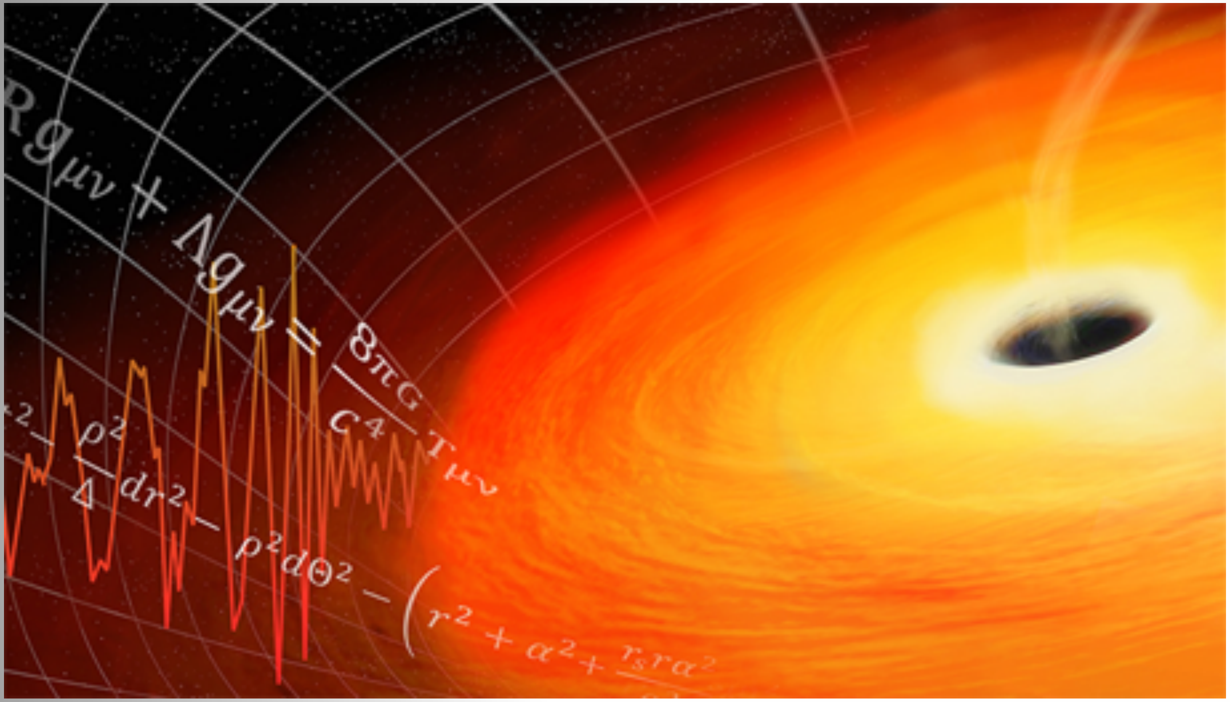
Discovering Black Holes and Gravitational Waves: Algorithms and Simulation



Scott Field
Department of Mathematics
U. Mass Dartmouth

ICERM Public Lecture 2019

Outline

- Gravitational wave science (highlights)
- What are gravitational waves?
  - Mathematical framework & intuition
- How to detect gravitational waves?
- Simulation of black holes and GWs
  - Computers & algorithms
- Ongoing work and challenges

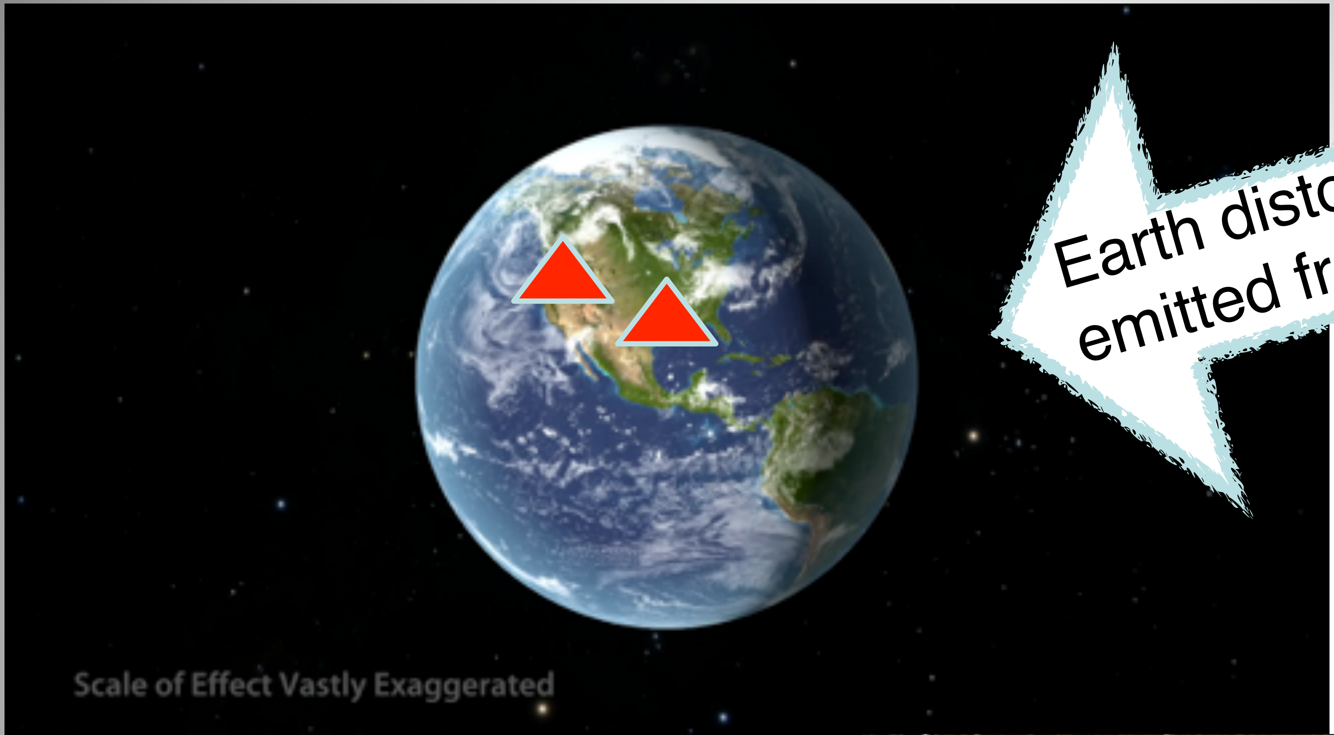
Common Acronyms

- GW = Gravitational Wave
- LIGO = Laser Interferometer Gravitational-Wave Observatory
- GR = General Relativity

References

1. Phys. Rev. Lett. 116, 061102 (2016)
2. Numerous figures pulled from the LIGO open science website
3. https://www.utdallas.edu/news/2015/2/26-31432_New-Insight-Found-in-Black-Hole-Collisions_story-sidebar.html
4. Holst, Sarbach, Tiglio, Vallisneri, “The emergence of gravitational wave science: 100 years of development of mathematical theory, detectors, numerical algorithms, and data analysis tools”
5. Ed Seidel’s APS April talk, 2018
6. Sarbach, Tiglio “Continuum and Discrete Initial-Boundary-Value Problems and Einstein's Field Equations”
7. Cervantes-Cota, Galindo-Uribarri, and Smoot, “A Brief History of Gravitational Waves”
8. Sormani, et al “The Mathematics of Gravitational Waves”, AMS Notices
9. Yvonne Choquet-Bruhat, “Beginnings of Cauchy problem”

Gravitational wave science (highlights)

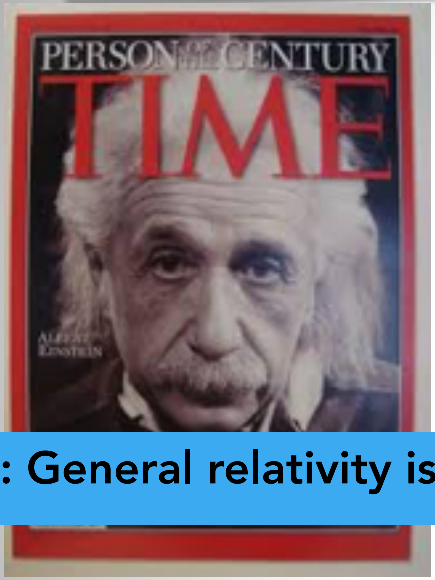


Earth distorted by GWs emitted from black holes

What: Two black holes
Where: Another galaxy

▲ = GW detectors

A 100 Year Research Problem



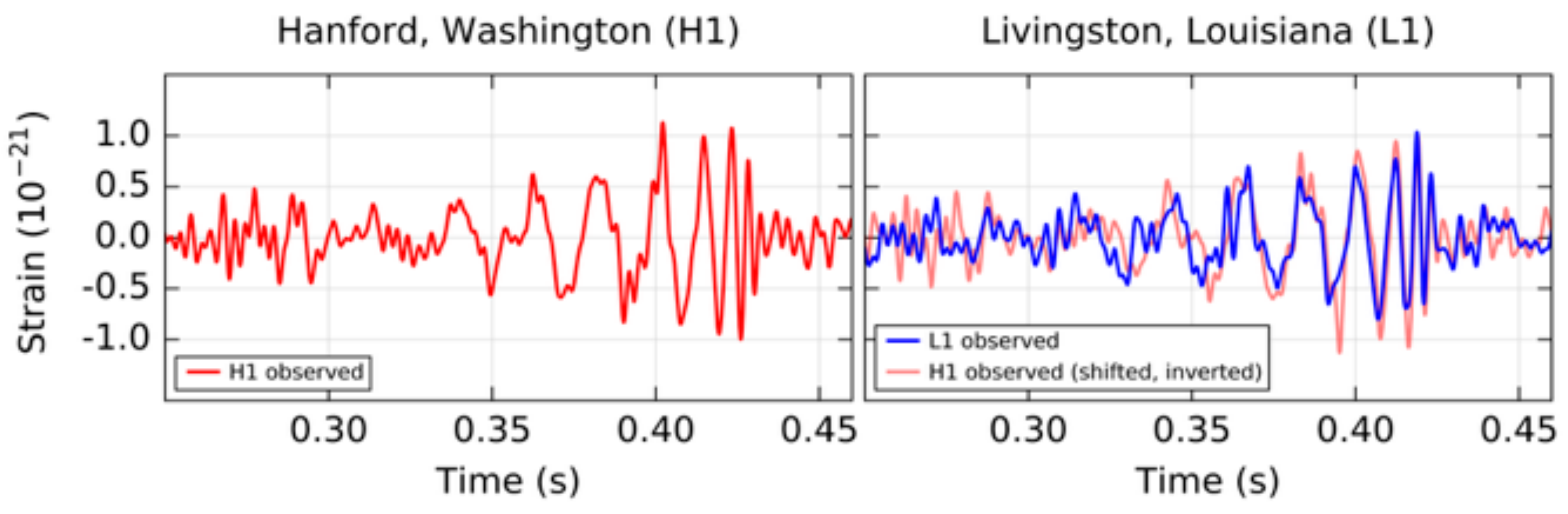
1915: General relativity is born

2005: First simulation of black holes and their emitted gravity waves (Frans Pretorius)

1950s - 1960s: Existence of solutions (Yvonne Choquet-Bruhat)
1957: Framework showing GWs can be measured (Felix Pirani)

2015: Gravitational waves observed by LIGO!

First Observation of GWs



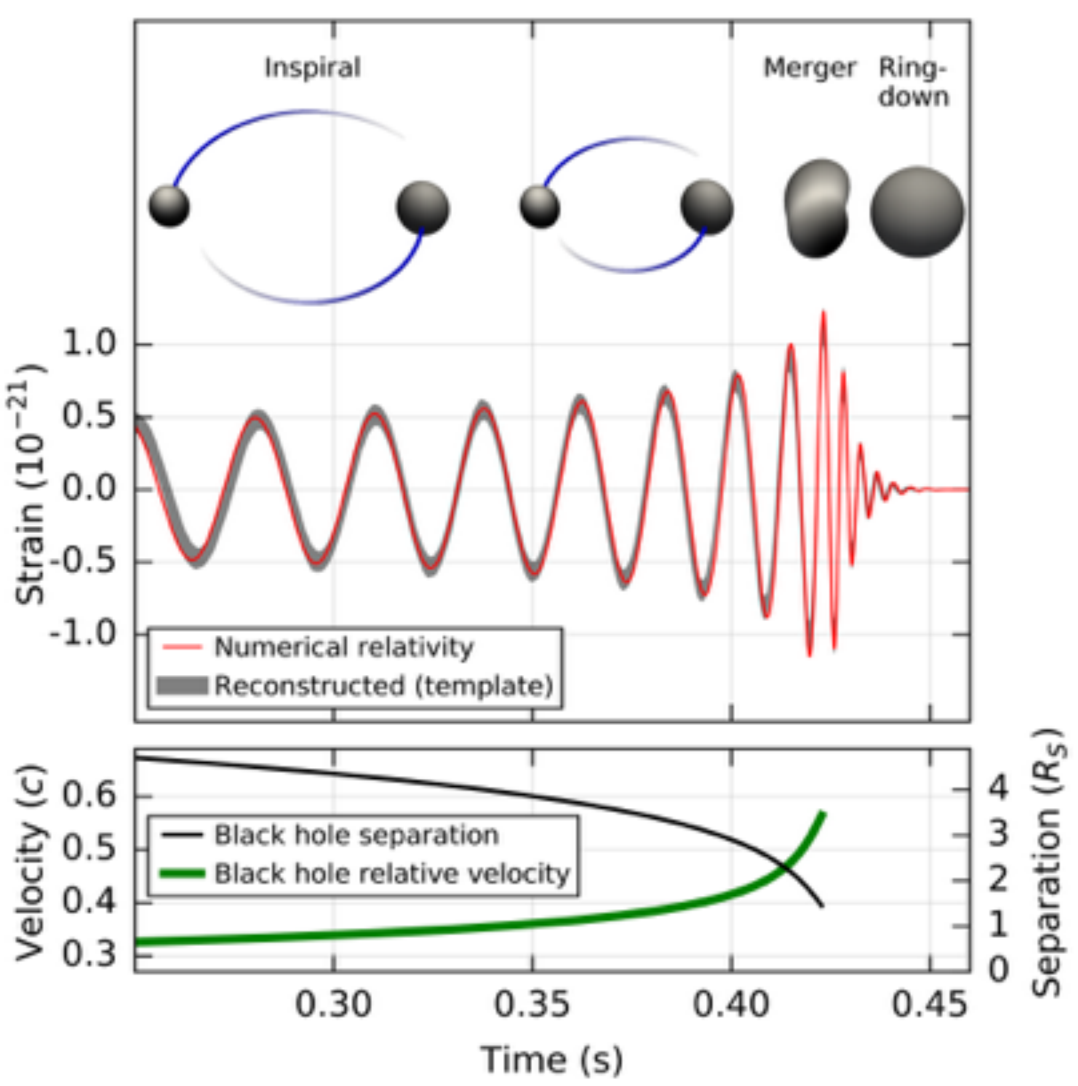
- On September 14, 2015 GWs passed through Earth
    - Scientific paper: Phys. Rev. Lett. 116, 061102
- Strain = GW signal measured by the LIGO detectors

Gravitational wave science (highlights)

Computer simulations are required to analyze the data.

Simulations are hard…

- Weeks of running on a supercomputer
- Advanced algorithms
- Advanced mathematical tools



Gravitational Waves Go Mainstream

The New York Times

Late Edition

$2.50

NEW YORK, FRIDAY, FEBRUARY 12, 2016

WITH FAINT CHIRP, SCIENTISTS PROVE EINSTEIN CORRECT

A RIPPLE IN SPACE-TIME

An Echo of Black Holes Colliding a Billion Light-Years Away

Scientists found gravitational waves in outer space.

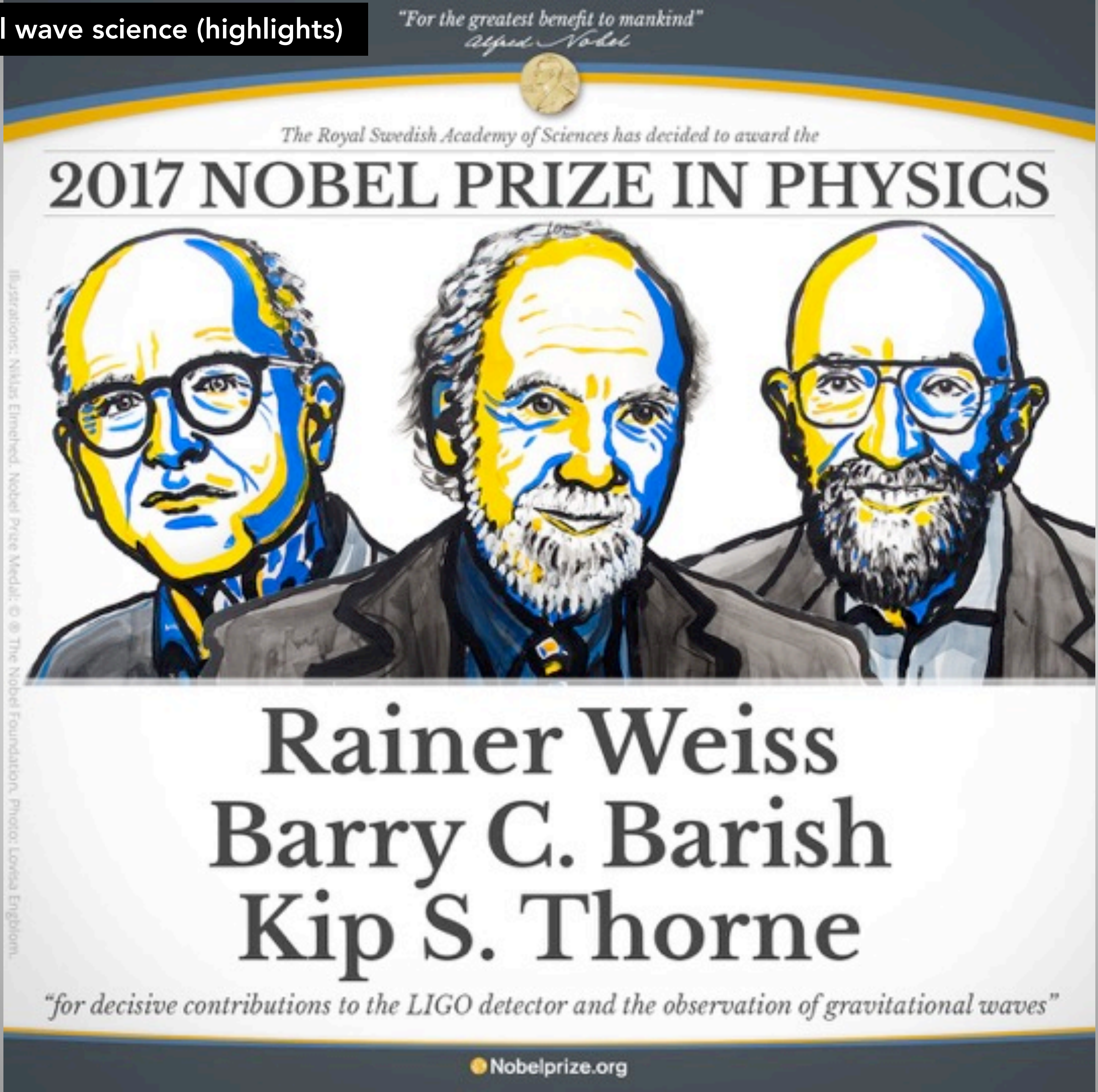
If only it were that easy to find an apartment in NYC with a walk-in closet.

Rent your own personal closet space:
manhattanministorage.com

manhattan mini storage
212-STORAGE

we're not scientists
but we totally got sp

Gravitational wave science (highlights)



"For the greatest benefit to mankind"
Alfred Nobel

The Royal Swedish Academy of Sciences has decided to award the

2017 NOBEL PRIZE IN PHYSICS

Rainer Weiss
Barry C. Barish
Kip S. Thorne

"for decisive contributions to the LIGO detector and the observation of gravitational waves"

Nobelprize.org

Illustrations: Niklas Elmehed. Nobel Prize Medal: © ® The Nobel Foundation. Photo: Lovisa Engblom.

Kip's 2016 visit (before his prize)

Bob Fisher, Richard Price

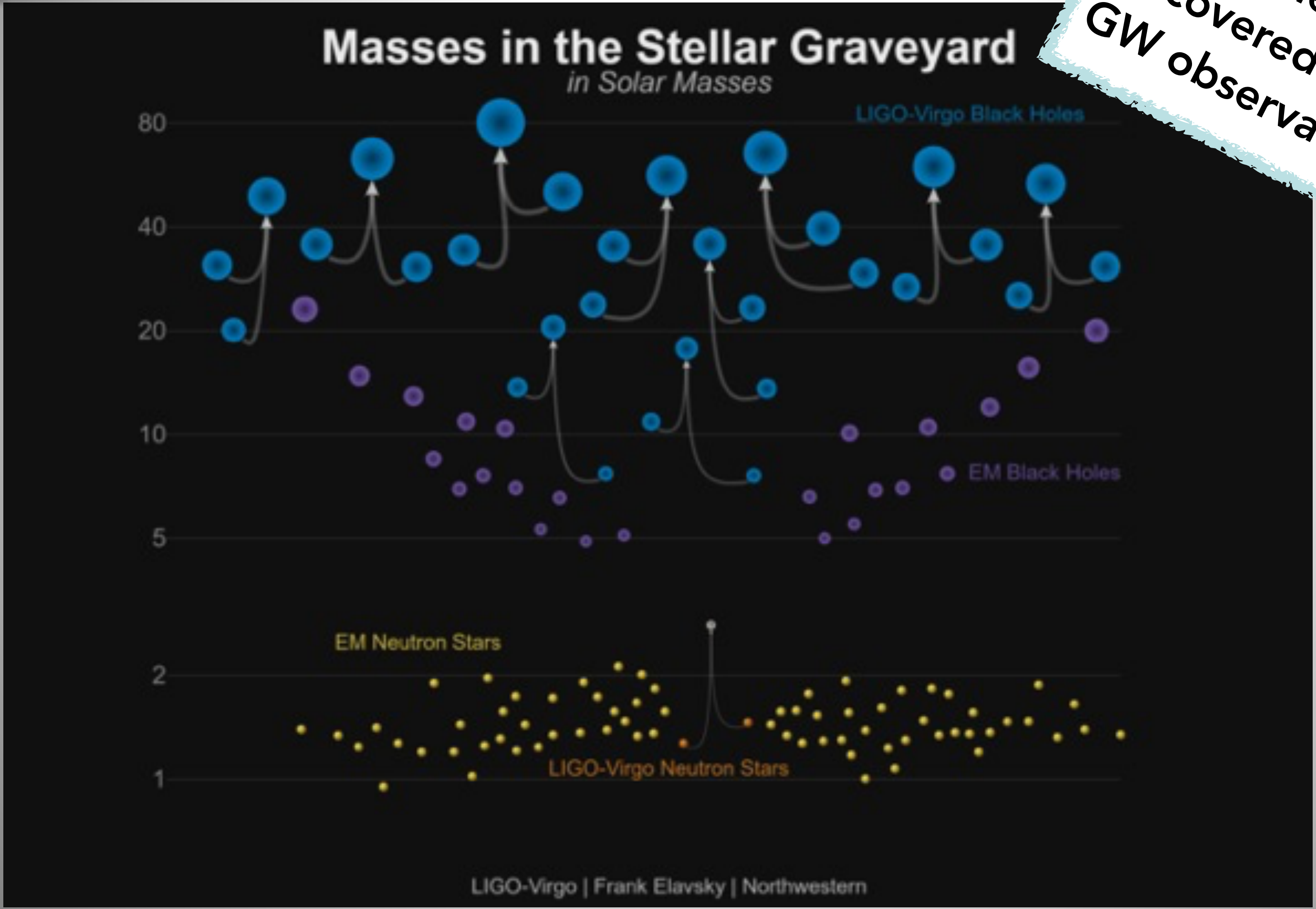
Kip

CSCVR directors, Sigal & Gaurav

PhD students, Zach & Tiffany

Black Hole Census

Black holes discovered through GW observations



LIGO-Virgo | Frank Elavsky | Northwestern

Gravitational wave science (highlights)

The observation of gravitational waves was an unprecedented ***experimental*** feat…

Figure 1. John Wheeler lecturing at a conference in Cambridge, UK, in 1971. Wheeler's style was to cover the blackboard with inspirational colored-chalk diagrams and phrases before the lecture, then work his way through them, one by one.

- … that required ***mathematical*** & ***computational*** breakthroughs

Engines of GW Science

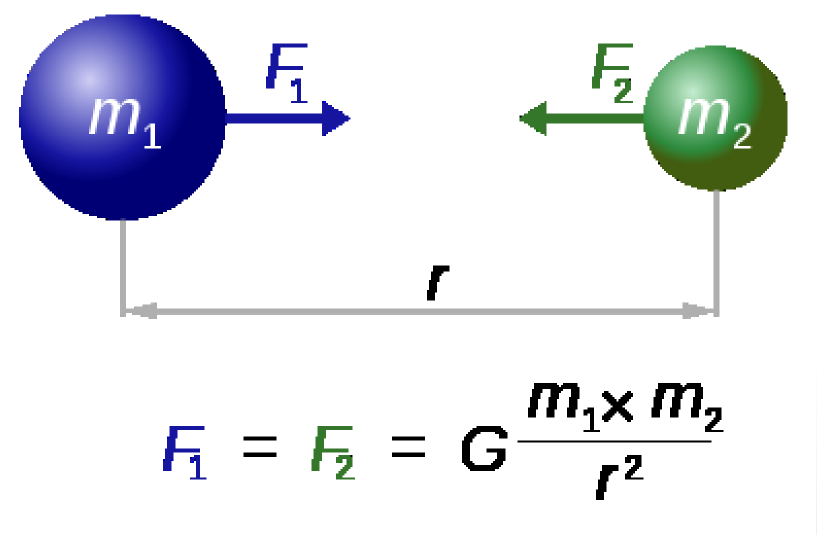
1. Astrophysical system to generate waves
    - Two black holes orbiting one another
2. Mathematical framework for computing the expected gravitational wave signal
3. Detectors to observe the signal
4. Algorithms and computers to solve equations
5. Data analysis tools to compare theory and observation

What are gravitational waves?

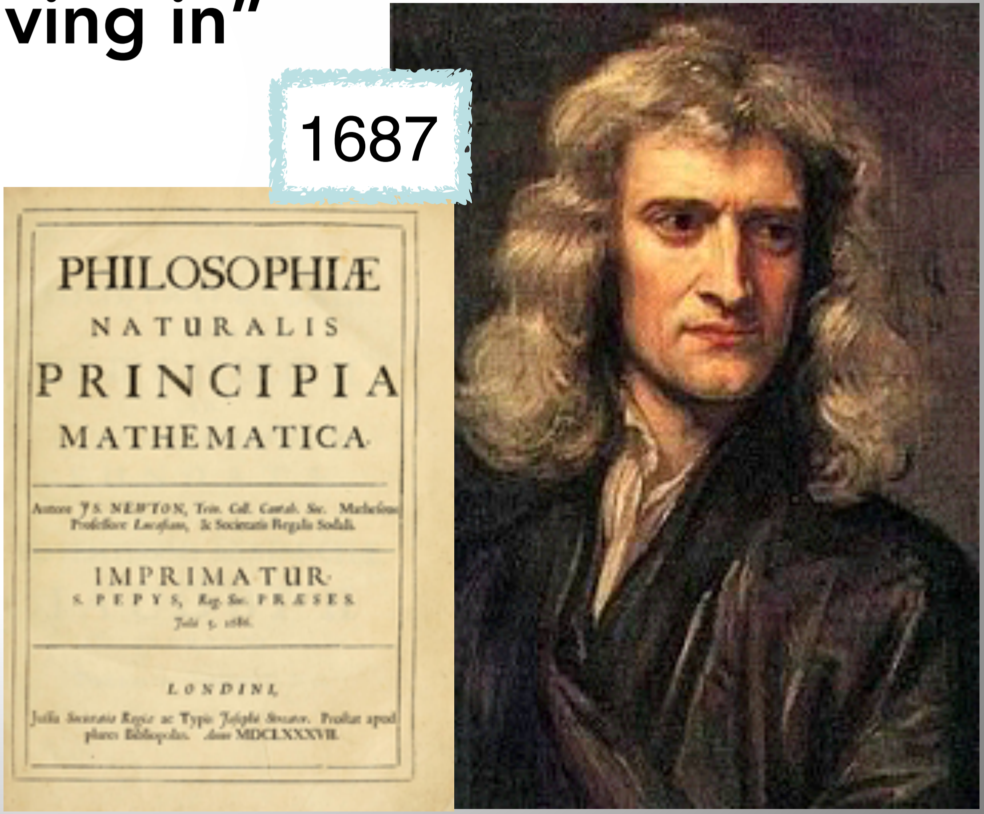
What is the mathematical theory that describes them?

What is Gravity? Newton’s Answer

- Gravity is a force between two objects
- No gravitational waves! Waves need a medium (e.g. water) to be “waiving in”



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$


1687

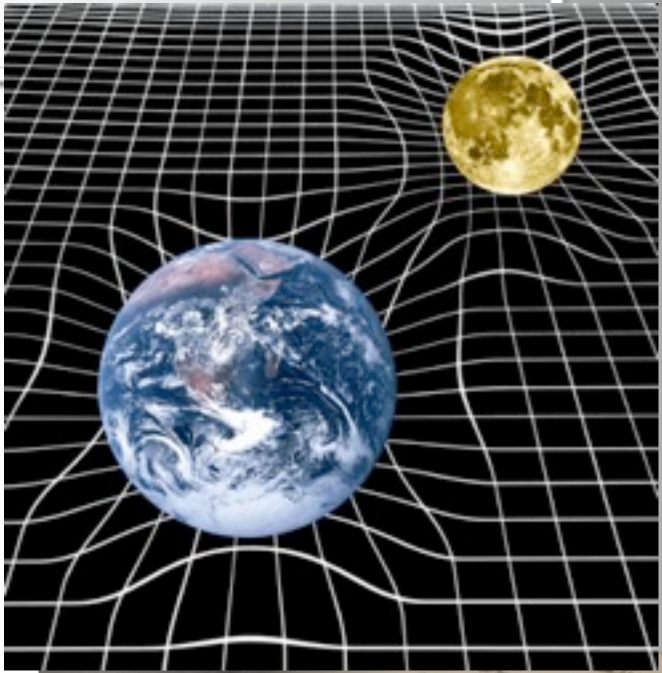
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

IMPRIMATUR
S. PEPYS, Reg. Soc. PRÆSES.

LONDINI,

Einstein's General Relativity

- Gravity is not a force in the usual sense of "push" or "pull"
- Mass causes space-time around it to bend or warp
- Path of objects (light included) is affected by this warped space-time
- The gravitational "force" is a manifestation of the bending of space and time

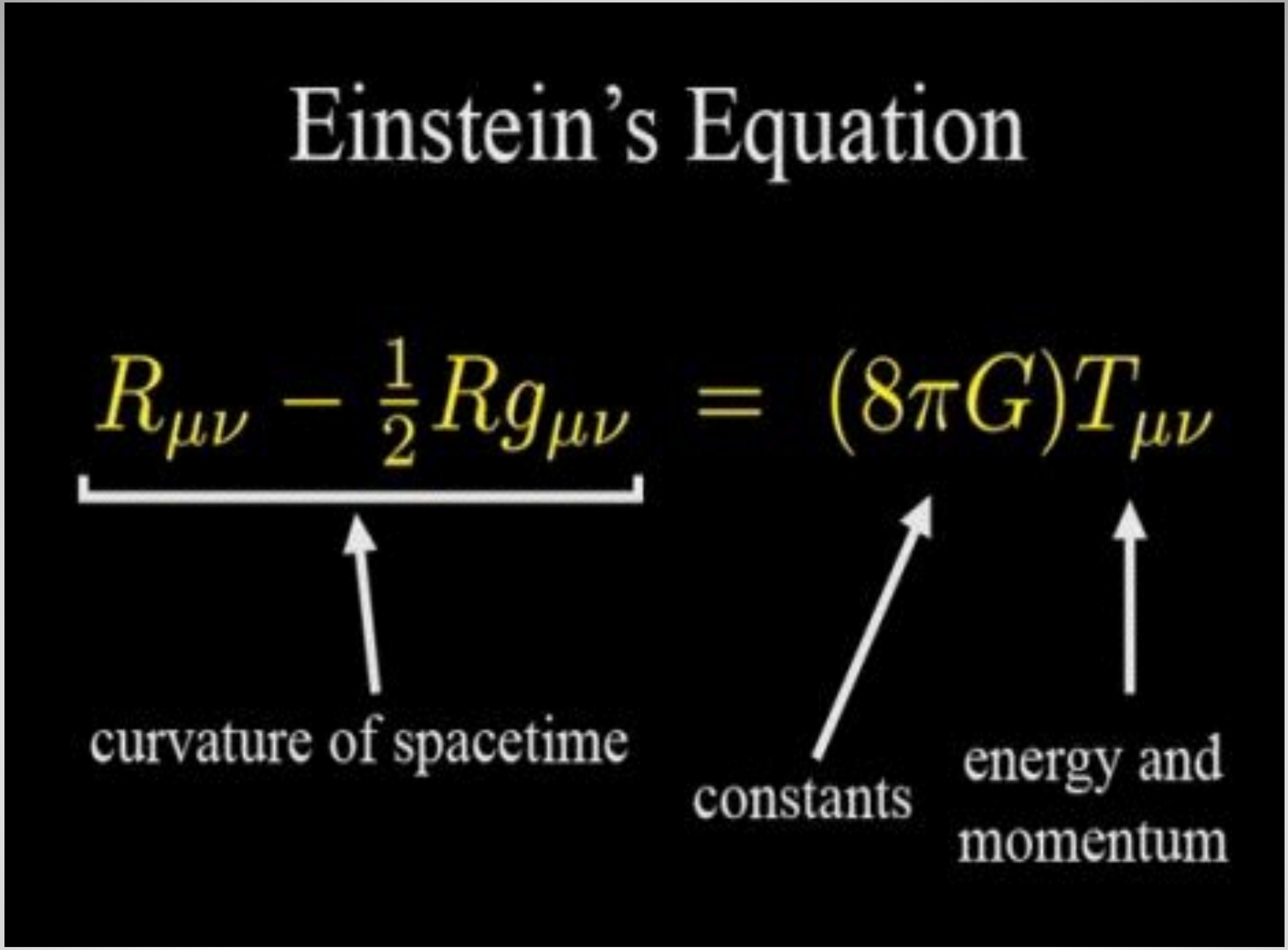


Example: Light Moving In Curved Space

- A black hole appears on an academic quad…

What are gravitational waves?

- Credit: Andy Bohn, et al.; SXS Collaboration



Einstein's Equation

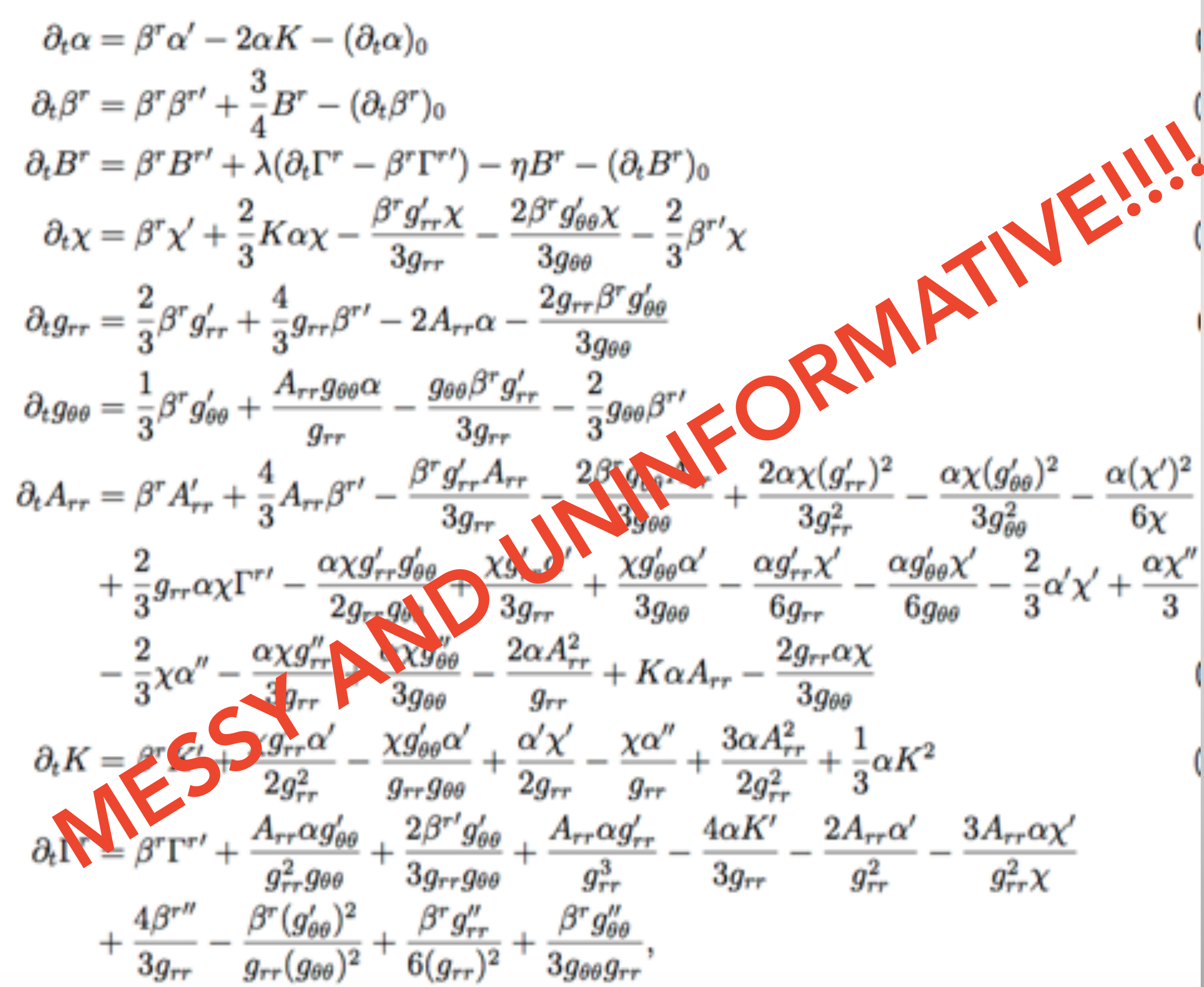
$$\underbrace{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}}_{\text{curvature of spacetime}} = (8\pi G)T_{\mu\nu}$$

constants

energy and momentum

- Secretly a partial differential equation (derivatives lurking in R; Ricci curvature)
- Solution to this equation describes the geometry of space and time
  - Gravitational lensing of an academic quad by a black hole
  - The distance from ICERM to Fenway Park changes when a GW passes by
  - ICERM employees age more quickly than Hemenway's empoylees

When solution has spherical symmetry…



$$\partial_t\alpha = \beta^r\alpha' - 2\alpha K - (\partial_t\alpha)_0$$

$$\partial_t\beta^r = \beta^r\beta^{r\prime} + \frac{3}{4}B^r - (\partial_t\beta^r)_0$$

$$\partial_t B^r = \beta^r B^{r\prime} + \lambda(\partial_t\Gamma^r - \beta^r\Gamma^{r\prime}) - \eta B^r - (\partial_t B^r)_0$$

$$\partial_t\chi = \beta^r\chi' + \frac{2}{3}K\alpha\chi - \frac{\beta^r g'_{rr}\chi}{3g_{rr}} - \frac{2\beta^r g'_{\theta\theta}\chi}{3g_{\theta\theta}} - \frac{2}{3}\beta^{r\prime}\chi$$

$$\partial_t g_{rr} = \frac{2}{3}\beta^r g'_{rr} + \frac{4}{3}g_{rr}\beta^{r\prime} - 2A_{rr}\alpha - \frac{2g_{rr}\beta^r g'_{\theta\theta}}{3g_{\theta\theta}}$$

$$\partial_t g_{\theta\theta} = \frac{1}{3}\beta^r g'_{\theta\theta} + \frac{A_{rr}g_{\theta\theta}\alpha}{g_{rr}} - \frac{g_{\theta\theta}\beta^r g'_{rr}}{3g_{rr}} - \frac{2}{3}g_{\theta\theta}\beta^{r\prime}$$

$$\begin{aligned}\partial_t A_{rr} = {}& \beta^r A'_{rr} + \frac{4}{3}A_{rr}\beta^{r\prime} - \frac{\beta^r g'_{rr}A_{rr}}{3g_{rr}} - \frac{2\beta^r g'_{\theta\theta}A_{rr}}{3g_{\theta\theta}} + \frac{2\alpha\chi(g'_{rr})^2}{3g_{rr}^2} - \frac{\alpha\chi(g'_{\theta\theta})^2}{3g_{\theta\theta}^2} - \frac{\alpha(\chi')^2}{6\chi} \\ & + \frac{2}{3}g_{rr}\alpha\chi\Gamma^{r\prime} - \frac{\alpha\chi g'_{rr}g'_{\theta\theta}}{2g_{rr}g_{\theta\theta}} + \frac{\chi g'_{rr}\alpha'}{3g_{rr}} + \frac{\chi g'_{\theta\theta}\alpha'}{3g_{\theta\theta}} - \frac{\alpha g'_{rr}\chi'}{6g_{rr}} - \frac{\alpha g'_{\theta\theta}\chi'}{6g_{\theta\theta}} - \frac{2}{3}\alpha'\chi' + \frac{\alpha\chi''}{3} \\ & - \frac{2}{3}\chi\alpha'' - \frac{\alpha\chi g''_{rr}}{3g_{rr}} - \frac{\alpha\chi g''_{\theta\theta}}{3g_{\theta\theta}} - \frac{2\alpha A_{rr}^2}{g_{rr}} + K\alpha A_{rr} - \frac{2g_{rr}\alpha\chi}{3g_{\theta\theta}}\end{aligned}$$

$$\partial_t K = \beta^r K' + \frac{\chi g'_{rr}\alpha'}{2g_{rr}^2} - \frac{\chi g'_{\theta\theta}\alpha'}{g_{rr}g_{\theta\theta}} + \frac{\alpha'\chi'}{2g_{rr}} - \frac{\chi\alpha''}{g_{rr}} + \frac{3\alpha A_{rr}^2}{2g_{rr}^2} + \frac{1}{3}\alpha K^2$$

$$\begin{aligned}\partial_t\Gamma^r = {}& \beta^r\Gamma^{r\prime} + \frac{A_{rr}\alpha g'_{\theta\theta}}{g_{rr}^2 g_{\theta\theta}} + \frac{2\beta^{r\prime}g'_{\theta\theta}}{3g_{rr}g_{\theta\theta}} + \frac{A_{rr}\alpha g'_{rr}}{g_{rr}^3} - \frac{4\alpha K'}{3g_{rr}} - \frac{2A_{rr}\alpha'}{g_{rr}^2} - \frac{3A_{rr}\alpha\chi'}{g_{rr}^2\chi} \\ & + \frac{4\beta^{r\prime\prime}}{3g_{rr}} - \frac{\beta^r(g'_{\theta\theta})^2}{g_{rr}(g_{\theta\theta})^2} + \frac{\beta^r g''_{rr}}{6(g_{rr})^2} + \frac{\beta^r g''_{\theta\theta}}{3g_{\theta\theta}g_{rr}},\end{aligned}$$

MESSY AND UNINFORMATIVE!!!!

Mathematical Structure of Equations

- System of coupled, nonlinear partial differential equations
- When written with first order derivatives of time and space, there are 52 equations with hundreds of terms!
- Paper-and-pencil solutions only known for simple cases; computers are needed

Does Einstein's equation of general relativity allow for gravitational waves?

Theoretical Justification for Gravitational Waves?

Existence of GWs was debated until the late 1950s

1. Existence of solutions? (Not obvious)
2. Equations are too hard to solve, so how can we say anything concrete about the possibility of gravitational waves?

Issue 1: Existence of Solutions

Under what conditions can we solve Einstein’s equation of general relativity?

Why this matters: If solutions don’t exist, it doesn’t make sense to ask a carry out computer simulations

Interlude: When Can We Solve an Equation?

Q: Solve for x

$$12 + 2x - 8 = 7x + 5 - 5x$$

Interlude: When Can We Solve an Equation?

Q: Solve for x

$$12 + 2x - 8 = 7x + 5 - 5x$$

A:

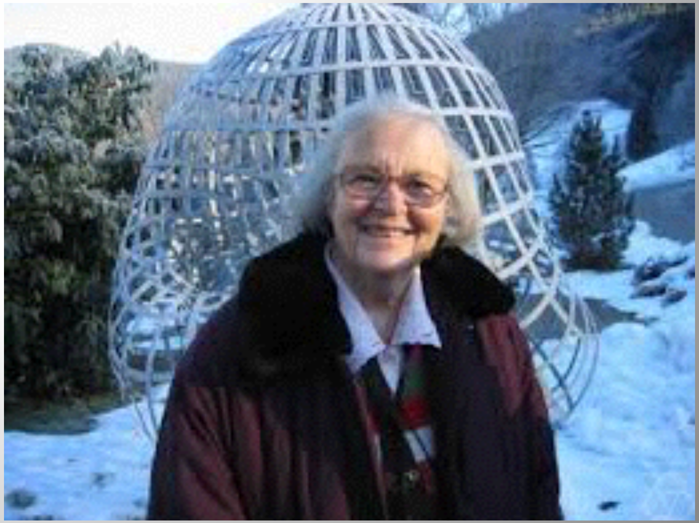
$$4 + 2x = 5 + 2x$$

$$4 = 5$$

Not all equations can be solved...

Roadmap to Solvability

- (1930's) Mathematical tools developed by Kurt Friedrichs, Hans Lewy, and Sergei Sobolev
- (1947) A graduate student, Yvonne Choquet-Bruhat (YCB), begins using these new tools to show the equations can be solved
- (1952) YCB *shows Einstein equations have solutions under restricted conditions*
- (1969) YCB + Robert Geroch *extend the results to general conditions*



Issue 2: gravitational waves?

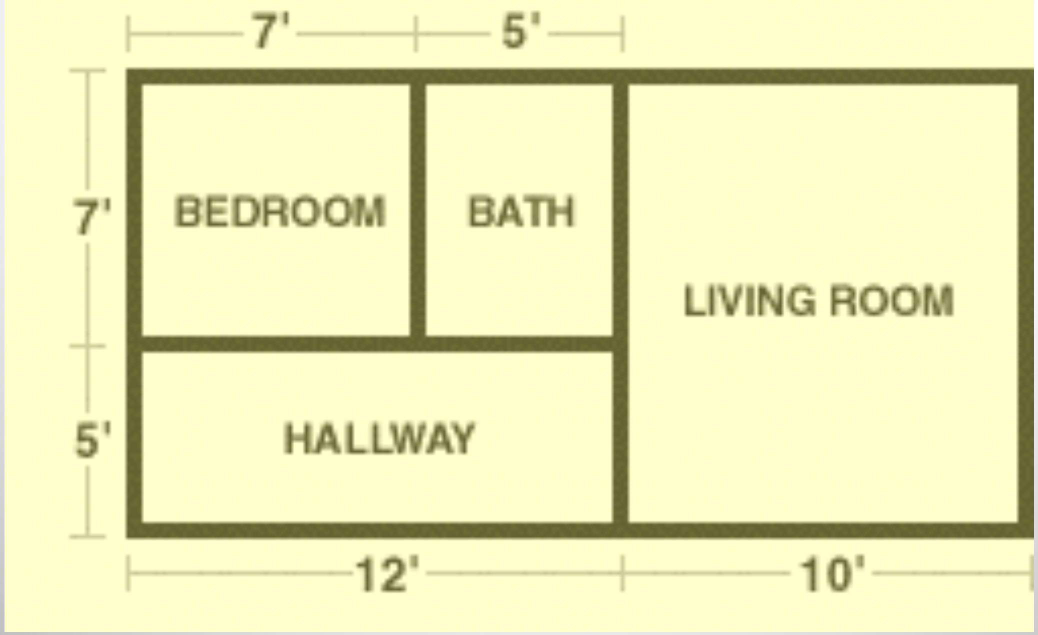
Do the Einstein equations admit solutions that can be interpreted as gravitational waves?

Why this matters: Why spend billions of dollars building gravitational wave detectors if they don't exist?

- (1916) Einstein finds approximate solutions that are waves; but he dismisses them as unphysical…

Interlude: When Can We Trust a Solution?

Q: You have a square bedroom thats 49 square feet. Whats the length of the wall?



Interlude: When Can We Trust a Solution?

Q: You have a square bedroom thats 49 square feet.
Whats the length of the wall?

A: X = wall’s length

$$x^2 = 49$$

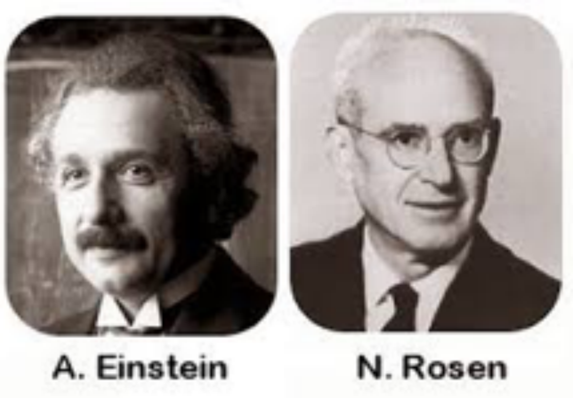
Mathematically, the wall could be 7 feet or -7 feet.
Physically, only 7 feet makes sense.

Not all solutions can be trusted…

Roadmap to Waves: Part I

(1934) The Einstein and Rosen paper

- (Preprint) "… I arrive at the interesting result that GWs do not exist"

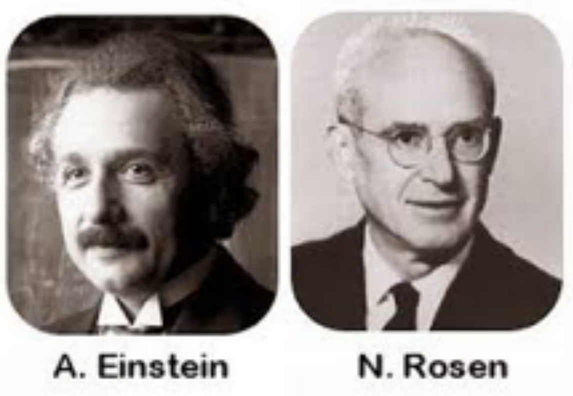

A. Einstein N. Rosen

(Preprint) 1934: "Do gravitational waves exist?"

Roadmap to Waves: Part I

(1934) The Einstein and Rosen paper

- (Preprint) "… I arrive at the interesting result that GWs do not exist"
- (Revision) "…The second part of the article was altered by me…as we had misinterpreted the results… I want to thank my colleague Professor Roberston…."



(Preprint) 1934: "Do gravitational waves exist?"

("Revised" paper) 1934: "On gravitational waves"

Roadmap to Waves: Part II

- (1957) The Chapel Hill conference
    - "Proof by discussion": Pirani derived an equation that could be used to measure gravitational waves. A thought experiment by Feynman and Bondi showed the waves could generate heat, and were therefore physical.

Felix Pirani

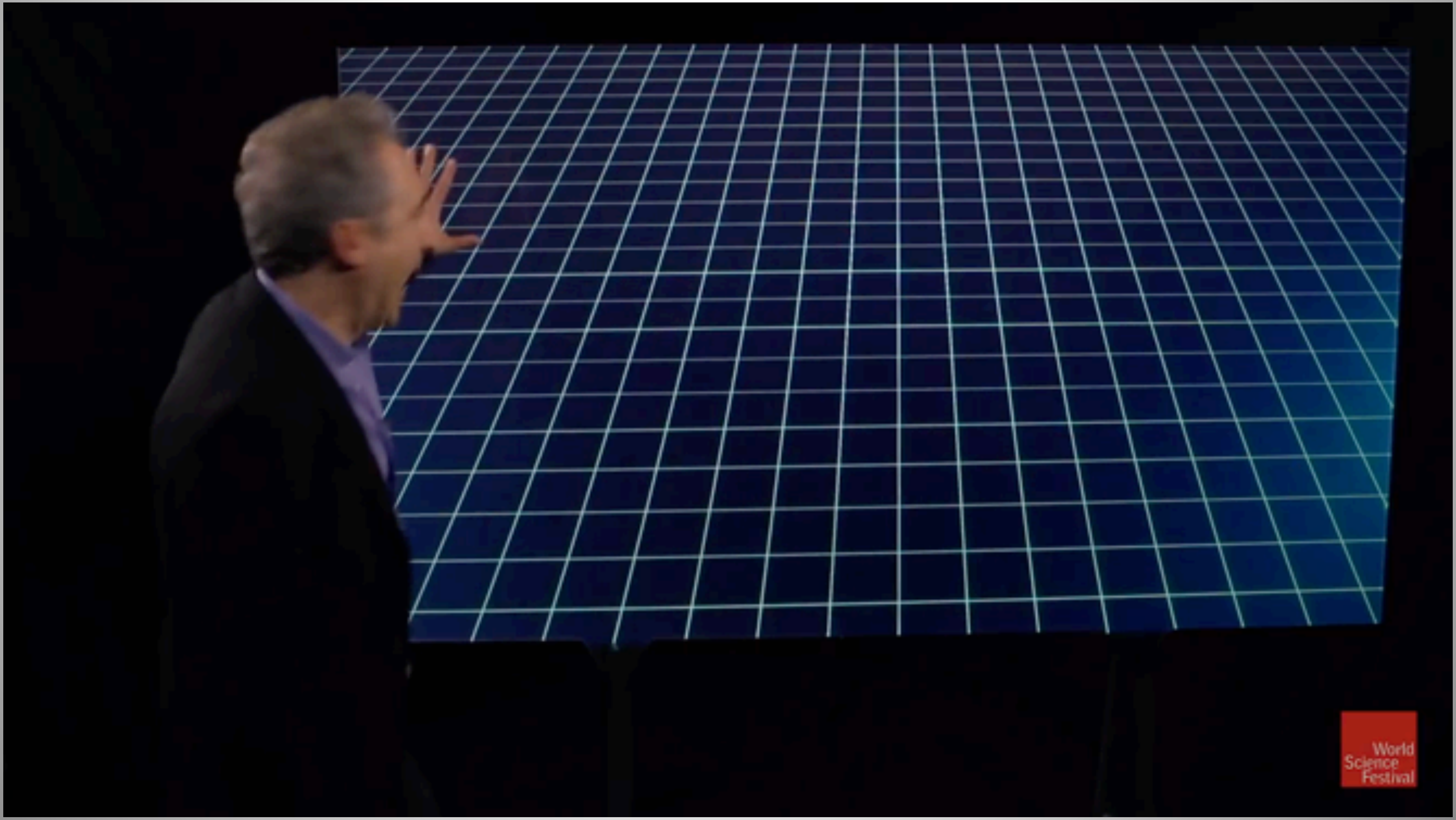
Richard Feynman, Hermann Bondi, Joseph Weber

Thanks to the hard work of many researchers from 1915 to the mid-1960s we now know

1. It makes sense to solve Einstein’s equations of general relativity under general conditions
2. Gravitational waves are one particularly import feature of the solutions

Lets see what these waves look like…

Generation and propagation of gravitational waves



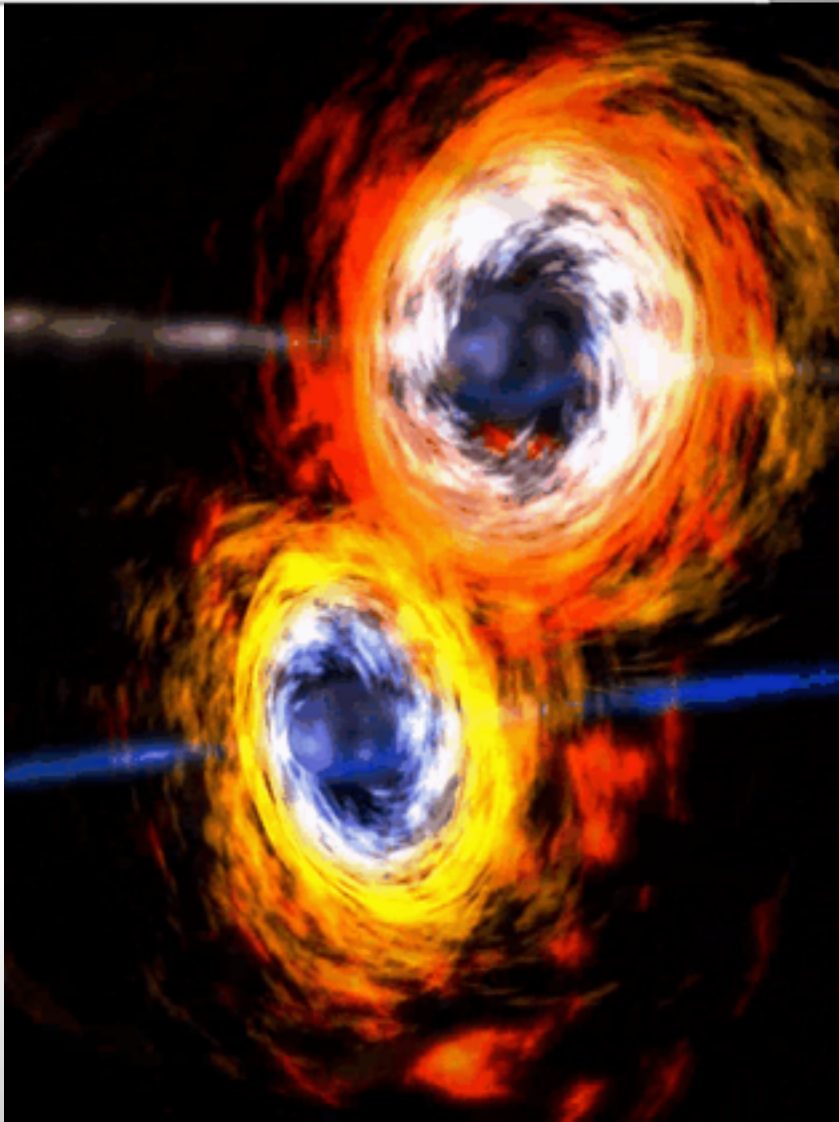
How to observe gravitational waves?

General relativity predicts their existence. How to test the prediction?

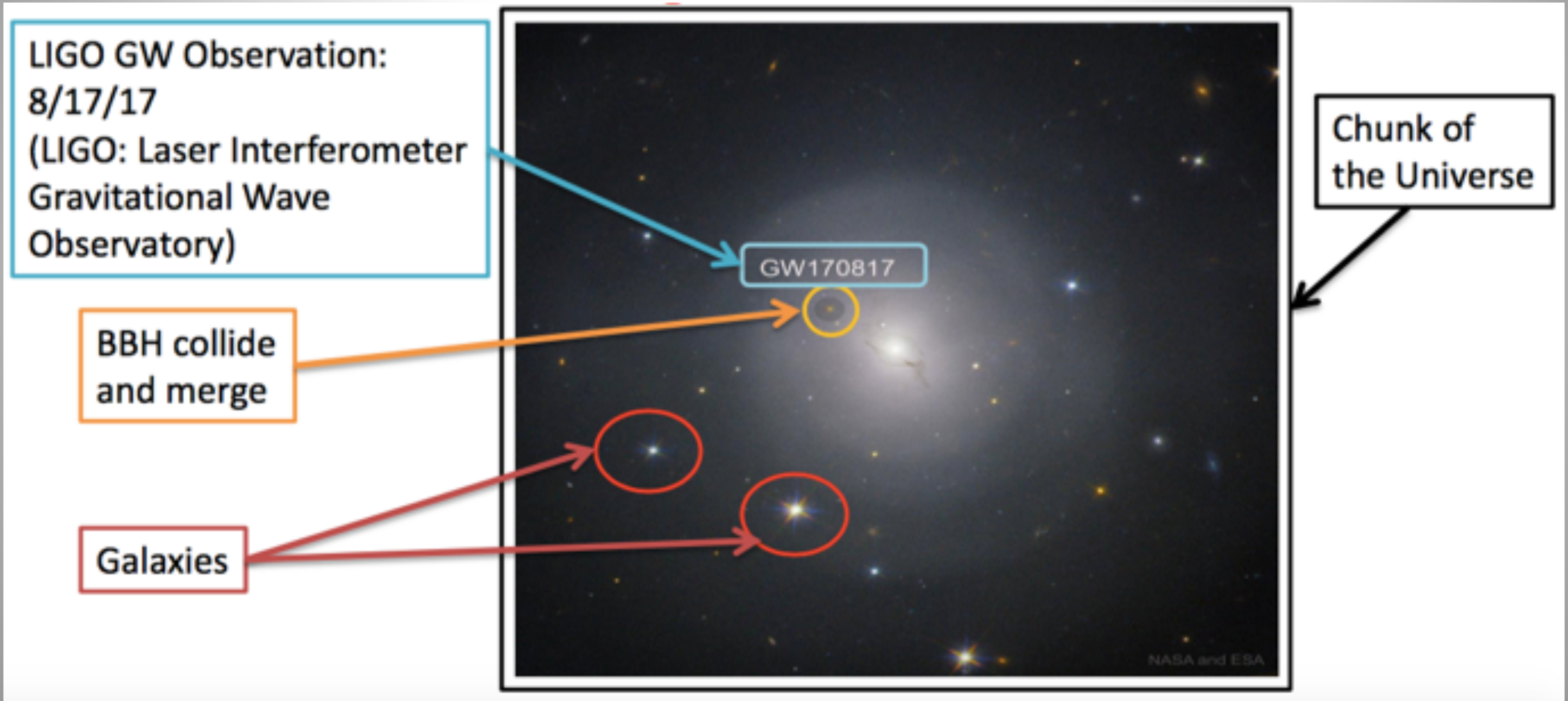
Strong sources of gravitational waves

The most promising sources of gravitational waves are those that move dense objects at high accelerations.

Examples are supernovae and collisions of compact objects like neutron stars and black holes.



Real-world example



* *Not* seen with an optical (traditional) telescope

Fig. by Kimberly Matsuda, Mathematics undergrad

Real-world example

- Use the gravitational wave signal to answer scientific questions
    - Properties of the black hole system (e.g. masses)
    - Is Einstein's theory of relativity correct?
    - Number of space dimensions
    - Speed of gravity waves
    - Populations of black holes

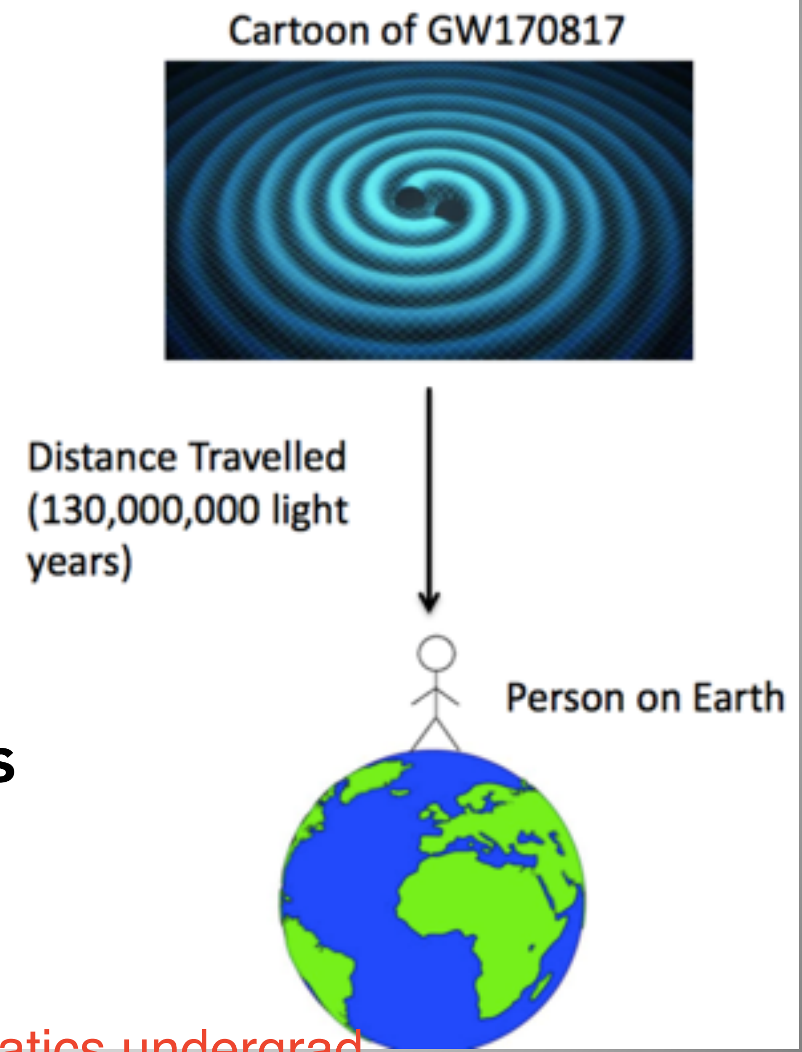
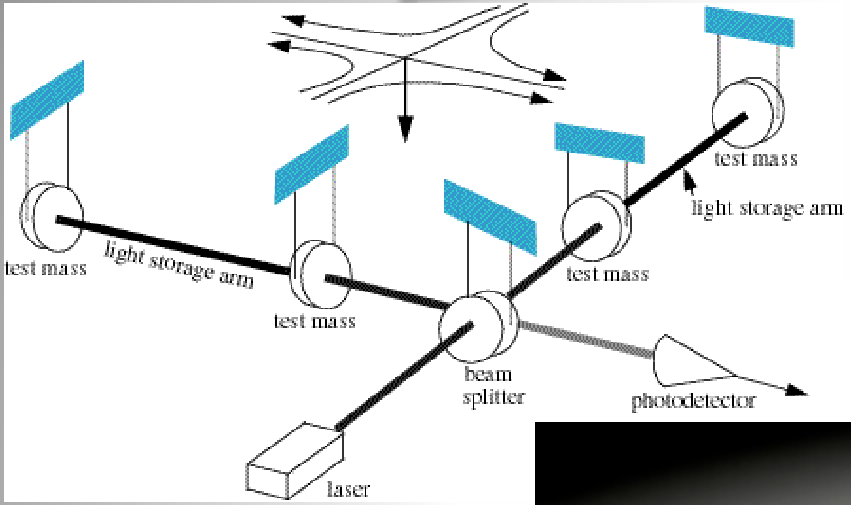
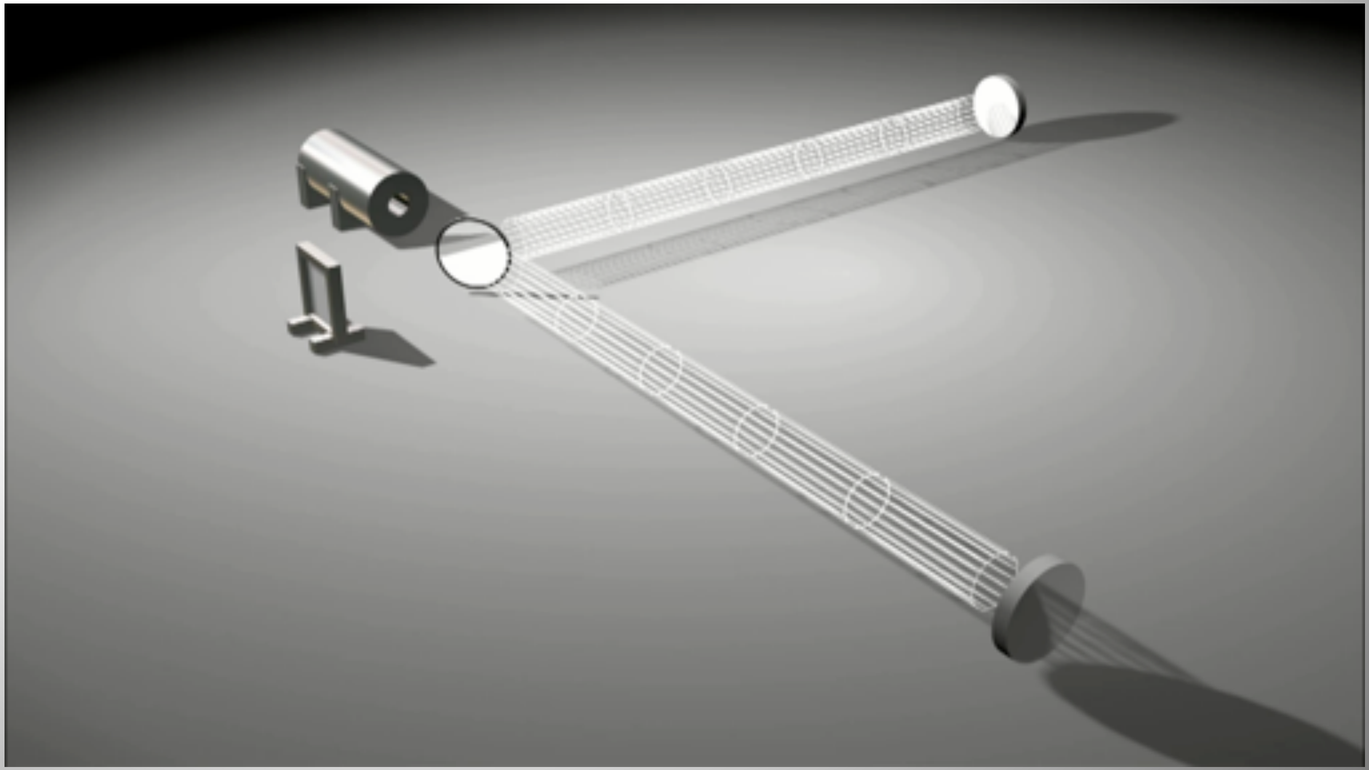


Fig. by Kimberly Matsuda, Mathematics undergrad

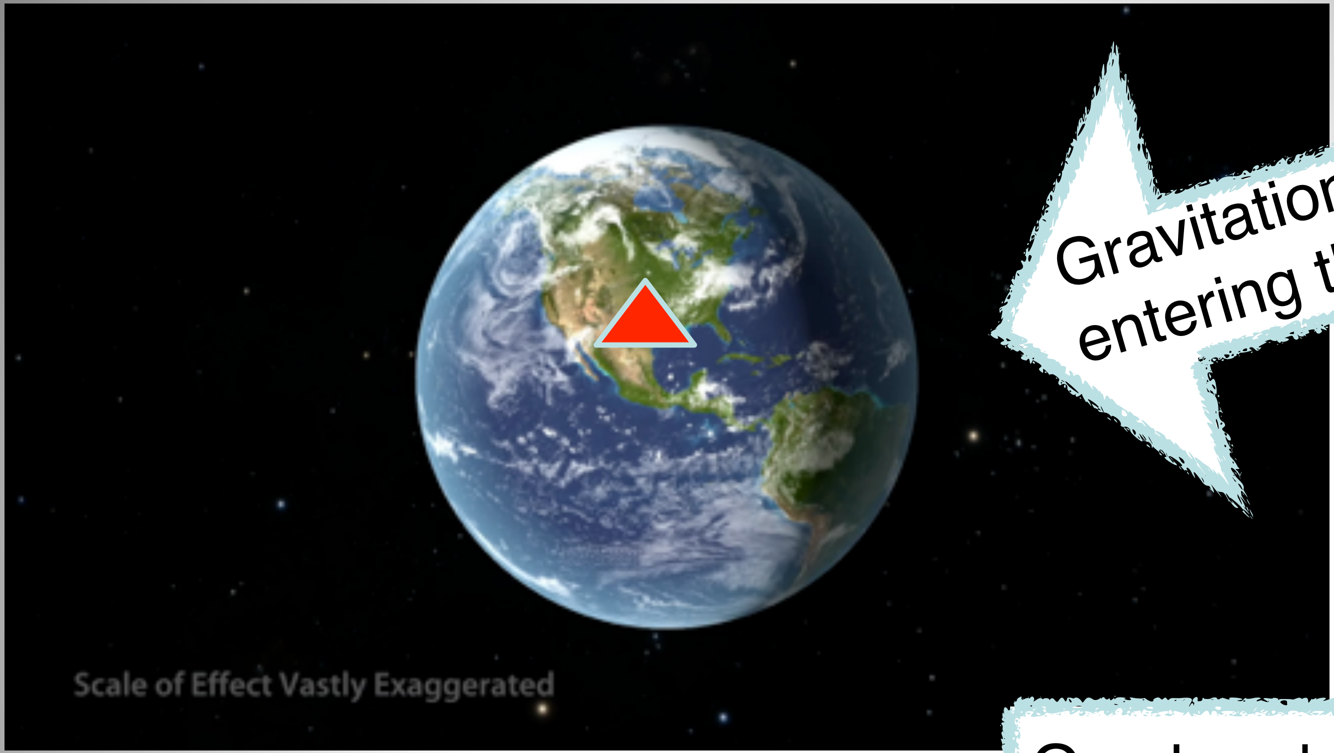
Detectors on Earth

How the detector works

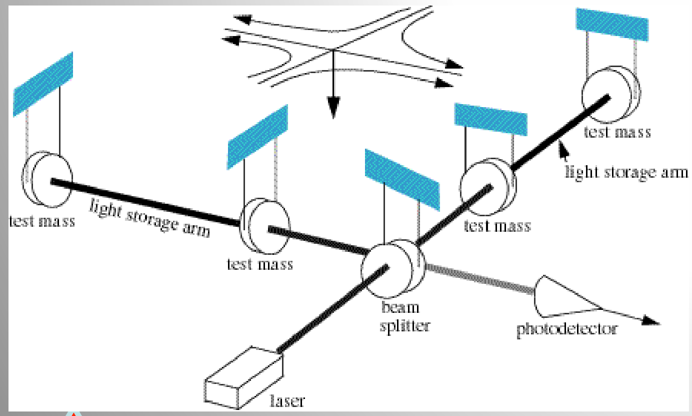




How to observe gravitational waves?

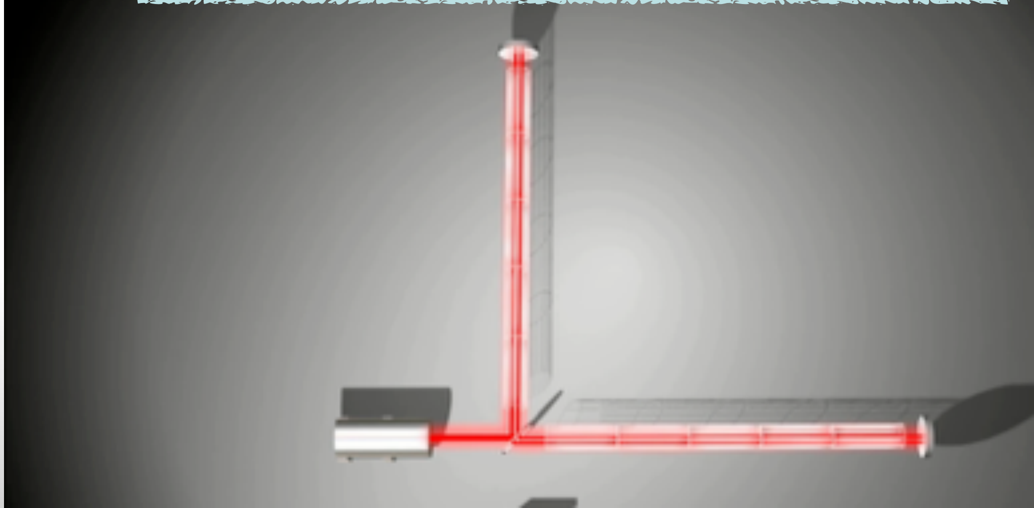


Gravitational waves are entering the detector



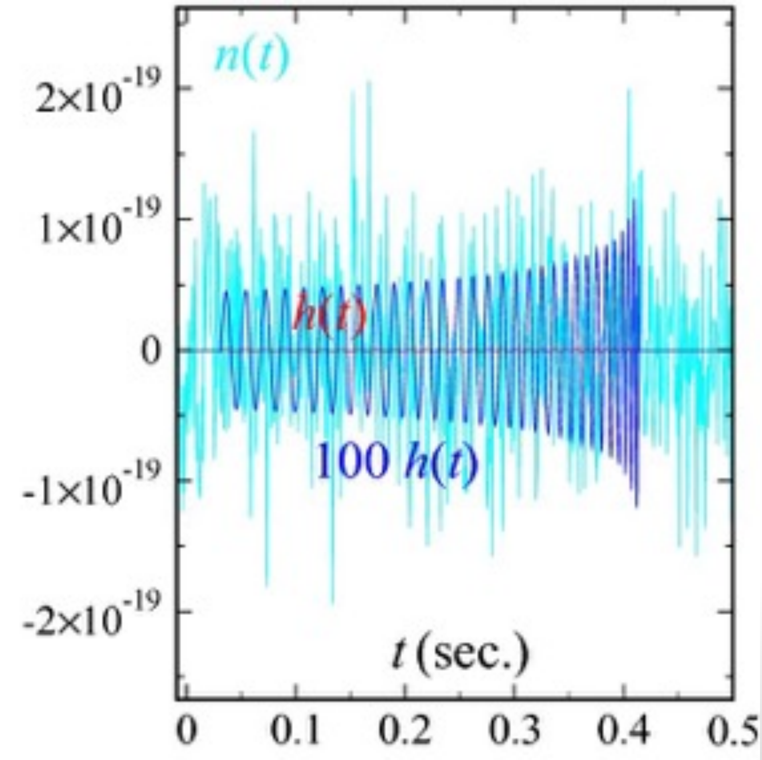
▲ = GW detector

Overhead view of detector



Measuring small changes

- No GWS: distance between mirrors is 4 kilometers
- GW causes small, time-dependent changes $\Delta L \approx 10^{-18} km$



- Smaller than the size of a proton
- Gravitational wave detection requires complicated data analysis algorithms

Role of Computational Models

- Signal is weak, buried in detector noise
- Precise computer models are used to generate thousands / millions of "template" signals from likely sources
- Comparison to templates allow for detection and parameter estimation
- Computational modeling is absolutely essential in the discovery process!

Solving Einstein’s equation on a computer

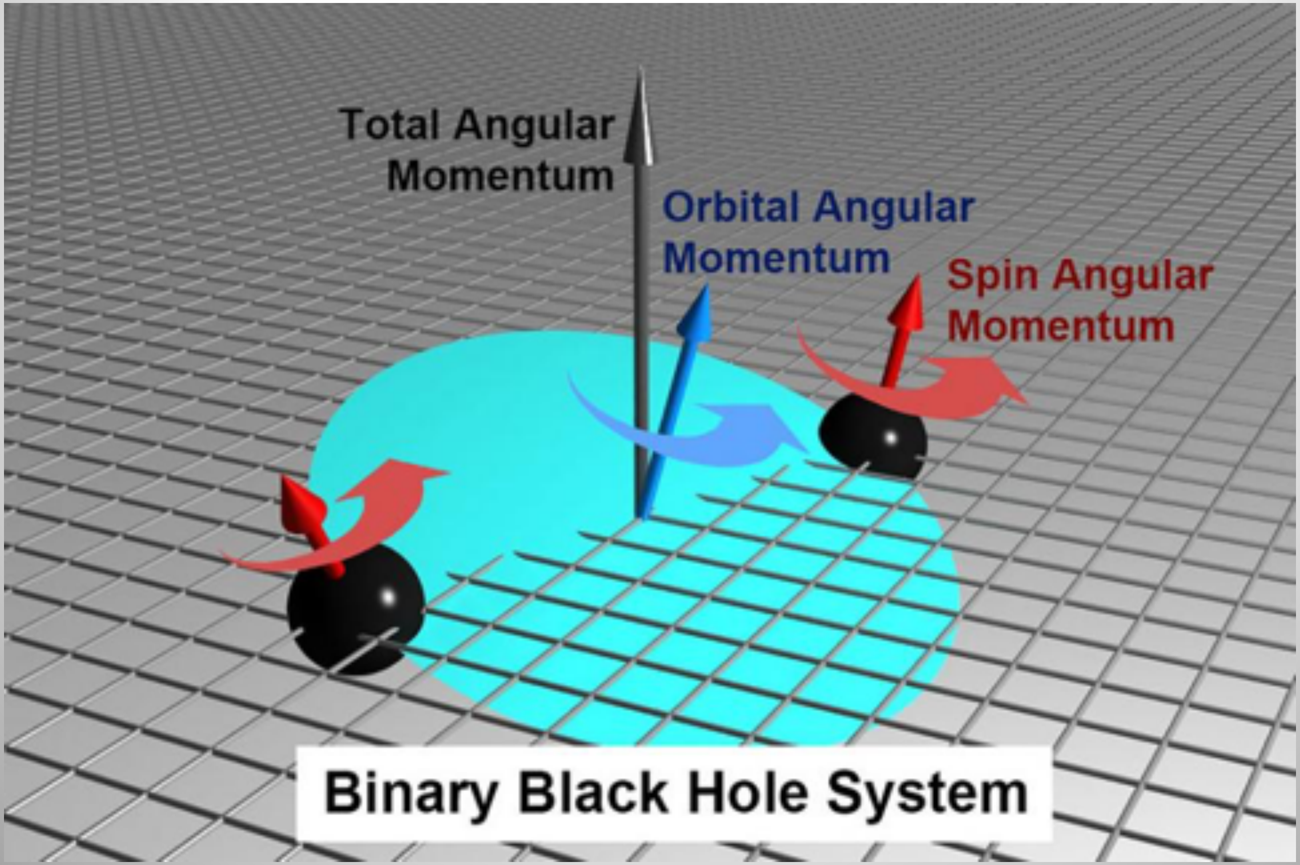
(A brief history of computational relativity)

What is computational relativity?

- Use algorithms, mathematical tools, and computational resources to find solutions of general relativity equations
- We specify the problem's setup: masses of the black holes, spins of the black holes, etc.
- Computer simulations solve the equations for this configuration to get various outputs like the gravitational wave signal

Two body problem (setup)

Input to the computer: black holes mass & spins



Just differential equations…

- "These are just differential equations, and people solve those all the time. Throw them on a computer. Just do it!"
    - Weather simulations
    - Airplane simulations
    - Rocket re-entry simulations

It took 50 years!

- (1957) Origins of computational relativity
  - Bryce DeWitt spends time at Lawrence Livermore National Lab working on fluid simulations. He and Charles Misner suggest the following:

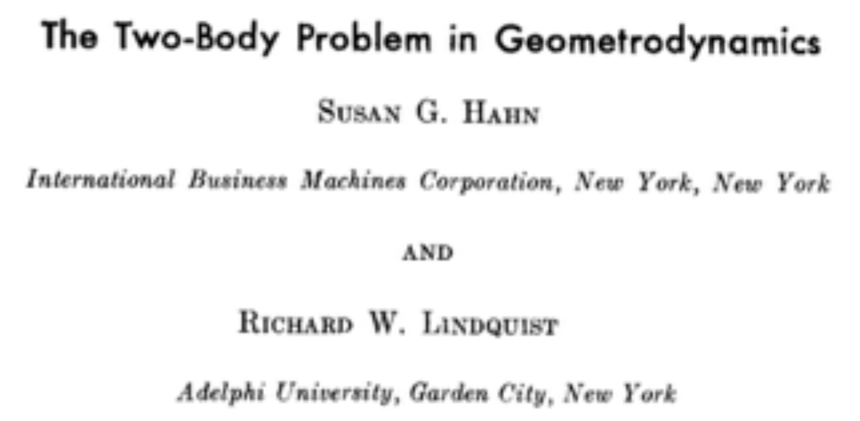
"First we assume that you have a computing machine better than anything we have now, and many programmers and a lot of money, and you want to look at a nice pretty solution of the Einstein equations…."

(2005) First stable simulation of two black holes

- Strongly hyperbolic formulations with constraint damping
- 2nd order finite difference (still predominate use)

High-performance computing in 1964

- First attempt carried out by Hahn and Lindquist
- Hahn, a student of Peter Lax, had access to the IBM 7090 supercomputer
  - 1 MegaFLOPs
  - Cost 3 million
- 51x51 mesh points
- Crashes in 50 steps
- 4 minutes/step



The Two-Body Problem in Geometrodynamics

Susan G. Hahn

International Business Machines Corporation, New York, New York

AND

Richard W. Lindquist

Adelphi University, Garden City, New York

An unsolicited proposal — 1983

- (1982) Peter Lax's report on supercomputing in the US: Why are there no supercomputers available to US academics
    - Simulations done in Germany or classified project
- Larry Smarr, who works in computational relativity, is using supercomputers through a friend at Livermore Lab.
- Larry writes the first *unsolicited* NSF proposal "A center for Scientific & Engineering Supercomputing" to be funded
    - The first supercomputing center network is born
        - Cornell, NCSA, Pittsburgh, San Diego, Princeton
        - Many of the first simulations are computational relativity

Progress, but somethings wrong

- (early 1990s) Despite new supercomputing centers (thanks to Larry) and decades of research effort, no one can evolve a binary black hole system
    - Algorithms or formulations are unstable — codes crash
- (1993) LIGO is funded to detect gravitational waves; but we don't know what they look like!
- (Late 1990s) NSF funds the Binary Black Hole Grand Challenge to support new mathematical, numerical, and HPC techniques to solve the problem

Grand Challenge: Why does the code “blow up”?

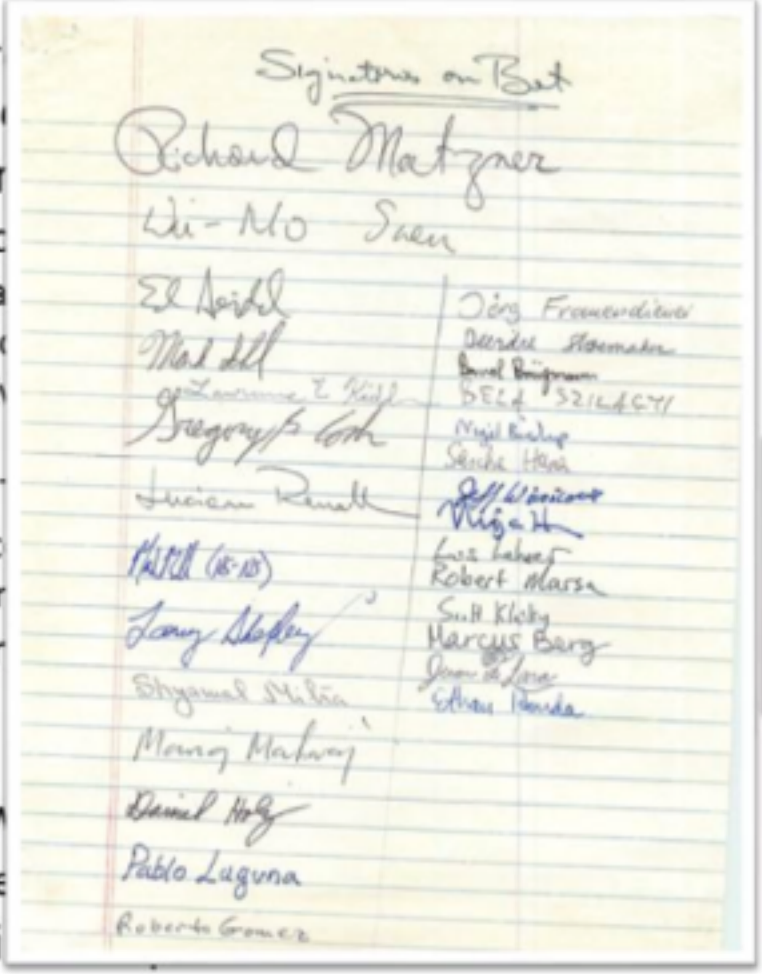
- Yvonne Choquet-Bruhat proved that solutions exist
- Whats the right way to instruct a computer to find them?
- There are many wrong ways, which lead to uncontrolled errors; the computer stops working
- This “blue screen of death” is a familiar situation for anyone who has used a computer

```
FATAL ERROR

A problem has been detected and system has been shut down to prevent damage
to your computer.

THE FOLLOWING FILES ARE MISSING:
REASON_TO_USE

If this is a laptop i'm pretty positive you're either dropped it a few times
or spilled a drink on it.

Technical information:

*** STOP: [illegible]

Press any key to continue_
```

Grand Challenge Collaborations

- Complex problems r
  scales: experiment
- NSF Black Hole Grar
  - Brought together c
    - University of Texa
    - NCSA/Illinois (Sei
    - North Carolina (E
    - Syracuse (G. Fox)
    - Cornell (Shapiro,
    - Pittsburgh (Winic
    - Penn State (Lagur
  - Many lessons learr
    appreciated
- NASA Neutron Star
  - Illinois, Argonne, W
  - Deliverables focuse
- Later: EU Network i

Richard



Signatures on Bet

Richard Matzner

Wai-Mo Suen

Lawrence E. Kidder

Gregory B. Cook

Shyamal Mitra

Daniel Holz

Pablo Laguna

Roberto Gomez

Jörg Frauendiener

Deirdre Shoemaker

BELA SZILAGYI

Nigel Bishop

Sascha Husa

Jeff Winicour

Luis Lehner

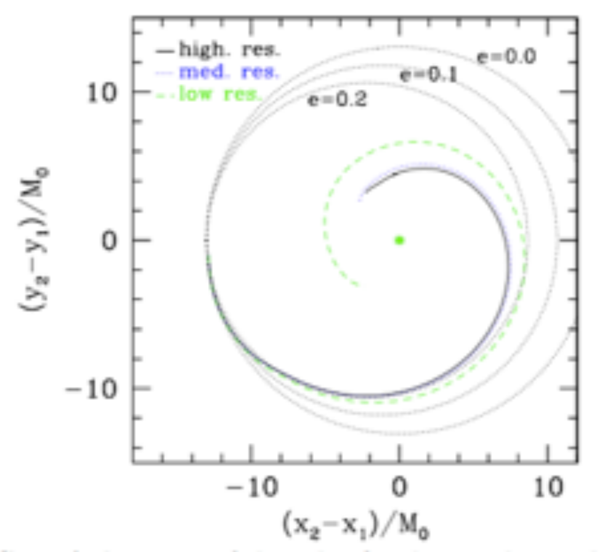
Robert Marsa

Scott Klasky

Marcus Berg

Ethan Honda

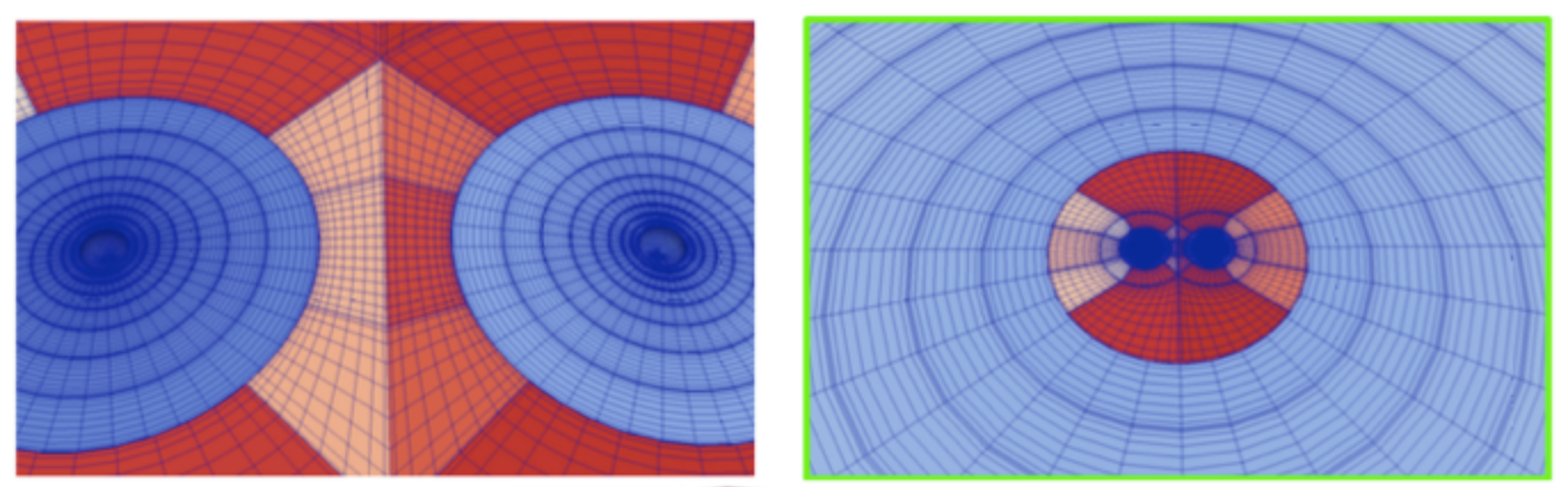
The 2005 Breakthrough



- Frans Pretorius' talk at the Banff mathematical research institute
  - First stable simulations of binary black holes; first numerical gravitational waves
- Key reasons it worked:
  - Constraint damping; originally proposed by mathematician Heinz-Otto Kriess
  - Adaptive mesh refinement
  - Removed the black hole singularity from the grid
  - Numerical dissipation
- Today: many research groups have their own codes

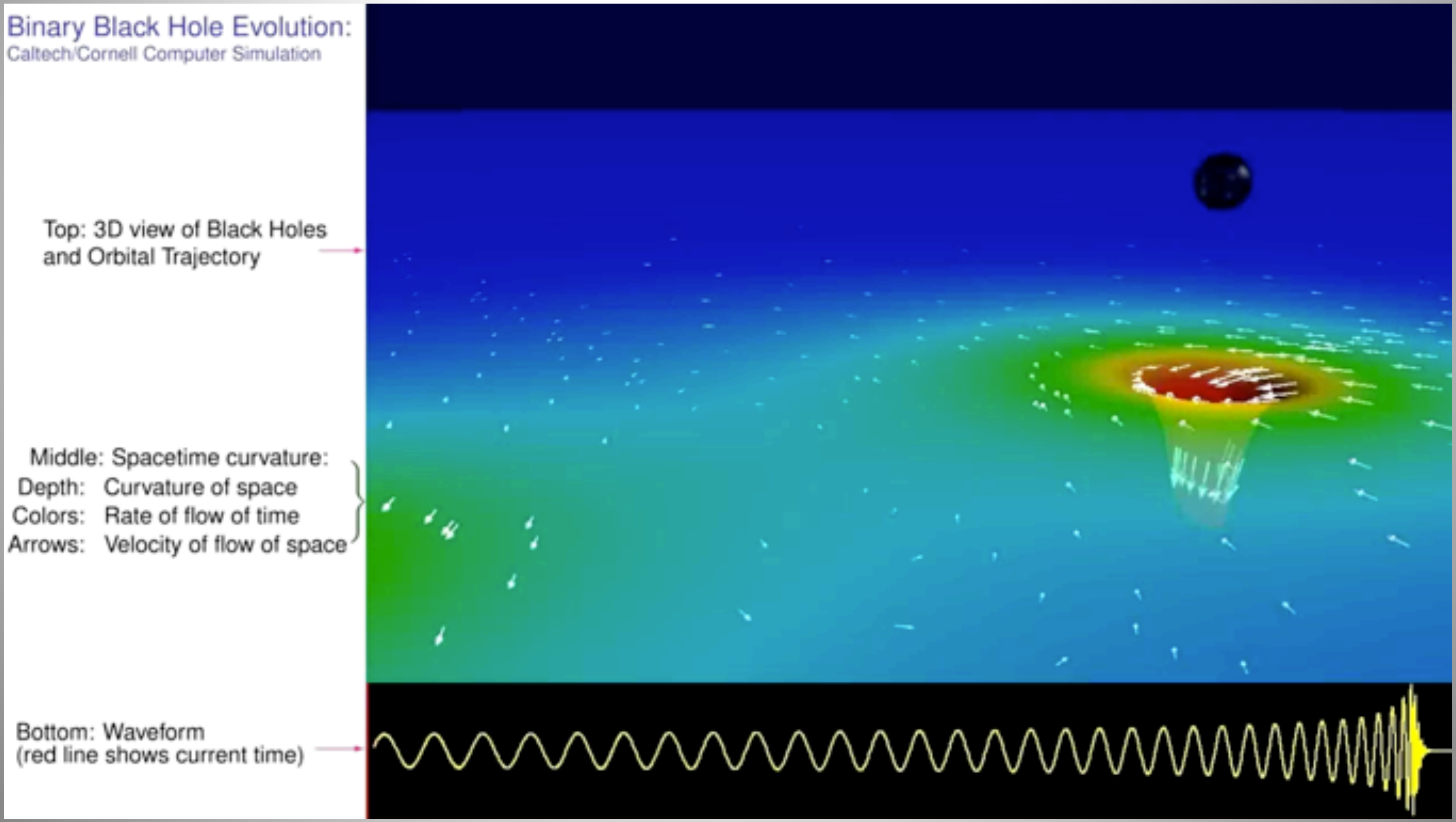
Spectral Einstein Code (SpEC)

- SpEC uses a multi-domain grid
- High-order basis functions
- Parallelization by domain



Brief history of computational relativity

- Typical simulation
    - 100 cores; 2 - 4 weeks
    - Runs on Blue Waters, Stampede, Comet, etc…



Key contributions to gravitational wave science

- Two-body binary black hole problem
  - 8D parameter space (each hole has mass and spin)
- Simulations are used to…
  - Building high-fidelity gravitational wave models
  - Compare directly to observed GW datasets
- Need a good model for answering science questions
  - Final mass and spin of the merged black holes

Ongoing Work* and Future Directions

* Biased towards U. Mass Dartmouth

Urgent need for solutions

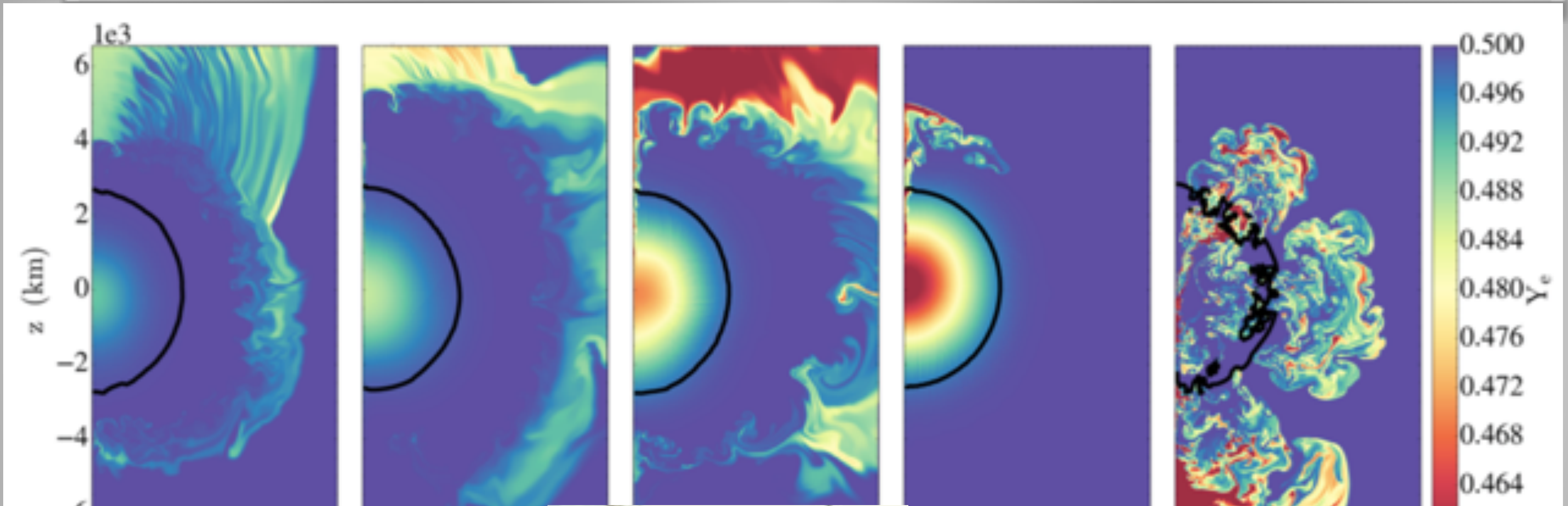
- Over the next decade, gravitational-wave detectors will be observing more events, with higher signal-to-noise ratios, and longer durations
- Heavy demands will be placed on simulation codes, models, and data analysis efforts

Multi-disciplinary Approach

Center for Scientific Computing & Visualization Research

- Research groups…
  - Gravity theory
  - Computational astrophysics
  - Numerical analysis
  - Data science
- PhD students…
  - Yun Hao, Rahul Kashyap, Caroline Mallary, Ed McClain, Alec Yonika, Gustavo Reynoso, Vishal Tiwari
- Masters students …
  - Joel Baer, Gabriel Casabona, Connor Kenyon, Nishad Muhammed, Nur Rifat, Feroz Shaik
- Undergraduate students…
  - Dwyer Deighan, Chris Gilbert, Kim Matsuda, Owen Tower

White dwarf mergers and explosions



Rahul Kashyap

Gabriel Casaba

Robert Fisher

Novel HPC solutions

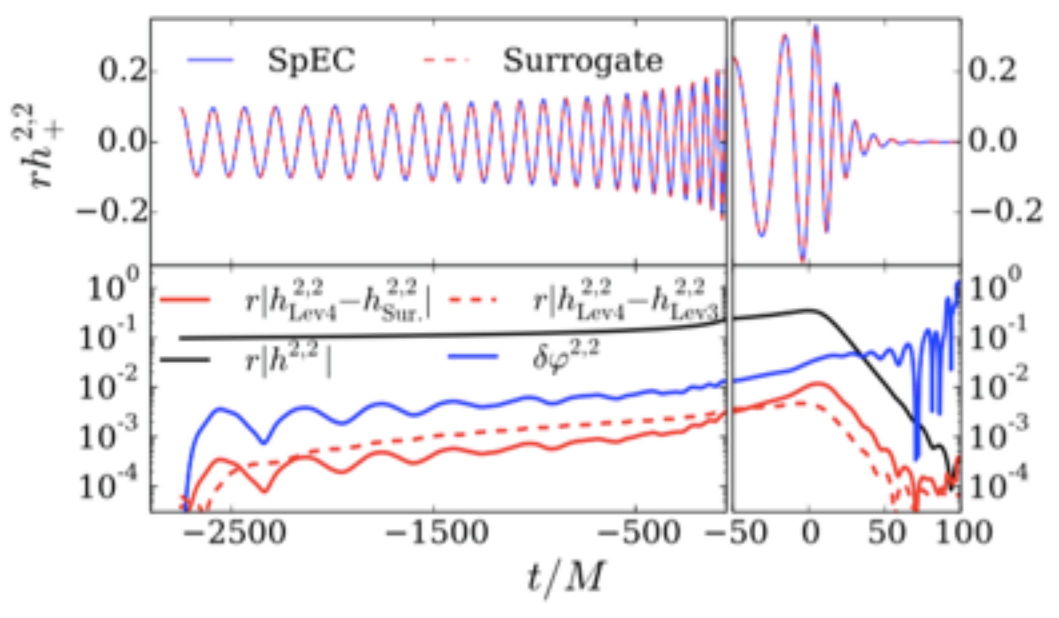
- Using GPUs and playstation to accelerate simulations of perturbations of rotating black holes

Alec Yonika

Gaurav Khanna

Fast computational models

- Recall a single computational relativity simulations takes 2 - 4 weeks. Can we use these in real-time data analysis studies?
- Yes! Train a fast-to-evaluate model directly from the numerical data
- Evaluation of model is fast (<< 1 second) and as accurate as the numerical gravitational wave model

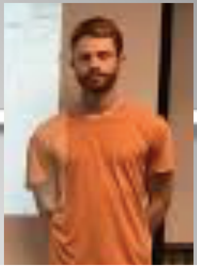


Nur Rifat

Feroz Sheik

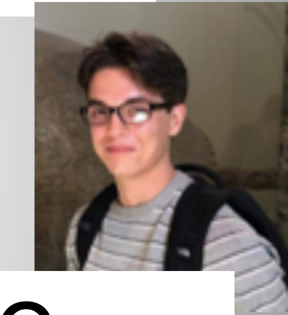
Additional Projects

- Classifying gravitational waves with convolutional neural networks
- Accurate models for gravitational wave propagation
- Discontinuous Galerkin methods for extreme mass ratio binary black holes and relativistic hydrodynamics



Dwyer

Kim



Owen

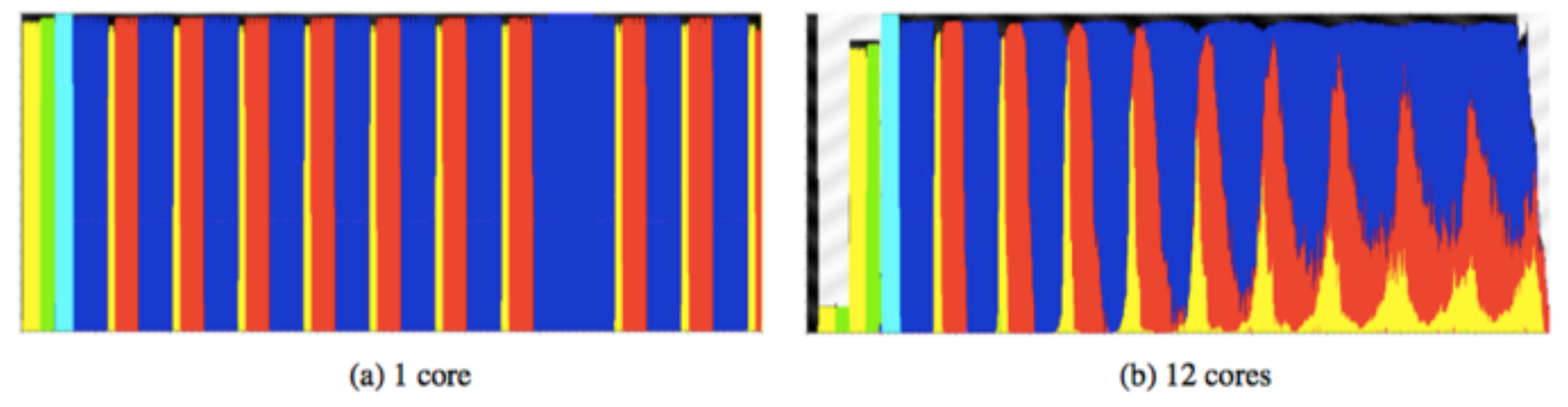
Aakash

Ed

Final thoughts

- The computation of gravitational waves has a rich history, deeply interconnected with (applied) mathematics and high-performance computing
- Recent set of gravitational wave detections has underscored the need for fast, accurate numerical computations
- Meeting observational demands will rely heavily on the efficient usage HPC resources & improved numerical methods
    - ICERM will host a semester long program addressing some of these issues in 2020 (Fall)

SpECTRE: A new code



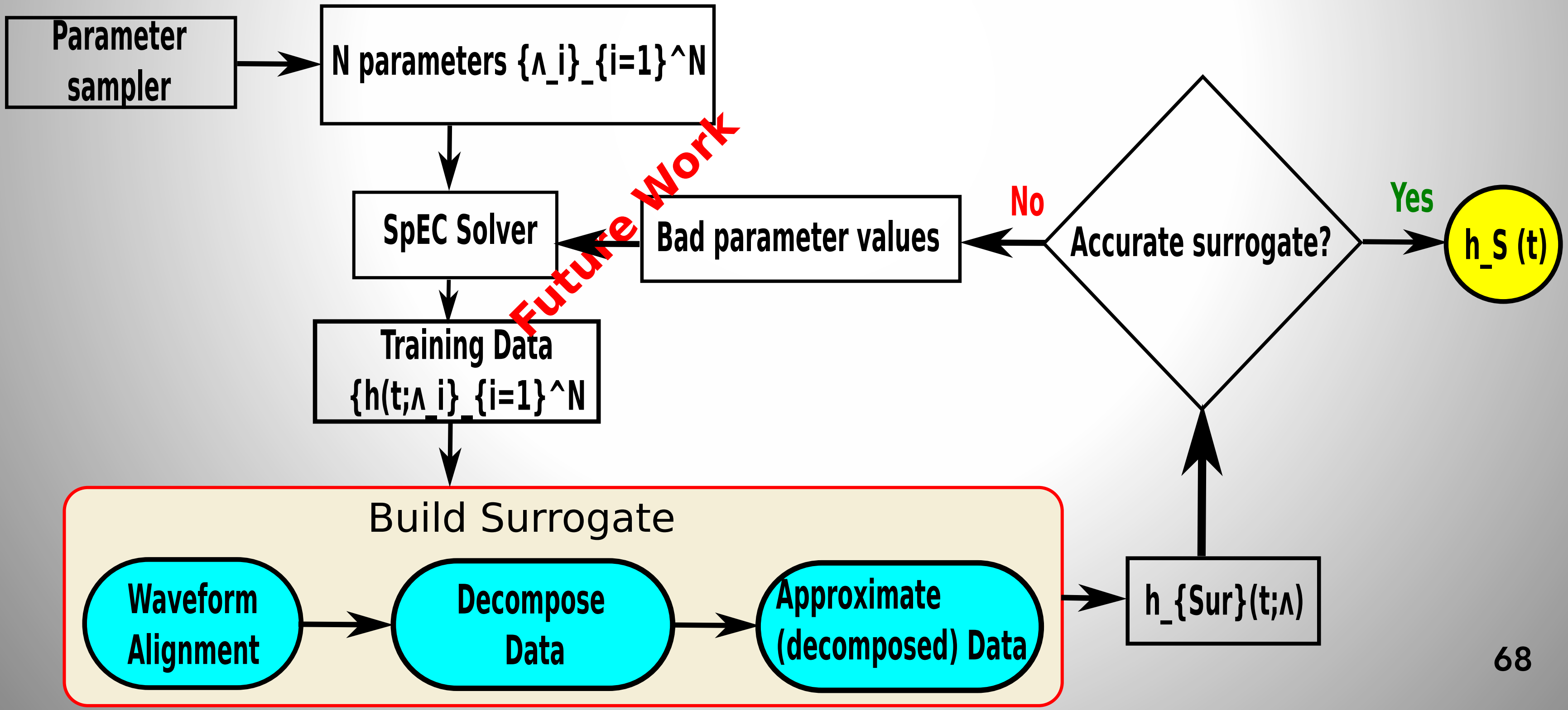
(a) 1 core

(b) 12 cores

- Total amount of time spent executing each task summed across all processors in a given time interval. The vertical axis shows the combined processor utilization (from 0% to 100%) and the horizontal axis shows the wall time.
- Black: Charm++ RTS
- white: idle cores
- The additional colors show SpECTRE tasks.

Fast computational models

- Recall a single computational relativity simulations takes 2 - 4 weeks. Can we use these in real-time data analysis studies?
- Yes! Train a fast-to-evaluate model directly from the numerical data



SpECTRE: A new code

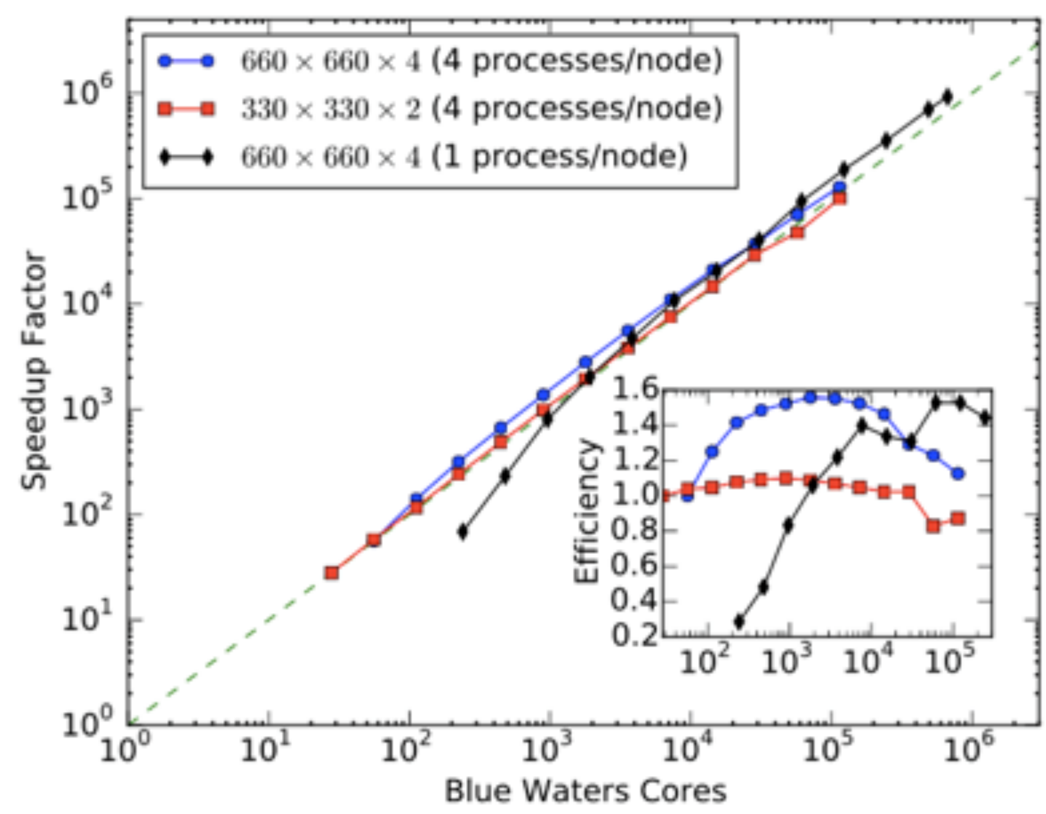
Issues

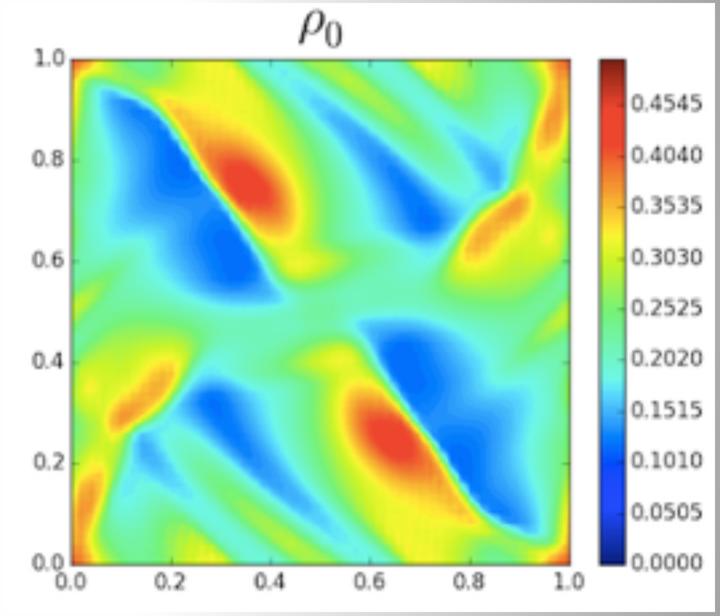
- Moore’s law is dead
- Next-generation systems will have millions of cores
- Cores idle during communication (waiting for data)
- Load balancing & synchronization with current codes

Proposed solution

- Discontinuous Galerkin method
- Task-based parallelism (using the Charm++ library)

SpECTRE: A new code





- **Strong scaling on Blue Waters; fixed grid size**
- **Problem: Orszag-Tang vortex test case**

Challenges and Opportunities

- When matter fields are included (stars), simulations are often neither accurate nor efficient enough to meet observational needs
  - Essentially no use of modern techniques like discontinuous Galerkin methods, finite element, reduced basis, …
  - Star detonation is a multi-scale, multi-physics process. Needed for even qualitatively correct results
- Simulations are too slow for direct use in, say, Bayesian parameter estimation studies
- Long duration GW signals are inaccessible to current codes
- Building accurate computational models to enable high-precision science
- Future space-based detectors will lead to new opportunities and challenges