ICERM Homogeneous Dynamics Workshop

Problem Session

1. RH.

2. $\alpha m^2 + n^2$ gaps have exponential distribution ($\alpha = \sqrt{2}$ or $\alpha \in \mathbb{R}_+ \setminus \mathbb{Q}$ or a.e. α? W.A.) (ordered by $|n|,|m| < N$), (ordered by values). (If $(\alpha n + m)^2 \ldots$) (If indefinite & irrational, this is effective Oppenheim.)

3. $\{n^{1/3}\}_{n \in \mathbb{Z}_+}$ gaps have exponential distribution. $1/3 \neq 1/2$.

4. $\alpha m^2 + n^2 + \ell^2$, $\dfrac{\alpha > 0}{\alpha < 0}$, $\alpha \in \mathbb{R} \setminus \mathbb{Q}$. consecutive gaps? [Bourgain-Rudnick-Radziwill?] [Davenport-Lewis?]

5. $\{n^2 \alpha\}$ (mod 1). 2-pt correlation $\rightarrow$ uniform? ($\alpha = \sqrt{2}$?).

6. Hyperbolic 3-mfld, ~~closed~~ inf vol, no arithmetic cover, but ∞ly many (or 1?) closed immersed surfaced.

7. (Kleinbock) $G = SL_3(\mathbb{R})$, $\Gamma < G$ inf. covol, $H = SO(2,1) < G$.
Can one prove H-orbits are closed or dense in $\Gamma \backslash G$? (Rigidity)
Values of quad forms with variables restricted on Γe_1, say.

$\Gamma \leqslant SO(2,1)$

(8) $(2,2)$ form $= (2) - (2)$ (gap).

~~gap~~ value distr of $Q(a) - Q(m)$,

$(2,2)$ form

Let Q be binary pos def quadratic,

~~on average~~ for generic Q in .

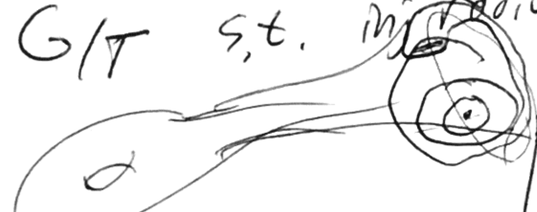
$$\int_Q \#\{a,b \in B_T \mid Q(a) - Q(b) \in I\} = M_T + O(E_T(I)).$$

How small can $I \subset \mathbb{R}$ interval get relative to T s.t. $E_T = o(M_T)$?

OR a.e. Q? (Mohammadi)

(9) (Margulis) $G = SL_3(\mathbb{R}) > \Gamma$ discrete, $\mathbb{Z}$-dense, infinite volume.

$\exists ?\ g_1, g_2, \ldots \in G/\Gamma$ s.t. inj radius in $\Gamma\backslash G$ at $g_j \to \infty$.



($\Rightarrow$ Margulis Normal Subgp Thm!).

(10) Mahler: every $x \in C \setminus \mathbb{Q}$ (middle third Cantor set) is transcendental?