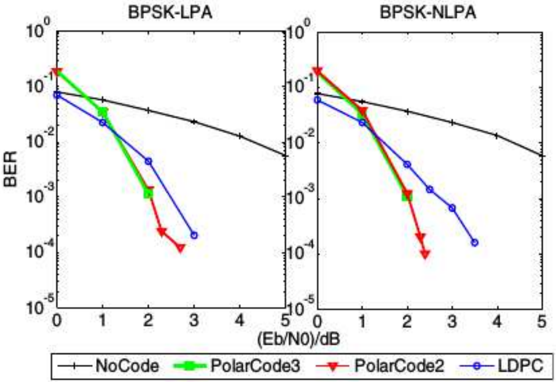


Millimeter Wave 60 GHz Communications

- Recent IEEE 802.11.ad Wi-Fi standard operates at 60 GHz ISM band and uses an LDPC code with block length 672 bits, rates 1/2, 5/8, 3/4, 13/16.
- Two papers compare polar codes that study polar coding for 60 GHz applications:
  - Z. Wei, B. Li, and C. Zhao, "On the polar code for the 60 GHz millimeter-wave systems," EURASIP, JWCN, 2015.
  - Youn Sung Park, "Energy-Effcient Decoders of Near-Capacity Channel Codes," PhD Dissertation, The University of Michigan, 2014.

Millimeter Wave 60 GHz Communications

Wei et al compare polar codes with the LDPC codes used in the standard using a nonlinear channel model



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Millimeter Wave 60 GHz Communications

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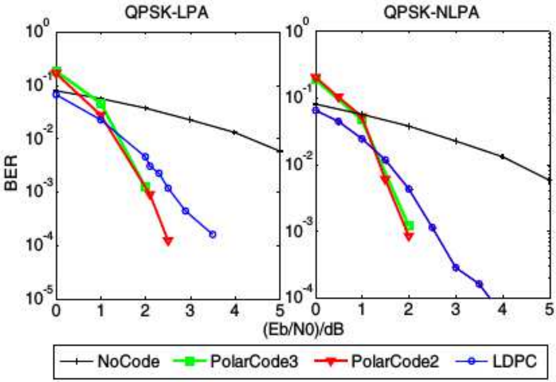

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Millimeter Wave 60 GHz Communications

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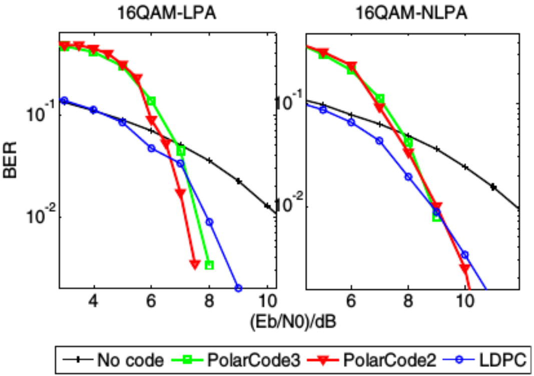

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Polar codes vs IEEE 802.11ad LDPC codes

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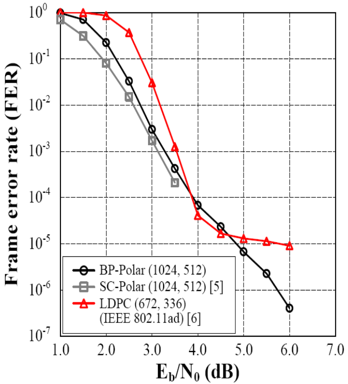


(Park's result on LDPC conflicts with reference IEEE 802.11-10/0432r2. Whether there exists an error floor as shown needs to be confirmed independently.)

Source: Youn Sung Park, "Energy-Effcient Decoders of Near-Capacity Channel Codes," PhD Dissertation, The University of Michigan, 2014.

Polar codes vs IEEE 802.11ad LDPC codes

In terms of implementation complexity and throughput, Park (2014) gives the following figures.

	LPDC			Polar	
Throughput Gb/s	0.5	6	9	0.779	4.676
Energy efficiency (pJ/b)	21	61.7	89.5	23.8	102.1
Area efficiency (Gb/s/mm2)	0.31	3.75	5.63	0.528	3.168

Source: Youn Sung Park, “Energy-Efficient Decoders of Near-Capacity Channel Codes,” PhD Dissertation, The University of Michigan, 2014.

Optical access/transport network

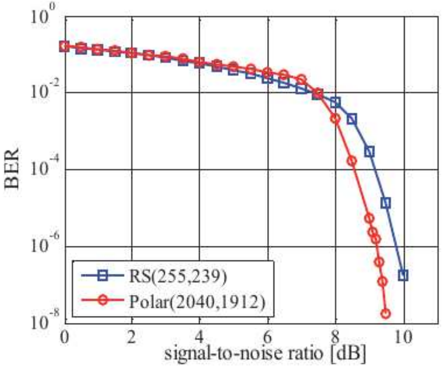
- 10-100 Gb/s at 1E-12 BER
- OTU4 (100 Gb/s Ethernet) and ITU G.975.1 standards use Reed-Solomon (RS) codes
- The challenge is to provide high reliability at low hardware complexity.

Polar codes for optical access/transport

There have been some studies of polar codes fore optical transmission.

- A. Eslami and H. Pishro-Nik, "A practical approach to polar codes," ISIT 2011. (Considers a polar-LDPC concatenated code and compares it with OTU4 RS codes.)
- Z. Wu and B. Lankl, "Polar codes for low-complexity forward error correction in optical access networks," ITG-Fachbericht 248: Photonische Netze - 05, 06.05.2014, Leipzig. (Compares polar codes with G.975.1 RS codes.)
- T. Ahmad, "Polar codes for optical communications", MS Thesis, Bilkent University, May 2016.
- L. Beygi, E. Agrell, J. M. Kahn, and M. Karlsson, "Coded modulation for fiber-optic networks," IEEE Sig. Proc. Mag., Mar. 2014. (Coded modulation for optical transport.)

Comparison of polar codes with G.975.1 RS codes



Source: Z. Wu and B. Lankl, above reference.

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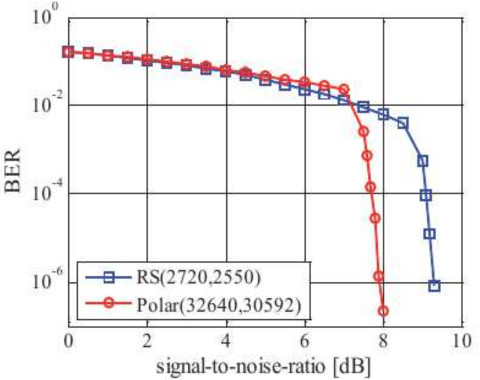
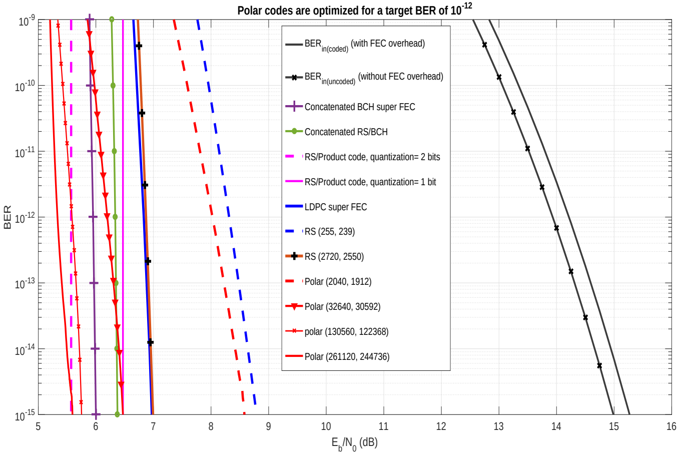


Source: Z. Wu and B. Lankl, above reference.

Comparison of polar codes with all codes in G.975.1

In a recent MS thesis, T. Ahmad compared polar codes with G.975.1 codes.



Comparison of polar codes with all codes in G.975.1

The conclusion of Ahmad (2016) is that polar codes perform better than all G.975.1 FEC schemes.

FEC Code	BER_{in}	**NCG (dB)**	**CG (dB)**	**Q (dB)**	$\frac{Eb}{No}$ **(dB)**
RS (255, 239)	1.82E-04	5.62	5.90	11.04	8.31
LDPC super FEC code	1.33E-03	7.10	7.39	9.56	6.83
RS (2720, 2550)	1.26E-03	7.06	7.34	9.60	6.87
Conc. RS/CSOC code(24.5%OH)	5.80E-03	7.95	8.90	8.04	5.31
Concatenated BCH code	3.30E-03	7.98	8.26	8.68	5.95
Conc. RS/BCH code	2.26E-03	7.63	7.91	9.06	6.34
Conc. RS/Product code	4.60E-03	8.40	8.68	8.30	5.57
Polar (2040, 1912)	2.81E-04	5.91	6.19	10.75	8.02
Polar (32640, 30592)	2.60E-03	7.74	8.02	8.92	6.20
Polar (130560, 122368)	4.61E-03	8.35	8.63	8.31	5.58
Polar (261120, 244736)	5.72E-03	8.60	8.89	8.06	5.33

Comparison of polar codes with 3rd Generation FEC for optical transport

Ahmad's study finds that polar codes fall short of beating 3G FEC proposed for optical transport.

FEC code	NCG (dB)	Comments
Polar (32640, 27200)	10.07	Ahmad (2016)
Polar (130560, 108800)	10.79	Ahmad (2016)
Polar (261120, 217600)	11.07	Ahmad (2016)
Polar (522240, 435200)	11.30	Ahmad (2016)
CC-LDPC (10032, 4, 24)	11.50	3G FEC, 12 iterations
QC-LDPC (18360, 15300)	11.30	3G FEC, 12 iterations

Summary

- With list-decoding and CRC polar codes deliver comparable performance to LDPC and Turbo codes used in present wireless standards
- SoA in coding is already close to theoretical limits, leaving little margin for improvement
- Polar coding compared to SoA offers some advantages:
  - Universal: the same hardware can be used with different code lengths, rates, channels
  - Flexible: the code rate can be adjusted readily to any number between 0 and 1
  - Versatile: can be used in multi-terminal coding scenarios

FEC for 5G

- What is 5G?
- What will be new in terms of FEC?

What is 5G?

Andrews *et al.*[3] answer this question as follows.

- It willl *not* be an incremental advance over 4G.
- Will be characterized by
  - Very high frequencies and massive bandwidths with very large no of antennas
  - Extreme base station and device connectivity
  - Universal connectivity between 5G new air interfaces, LTE, WiFi, etc.

[3] Andrews et al., "What will 5G be?" JSAC 2014

Technical requirements for 5G

Again, according to Andrews *et al.*, 5G will have to meet the following requirements (not all at once):

- Data rates compared to 4G
  - Aggregate: 1000 times more capacity/km2 compared to 4G
  - Cell-edge: 100 - 1000 Mb/s/user with 95% guarantee
  - Peak: 10s of Gb/s/user
- Round-trip latency: Some applications (tactile Internet, two-way gaming, virtual reality) will require 1 ms latency compared to 10-15 ms that 4G can provide
- Energy and cost: Link energy consumption should remain the same as data rates increase, meaning that a 100-times more energy-efficient link is required
- No of devices: 10,000 more low-rate devices for M2M communications, along with traditional high-rate users

Key technology ingredients for 5G

It is generally agreed that the 1000x aggregate data rate increase will be possible through a combination of three types of gains.

- Densification of network access nodes
- Increased bandwidth (move to mm waves)
- Increased spectral efficiency through new communication techniques:
  - advanced MIMO
  - improved multi-access
  - better interference management
  - improved coding and modulation schemes

Energy challenge

- Latest smartphone batteries have a power rating of about 10 Wh (2550 mAh at 3.85 V) or an energy of 36 kJ = 3.6E13 nJ
- Typical energy consumption at the decoder today is 1-10 nJ/bit
- At 3.6 nJ/bit and a data rate of 1 Gb/s, the FEC consumes all battery power in 20 mins!
- Technology challenge: Build FEC with energy consumption less than 10 pJ/bit

Throughput challenge

Typical backbone (fronthaul/backhaul) requirement for the near future:

- 1 Tb/s data rate
- Frame Error Rate (FER) better than 1E-15
- Latency: 10-200 μsec
- Total power consumption: 20 W max (20 pJ/bit)

Outlook for applied FEC research

Applied FEC research will remain an active area for the next decade

- Diverse set of applications, diverse set of requirements
- More sophisticated coding techniques to serve multi-user techniques (relaying, etc.) are in demand
- One-size-fits-all type of solution impossible
- A trade-off will emerge given the conflicting demands for energy-efficiency, low-latency, flexibility, and near-optimal performance

Thank you!