ICERM Homogeneous Dynamics Workshop

Problem Session

1. RH.
   \(\alpha = \sqrt{2}\) or \(\alpha \in \mathbb{R} \setminus \mathbb{Q}\) or \(\alpha \notin \mathbb{Q}\)?
   (ordered by \(\ln(1 + (1/n))\), ordered by values)

2. \(x^2 + y^2\) gaps have exponential distribution

3. \(\sqrt{x^2 + y^2}\) gaps have exponential distribution
   (If \((x^2 + y^2)^{1/2}\)...
   (If infinite & irrational, this is effective Oppenheim.)

4. \(x^2 + y^2 + 1\), \(x > 0\), \(x \in \mathbb{R} \setminus \mathbb{Q}\), gaps.
   Consecutive

5. \(\{n \alpha\} \pmod{1}\), 2-point correlation \(\rightarrow\) uniform? \((\alpha = 5/2?)\)

6. Hyperbolic 3-manifolds, no arithmetic cover, but
   \(\infty\) many (or 1?) closed surfaces.

7. (Kleinbock) \(G = SL_3(\mathbb{R})\), \(\Gamma \leq G\) inf. cov., \(H = SO(2, 1)<G\).
   Can one prove \(H\)-orbits are closed or dense on \(\mathbb{R}\)? (Rigidity)
   Values of quad forms restricted on \(\Gamma\cdot e_i\), say.
$T \leq S_0(2,1)$

8) $(2,2)$ form $= (2) - (2)$ (gap).

Let $Q$ be binary pos def quadratic.

For general $Q$ in $\mathbb{R}^n$.

$S = \{ q, q \in \mathbb{R}^1 \mid Q(q)-Q(0) \in T \}$

$\sim M + O(E(T))$.

How small can $D(T)$ get relative to $T$ s.t. $ET = o(M)$?

Or are $Q$?

(Mohammadi)

9) $(\text{Margulis})$ $G = SL_3(\mathbb{R}) \supset T$ discrete, $Z$-dense, $\mathbb{H}$ infinite volume.

For $g_1, g_2, \ldots \in G/T$ s.t. min radius of $\mathbb{H}$ at $g_i \to \infty$.

$\Rightarrow$ Margulis Normal Subgp Thm!

10) Mahler: Every $x \in \mathbb{C} \setminus \mathbb{Q}$ (middle third Cantor set) is transcendental?