Macroscopic modeling and simulation of crowd dynamics

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Outline of the talk

1. Macroscopic models

2. Numerical tests

3. Some rigorous results
Outline of the talk

1. Macroscopic models
2. Numerical tests
3. Some rigorous results
Mathematical modeling of pedestrian motion: frameworks

**Microscopic**

- individual agents
- ODEs system
- many parameters
- low and high densities
- comp. cost $\sim$ ped. number.

**Macroscopic**

- continuous fluid
- PDEs
- few parameters
- very high densities
- analytical theory
- comp. cost $\sim$ domain size
Pedestrians as "thinking fluid"\(^1\)

Averaged quantities:
- \(\rho(t, x)\) pedestrians density
- \(\bar{v}(t, x)\) mean velocity

Mass conservation

\[
\begin{align*}
\partial_t \rho + \text{div}_x (\rho \bar{v}) &= 0 \\
\rho(0, x) &= \rho_0(x)
\end{align*}
\]

for \(x \in \Omega \subset \mathbb{R}^2, t > 0\)

---

\(^1\)R.L. Hughes, Transp. Res. B, 2002
Macroscopic models

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Two classes

- **1st order models**: velocity given by a phenomenological speed-density relation \(\bar{v} = V(\rho)\bar{v}\)
- **2nd order models**: velocity given by a momentum balance equation

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Density must stay non-negative and bounded: \(0 \leq \rho(t, x) \leq \rho_{\text{max}}\)

Different from fluid dynamics:
- preferred direction
- no conservation of momentum / energy
- \(n \ll 6 \cdot 10^{23}\)

\(^1\)R.L. Hughes, Transp. Res. B, 2002
Continuum hypothesis

\[ n \ll 6 \cdot 10^{23} \text{ but ...} \]

Brown University, Main Green, 08.21.2017
Speed-density relation

Speed function $V(\rho)$:
- decreasing function wrt density
- $V(0) = v_{\text{max}}$ free flow
  \[ V(\rho_{\text{max}}) \approx 0 \text{ congestion} \]

Examples:

- $V(\rho)$
- $\rho V(\rho)$
Desired direction of motion $\vec{\mu}$

Pedestrians:
- seek the shortest route to destination
- try to avoid high density regions

\[ \vec{\nu} = -\frac{\nabla_x \phi}{|\nabla_x \phi|} \]
Desired direction of motion $\vec{\mu}$

Pedestrians:
- seek the shortest route to destination
- try to avoid high density regions

\[ \vec{\nu} = -\frac{\nabla_x \phi}{|\nabla_x \phi|} \]

The potential $\phi : \Omega \to \mathbb{R}$ is given by the Eikonal equation

\[
\begin{cases}
|\nabla_x \phi| = C(t, x, \rho) & \text{in } \Omega \\
\phi(t, x) = 0 & \text{for } x \in \Gamma_{outflow}
\end{cases}
\]

where $C = C(t, x, \rho) \geq 0$ is the running cost

$\implies$ the solution $\phi(t, x)$ represents the weighted distance of the position $x$ from the target $\Gamma_{outflow}$
Eikonal equation: level set curves for $|\nabla_x \phi| = 1$

In an empty space: potential is proportional to distance to destination
The fastest route ...

... needs not to be the shortest!
First order models

- Hughes’ model\(^1\)

\[ \vec{v} = -\frac{\nabla_x \phi}{|\nabla_x \phi|} \quad \text{s.t.} \quad |\nabla_x \phi| = \frac{1}{V(\rho)} \]

- minimize travel time avoiding high densities
- CRITICISM: instantaneous global information on entire domain

---

\(^1\)R.L. Hughes, Transp. Res. B, 2002
\(^3\)R.M. Colombo, Garavello and M. Lécureux-Mercier, M3AS, 2012
First order models

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  \[ \vec{v} = -\frac{\nabla_x \phi}{|\nabla_x \phi|} \quad \text{s.t.} \quad |\nabla_x \phi| = \frac{1}{V(\rho)} \]

  - minimize travel time avoiding high densities
  - CRITICISM: instantaneous global information on entire domain

- Dynamic model with memory effect\(^2\)
  \[ \vec{v} = -\frac{\nabla_x (\phi + \omega D)}{|\nabla_x (\phi + \omega D)|} \quad \text{s.t.} \quad |\nabla_x \phi| = \frac{1}{v_{\text{max}}}, \quad D(\rho) = \frac{1}{v(\rho)} + \beta \rho^2 \quad \text{discomfort} \]

  - minimize travel time based on knowledge of the walking domain
  - temper the behavior locally to avoid high densities

---

\(^1\) R.L. Hughes, Transp. Res. B, 2002
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First order models

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  \]
  
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  \]
  
  - minimize travel time based on knowledge of the walking domain
  - temper the behavior locally to avoid high densities

- Non-local flow:\(^3\)
  \[
  \vec{v} = V(\rho) \left( \vec{v} - \varepsilon \frac{\nabla (\rho \ast \eta)}{\sqrt{1 + |\nabla (\rho \ast \eta)|^2}} \right) \quad \text{with} \quad \vec{v} = -\frac{\nabla x \phi}{|\nabla x \phi|} \quad \text{s.t.} \quad |\nabla x \phi| = 1
  \]

\(^1\)R.L. Hughes, Transp. Res. B, 2002
\(^3\)R.M. Colombo, Garavello and M. Lécureux-Mercier, M3AS, 2012
Second order model

Momentum balance equation\(^4\) \(^5\)

\[
\partial_t (\rho \vec{v}) + \text{div}_x (\rho \vec{v} \otimes \vec{v}) + \nabla_x P(\rho) = \rho \frac{V(\rho)\vec{v} - \vec{v}}{\tau}
\]

where

- \(V(\rho) = v_{\text{max}} e^{-\alpha \left(\frac{\rho}{\rho_{\text{max}}}\right)^2}\)
- \(|\nabla_x \phi| = 1/V(\rho)|
- \(P(\rho) = p_0 \rho^\gamma, \ p_0 > 0, \ \gamma > 1 \) internal pressure
- \(\tau \) response time

\(^4\) Payne-Whitham, 1971
Question

Can macroscopic models reproduce characteristic features of crowd behavior?
Outline of the talk

1. Macroscopic models
2. Numerical tests
3. Some rigorous results
Numerical schemes used

- **Space meshes:** unstructured triangular / cartesian
- **Eikonal equation:** linear, finite element solver\(^6\) / fast-sweeping
- **First order models:** Lax-Friedrichs
- **Second order models:** explicit time integration with advection-reaction splitting (HLL scheme)
- **Non-local models:** dimensional splitting Lax-Friedrichs

\(^6\) [Bornemann-Rasch, 2006]
Corridor evacuation with two exits

Configuration at $t = 0$

Parameters choice:

- $\rho_0 = 3 \text{ped/m}^2$ initial density
- $\rho_{\text{max}} = 10 \text{ped/m}^2$ maximal density
- $v_{\text{max}} = 2 \text{m/s}$ desired speed
- $\tau = 0.61 \text{s}$ relaxation time
- $p_0 = 0.005 \text{ped}^{1-\gamma} \text{m}^{2+\gamma}/\text{s}^2$ pressure coefficient
- $\gamma = 2$ adiabatic exponent
- $\alpha = 7.5$ density-speed coefficient
- $\varepsilon = 0.8$ correction coefficient
- $\eta = [1 - (x/r)^2]^3[1 - (y/r)^2]^3$ convolution kernel, with $r = 15 \text{m}$
Corridor evacuation with two exits

\[ t = 20s \]

\[ |\nabla_x \phi| = 1 \]

\[ \nabla_x (\phi + \omega_D) \]

\[ |\nabla_x \phi| = 1/v(\rho) \]

second order

non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]
Corridor evacuation with two exits

$t = 40s$

$|\nabla_x \phi| = 1$

$\nabla_x (\phi + \omega D)$

$|\nabla_x \phi| = 1/\nu(\rho)$

second order

non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]
Corridor evacuation with two exits

\[ t = 60s \]

\[ |\nabla_x \phi| = 1 \]

\[ \nabla_x (\phi + \omega D) \]

\[ |\nabla_x \phi| = 1/v(\rho) \]

second order

non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]
Corridor evacuation with two exits

\[ t = 80s \]

\[ |\nabla_x \phi| = 1 \]

\[ \nabla_x (\phi + \omega D) \]

\[ |\nabla_x \phi| = 1/\nu(\rho) \]

second order

non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]
Room evacuation with obstacle

Configuration at \( t = 0 \)

Parameters choice:
- \( \rho_0 = 3 \text{ped/m}^2 \) initial density
- \( \rho_{\text{max}} = 6 \text{ped/m}^2 \) maximal density
- \( v_{\text{max}} = 2 \text{m/s} \) desired speed
- \( \tau = 0.61 \text{s} \) relaxation time
- \( p_0 = 0.005 \text{ped}^{1-\gamma} \text{m}^2 + \gamma \text{/s}^2 \) pressure coefficient
- \( \gamma = 2 \) adiabatic exponent
- \( \alpha = 7.5 \) density-speed coefficient
- \( \varepsilon = 0.8 \) correction coefficient
- \( \eta = [1 - (x/r)^2]^3 [1 - (y/r)^2]^3 \) convolution kernel, with \( r = 1.5 \text{m} \)
Macroscopic models

Room evacuation with obstacle

\[ t = 2s \]

\[ |\nabla_x \phi| = 1 \]

\[ \nabla_x (\phi + \omega D) \]

\[ |\nabla_x \phi| = 1/v(\rho) \]

second order

non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]
Room evacuation with obstacle

\[ t = 5s \]

\[ |\nabla_x \phi| = 1 \quad \nabla_x (\phi + \omega D) \]

\[ |\nabla_x \phi| = 1/v(\rho) \quad \text{second order} \]

non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]
Room evacuation with obstacle

\[ t = 8s \]

\[ |\nabla_x \phi| = 1 \]

\[ \nabla_x (\phi + \omega D) \]

\[ |\nabla_x \phi| = 1/v(\rho) \]

second order

non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]
Room evacuation with obstacle

\[ t = 11s \]

\[ |\nabla_x \phi| = 1 \]

\[ \nabla_x (\phi + \omega D) \]

\[ |\nabla_x \phi| = 1/v(\rho) \]

second order

non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]
Effect of the obstacle on the outflow

Time evolution of the total mass of pedestrians inside the room

\[ M(t) = \int_{\Omega} \rho(t, x) dx \]
Second order model: stop-and-go waves

\[ P(\rho) = 0.005\rho^2, \quad v_{\text{max}} = 2, \quad \rho_{\text{max}} = 7 \]

**Fig.** Time evolution of density profile at \( x = 64 \) (left exit)
Second order model: dependence on $p_0$

\[ P(\rho) = p_0 \rho^\gamma: \text{total evacuation time optimal for } p_0 \sim 0.5 \]

\[ P. Goatin (Inria) \quad \text{Macroscopic models} \quad \text{August 21-25, 2017} \quad 22 / 42 \]

with $v_{\max} = 2m/s$, $\rho_{\max} = 7\text{ped/m}^2$
Second order model: dependence on $v_{\text{max}}$

Total evacuation time

![Graph showing total evacuation time as a function of $v_{\text{max}}$.]

Social force models\textsuperscript{7} show a minimum for $v_{\text{max}} \simeq 1.4 \text{ m/s}$

$\Rightarrow$ faster-is-slower effect\textsuperscript{8}

Accounting for inter-pedestrian friction?

\textsuperscript{7}D. Helbing, I. Farkas and T. Vicsek, Nature, 2000

\textsuperscript{8}D.R. Parisi and C.O. Dorso, Physica A, 2007
Second order model: dependence on $v_{\text{max}}$

Total mass evolution

with $\rho_{\text{max}} = 7\text{ped}/m^2$, $\gamma = 2$, $p_0 = 0.005$
Evacuation optimization: Braess’ paradox\(^9\) ?

Problem: *clogging* at exit

Can obstacles reduce the evacuation time?

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\(^9\)Braess, D. *Über ein Paradoxon aus der Verkehrsplanung*, Unternehmensforschung, 12, pp. 258-268 (1968)
Evacuation optimization: Braess’ paradox?

Time evolution of the total mass of pedestrians inside the room

\[ P(\rho) = 0.001\rho^2 \]

- empty room
- five columns
Non-local model: lane formation\textsuperscript{10}

Two groups of pedestrians moving in opposite directions

\[
\begin{align*}
\partial_t U^1 + \text{div} & \left( c_1 U^1 (1 - U^1) \left( (1 - \epsilon_1 \frac{U^1 \ast \mu}{\sqrt{1 + \|U^1 \ast \mu\|^2}}) \vec{v}^1(x, y) - \epsilon_2 \frac{\nabla U^2 \ast \mu}{\sqrt{1 + \|\nabla U^2 \ast \mu\|^2}} \right) \right) = 0, \\
\partial_t U^2 + \text{div} & \left( c_2 U^2 (1 - U^2) \left( (1 - \epsilon_1 \frac{U^2 \ast \mu}{\sqrt{1 + \|U^2 \ast \mu\|^2}}) \vec{v}^2(x, y) - \epsilon_2 \frac{\nabla U^1 \ast \mu}{\sqrt{1 + \|\nabla U^1 \ast \mu\|^2}} \right) \right) = 0.
\end{align*}
\]

where

\[
c_1 = c_2 = 4 \quad \text{crowding factor}
\]
\[
\epsilon_1 = 0.3, \quad \epsilon_2 = 0.7,
\]

\text{can be derived as mean-field and hydrodynamic limit of microscopic model [Göttlich-Klar-Tiwari, JEM 2015]}

\textsuperscript{10}R.M. Colombo and M. Mercier, Acta Mathematica Scientia, 2011
Lane formation in bidirectional flows

[Aggarwal-Colombo-Goatin, SINUM 2015; Aggarwal-Goatin, BBMS 2016]
Lane formation in crossing flows

[Aggarwal-Colombo-Goatin, SINUM 2015; Aggarwal-Goatin, BBMS 2016]
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The 1D case: statement of the problem

We consider the initial-boundary value problem

\[
\rho_t - \left( \rho(1 - \rho) \frac{\phi_x}{|\phi_x|} \right)_x = 0 \quad x \in \Omega = (-1, 1], \ t > 0
\]

\[
|\phi_x| = c(\rho)
\]

with initial density \( \rho(0, \cdot) = \rho_0 \in BV(0, 1]\) and \textit{absorbing} boundary conditions

\[
\rho(t, -1) = \rho(t, 1) = 0 \quad \text{(weak sense)}
\]

\[
\phi(t, -1) = \phi(t, 1) = 0
\]
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and absorbing boundary conditions

\[ \rho(t, -1) = \rho(t, 1) = 0 \quad \text{(weak sense)} \]

\[ \phi(t, -1) = \phi(t, 1) = 0 \]

General cost function \( c: [0, 1[ \to [1, +\infty[ \) smooth s.t. \( c(0) = 1 \) and \( c'(\rho) \geq 0 \)

(e.g. \( c(\rho) = 1/v(\rho) \))
The 1D case: statement of the problem

The problem can be rewritten as

$$\rho_t - \left( \text{sgn}(x - \xi(t)) \ f(\rho) \right)_x = 0$$

where the turning point is given by

$$\int_{-1}^{\xi(t)} c(\rho(t, y)) \ dy = \int_{\xi(t)}^{1} c(\rho(t, y)) \ dy$$
The 1D case: statement of the problem

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where the turning point is given by

$$\int_{-1}^{\xi(t)} c(\rho(t,y)) \, dy = \int_{\xi(t)}^{1} c(\rho(t,y)) \, dy$$

→ the discontinuity point $\xi = \xi(t)$ is not fixed a priori, but depends non-locally on $\rho$
The 1D case: available results

- **existence and uniqueness** of Kruzkov’s solutions for an elliptic regularization of the eikonal equation and $c = 1/v$
  [DiFrancesco-Markowich-Pietschmann-Wolfram, JDE 2011]

- **Riemann solver** at the turning point for $c = 1/v$

- **entropy condition and maximum principle**
  [ElKhatib-Goatin-Rosini, ZAMP 2012]

- **wave-front tracking algorithm** and convergence of finite volume schemes
  [Goatin-Mimault, SISC 2013]

- **existence** for data with small $L^\infty$ and $TV$ norms and $c = 1/v$
  [Amadori-Goatin-Rosini, JMAA 2013]

- **local** version
  [Carrillo-Martin-Wolfram, M3AS 2016]

- **extension to graphs**
  [Camilli-Festa-Tozza, NHM 2017]
The 1D case: entropy condition

**Definition: entropy weak solution (ElKhatib-Goatin-Rosini, 2012)**

\( \rho \in C^0 \left( \mathbb{R}^+; L^1(\Omega) \right) \cap BV \left( \mathbb{R}^+ \times \Omega; [0, 1] \right) \) s.t. for all \( k \in [0, 1] \) and \( \psi \in C_c^\infty (\mathbb{R} \times \Omega; \mathbb{R}^+) \):

\[
0 \leq \int_0^{+\infty} \int_{-1}^1 (|\rho - k| \psi_t + \Phi(t, x, \rho, k) \psi_x) \, dx \, dt + \int_{-1}^1 |\rho_0(x) - k| \psi(0, x) \, dx
\]

\[
+ \text{sgn}(k) \int_0^{+\infty} (f(\rho(t, 1-)) - f(k)) \psi(t, 1) \, dt
\]

\[
+ \text{sgn}(k) \int_0^{+\infty} (f(\rho(t, -1+)) - f(k)) \psi(t, -1) \, dt
\]

\[
+ 2 \int_0^{+\infty} f(k) \psi(t, \xi(t)) \, dt.
\]

where \( \Phi(t, x, \rho, k) = \text{sgn}(\rho - k) (F(t, x, \rho) - F(t, x, k)) \)
The 1D case: maximum principle

**Proposition (ElKhatib-Goatin-Rosini, 2012)**

Let \( \rho \in C^0 \left( \mathbb{R}^+ ; \text{BV}(\Omega) \cap L^1(\Omega) \right) \) be an entropy weak solution. Then

\[
0 \leq \rho(t, x) \leq \| \rho_0 \|_{L^\infty(\Omega)}.
\]

Characteristic speeds satisfy

\[
f' \left( \rho^+ (t) \right) \leq \dot{\xi}(t), \text{ if } \rho^- (t) < \rho^+ (t),
\]
\[
-f' \left( \rho^- (t) \right) \geq \dot{\xi}(t), \text{ if } \rho^- (t) > \rho^+ (t).
\]
The 1D case: wave-front tracking [Goatin-Mimault, SISC 2013]

Riemann-type initial data:

\[ \Delta \rho = 2^{-4} \]

\[ \Delta \rho = 2^{-10} \]

Code freely available at:
http://www-sop.inria.fr/members/Paola.Goatin/wft.html
The 1D case: wave-front tracking [Goatin-Mimault, SISC 2013]

Density profile at $t = 0.8$:

\[ \Delta \rho = 2^{-4} \]

\[ \Delta \rho = 2^{-10} \]
The 1D case: numerical convergence of WFT [Goatin-Mimault, SISC 2013]

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\Delta \rho$</th>
<th>$\epsilon_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$2^{-5}$</td>
<td>$4.280e-2$</td>
</tr>
<tr>
<td>6</td>
<td>$2^{-6}$</td>
<td>$2.164e-2$</td>
</tr>
<tr>
<td>7</td>
<td>$2^{-7}$</td>
<td>$6.141e-3$</td>
</tr>
<tr>
<td>8</td>
<td>$2^{-8}$</td>
<td>$5.048e-3$</td>
</tr>
<tr>
<td>9</td>
<td>$2^{-9}$</td>
<td>$1.755e-3$</td>
</tr>
<tr>
<td>10</td>
<td>$2^{-10}$</td>
<td>$2.091e-3$</td>
</tr>
<tr>
<td>11</td>
<td>$2^{-11}$</td>
<td>$4.305e-4$</td>
</tr>
<tr>
<td>12</td>
<td>$2^{-12}$</td>
<td>$4.347e-4$</td>
</tr>
</tbody>
</table>

Table: $L^1$-error $\epsilon_\nu$ for wave-front tracking method between two subsequent discretization meshes $2^{-\nu}$ and $2^{-\nu-1}$. The comparison is done on a cartesian grid with $\Delta x = 10^{-3}$ and $\Delta t = 0.5\Delta x$. 
The 1D case: comparison WFT vs FV [Goatin-Mimault, SISC 2013]

Wave-front tracking with $\Delta \rho = 2^{-10}$ and finite volumes with $\Delta x = 1/1500$
The 1D case: comparison WFT vs FV [Goatin-Mimault, SISC 2013]

<table>
<thead>
<tr>
<th>$\Delta x$</th>
<th>$Err_G$</th>
<th>$\ln(Err_G)/\ln(\Delta x)$</th>
<th>$Err_R$</th>
<th>$\ln(Err_R)/\ln(\Delta x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/50</td>
<td>7.24e−2</td>
<td>−0.66</td>
<td>7.44e−2</td>
<td>−0.67</td>
</tr>
<tr>
<td>1/100</td>
<td>4.56e−2</td>
<td>−0.66</td>
<td>4.68e−2</td>
<td>−0.67</td>
</tr>
<tr>
<td>1/250</td>
<td>2.49e−2</td>
<td>−0.66</td>
<td>2.55e−2</td>
<td>−0.67</td>
</tr>
<tr>
<td>1/500</td>
<td>1.52e−2</td>
<td>−0.67</td>
<td>1.55e−2</td>
<td>−0.67</td>
</tr>
<tr>
<td>1/1000</td>
<td>9.03e−3</td>
<td>−0.68</td>
<td>9.12e−2</td>
<td>−0.68</td>
</tr>
<tr>
<td>1/1500</td>
<td>6.66e−3</td>
<td>−0.69</td>
<td>6.62e−3</td>
<td>−0.68</td>
</tr>
</tbody>
</table>

Table: $L^1$-norm of the error for Godunov and Rusanov schemes compared to wave-front tracking with $\Delta \rho = 2^{-10}$. 
Non-local fluxes in 2D

Multi-D integro-differential systems

\[
\partial_t U + \text{div}_x F(t, x, U, U \ast \eta) = 0
\]

with \( t \in \mathbb{R}^+, \ x \in \mathbb{R}^d, \ U(t, x) \in \mathbb{R}^N, \ \eta(x) \in \mathbb{R}^{m \times N} \)

**Theorem [Aggarwal-Colombo-Goatin, SINUM 2015]**

For any initial datum \( U_o \in (L^1 \cap L^\infty \cap BV)(\mathbb{R}^2; \mathbb{R}^N_+) \), there exists a solution \( U \in C^0(\mathbb{R}_+; L^1(\mathbb{R}^2; \mathbb{R}^N_+)) \). Moreover, for all \( k \in \{1, \ldots, N\} \) and for all \( t \in \mathbb{R}_+ \), the following bounds hold:

\[
\| U(t) \|_{L^\infty(\mathbb{R}^2; \mathbb{R}^N)} \leq e^{C(t+1)} \| U_o \|_{L^1} \| U_o \|_{L^\infty(\mathbb{R}^2; \mathbb{R}^N)},
\]

\[
\| U^k(t) \|_{L^1(\mathbb{R}^2; \mathbb{R})} = \| U^k_o \|_{L^1(\mathbb{R}^2; \mathbb{R})},
\]

\[
\text{TV}(U^k(t)) \leq e^{K_1 t} \text{TV}(U^k_o) + K_2 (e^{K_1 t} - 1),
\]

\[
\| U(t + \tau) - U(t) \|_{L^1(\mathbb{R}^2; \mathbb{R}^N)} \leq C(t) \tau.
\]
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- global description of spatio-temporal evolution
- mathematical tools for well-posedness and numerical approximation
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Thank you for your attention!