Discovering Black Holes and Gravitational Waves: Algorithms and Simulation

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Outline

• Gravitational wave science (highlights)
• What are gravitational waves?
  • Mathematical framework & intuition
• How to detect gravitational waves?
• Simulation of black holes and GWs
  • Computers & algorithms
• Ongoing work and challenges
Common Acronyms

- **GW** = Gravitational Wave
- **LIGO** = Laser Interferometer Gravitational-Wave Observatory
- **GR** = General Relativity
2. Numerous figures pulled from the LIGO open science website
4. Holst, Sarbach, Tiglio, Vallisneri, “The emergence of gravitational wave science: 100 years of development of mathematical theory, detectors, numerical algorithms, and data analysis tools”
5. Ed Seidel’s APS April talk, 2018
6. Sarbach, Tiglio “Continuum and Discrete Initial-Boundary-Value Problems and Einstein's Field Equations”
7. Cervantes-Cota, Galindo-Uribarri, and Smoot, “A Brief History of Gravitational Waves”
Gravitational wave science (highlights)

What: Two black holes
Where: Another galaxy
Earth distorted by GWs emitted from black holes

What: Two black holes
Where: Another galaxy

= GW detectors
A 100 Year Research Problem

1915: General relativity is born

2005: First simulation of black holes and their emitted gravity waves (Frans Pretorius)

1950s - 1960s: Existence of solutions (Yvonne Choquet-Bruhat)

1957: Framework showing GWs can be measured (Felix Pirani)

2015: Gravitational waves observed by LIGO!
• On September 14, 2015 GWs passed through Earth
  • **Scientific paper**: Phys. Rev. Lett. 116, 061102
• Strain = GW signal measured by the LIGO detectors
Computer simulations are required to analyze the data.

Simulations are hard...
- Weeks of running on a supercomputer
- Advanced algorithms
- Advanced mathematical tools
Gravitational wave science (highlights)

Gravitational Waves Go Mainstream

The New York Times

Scientists found gravitational waves in outer space.
If only it were that easy to find an apartment in NYC with a walk-in closet.

Rent your own personal closet space: manhattanministorage.com

we're not scientists, but we totally got space.
Gravitational wave science (highlights)
Kip’s 2016 visit (before his prize)

Bob Fisher, Richard Price

CSCVR directors, Sigal & Gaurav

PhD students, Zach & Tiffany

Physics of Interstellar @ UMassD
What are gravitational waves?

Black Hole Census

Black holes discovered through GW observations

Gravitational wave science (highlights)
The observation of gravitational waves was an unprecedented experimental feat...

... that required mathematical & computational breakthroughs
Engines of GW Science

1. Astrophysical system to generate waves
   • Two black holes orbiting one another

2. Mathematical framework for computing the expected gravitational wave signal

3. Detectors to observe the signal

4. Algorithms and computers to solve equations

5. Data analysis tools to compare theory and observation
What are gravitational waves?

What is the mathematical theory that describes them?
What is Gravity? Newton’s Answer

- Gravity is a force between two objects.
- **No** gravitational waves! Waves need a medium (e.g. water) to be “waiving in”.

\[ F_1 = F_2 = G \frac{m_1 \times m_2}{r^2} \]
Einstein’s General Relativity

- Gravity is not a force in the usual sense of “push” or “pull”
- Mass causes space-time around it to bend or warp
- Path of objects (light included) is affected by this warped space-time
- The gravitational “force” is a manifestation of the bending of space and time

What are gravitational waves?
1915
What are gravitational waves?

Example: Light Moving In Curved Space

- A black hole appears on an academic quad...
What are gravitational waves?

- Credit: Andy Bohn, et al.; SXS Collaboration
• Secretly a partial differential equation (derivatives lurking in R; Ricci curvature)
• Solution to this equation describes the geometry of space and time
  • Gravitational lensing of an academic quad by a black hole
  • The distance from ICERM to Fenway Park changes when a GW passes by
• ICERM employees age more quickly than Hemenway’s employees
What are gravitational waves?

When solution has spherical symmetry...
Mathematical Structure of Equations

- System of coupled, nonlinear partial differential equations
- When written with first order derivatives of time and space, there are 52 equations with hundreds of terms!
- Paper-and-pencil solutions only known for simple cases; computers are needed

Does Einstein’s equation of general relativity allow for gravitational waves?
Theoretical Justification for Gravitational Waves?

Existence of GWs was debated until the late 1950s

1. Existence of solutions? (Not obvious)
2. Equations are too hard to solve, so how can we say anything concrete about the possibility of gravitational waves?
Issue 1: Existence of Solutions

Under what conditions can we solve Einstein’s equation of general relativity?

Why this matters: If solutions don’t exist, it doesn’t make sense to ask a carry out computer simulations.
What are gravitational waves?

Interlude: When Can We Solve an Equation?

Q: Solve for x

\[12 + 2x - 8 = 7x + 5 - 5x\]
Q: Solve for $x$

\[ 12 + 2x - 8 = 7x + 5 - 5x \]

A: \[ 4 + 2x = 5 + 2x \]

\[ 4 = 5 \]

Not all equations can be solved...
What are gravitational waves?

Roadmap to Solvability

• (1930’s) Mathematical tools developed by Kurt Friedrichs, Hans Lewy, and Sergei Sobolev.

• (1947) A graduate student, Yvonne Choquet-Bruhat (YCB), begins using these new tools to show the equations can be solved.

• (1952) YCB shows Einstein equations have solutions under restricted conditions.

• (1969) YCB + Robert Geroch extend the results to general conditions.
Do the Einstein equations admit solutions that can be interpreted as gravitational waves?

Why this matters: Why spend billions of dollars building gravitational wave detectors if they don’t exist?

• (1916) Einstein finds approximate solutions that are waves; but he dismisses them as unphysical...
Q: You have a square bedroom that's 49 square feet. What's the length of the wall?
Q: You have a square bedroom that's 49 square feet. What's the length of the wall?

A: \( x = \text{wall's length} \quad x^2 = 49 \)

Mathematically, the wall could be 7 feet or -7 feet. Physically, only 7 feet makes sense.

Not all solutions can be trusted…
What are gravitational waves?

Roadmap to Waves: Part I

(1934) The Einstein and Rosen paper

- (Preprint) “... I arrive at the interesting result that GWs do not exist”

(Preprint) 1934: “Do gravitational waves exist?”
(1934) The Einstein and Rosen paper

- (Preprint) “… I arrive at the interesting result that GWs do not exist”
- (Revision) “…The second part of the article was altered by me... as we had misinterpreted the results... I want to thank my colleague Professor Roberston....”

(Preprint) 1934: “Do gravitational waves exist?”
(“Revised” paper) 1934: “On gravitational waves”
• (1957) The Chapel Hill conference

• “Proof by discussion”: Pirani derived an equation that could be used to measure gravitational waves. A thought experiment by Feynman and Bondi showed the waves could generate heat, and were therefore physical.
Thanks to the hard work of many researchers from 1915 to the mid-1960s we now know

1. It makes sense to solve Einstein’s equations of general relativity under general conditions

2. Gravitational waves are one particularly important feature of the solutions

Let's see what these waves look like…
What are gravitational waves?

Generation and propagation of gravitational waves
How to observe gravitational waves?

General relativity predicts their existence. How to test the prediction?
The most promising sources of gravitational waves are those that move dense objects at high accelerations. Examples are supernovae and collisions of compact objects like neutron stars and black holes.
How to observe gravitational waves?

Real-world example

* Not seen with an optical (traditional) telescope

Fig. by Kimberly Matsuda, Mathematics undergrad
How to observe gravitational waves?

Real-world example

• Use the gravitational wave signal to answer scientific questions
  • Properties of the black hole system (e.g. masses)
  • Is Einstein’s theory of relativity correct?
  • Number of space dimensions
  • Speed of gravity waves
  • Populations of black holes

Fig. by Kimberly Matsuda, Mathematics undergrad
How to observe gravitational waves?

Detectors on Earth
How to observe gravitational waves?

How the detector works
How to observe gravitational waves?

Gravitational waves are entering the detector

Overhead view of detector

△ = GW detector
How to observe gravitational waves?

**Measuring small changes**

- No GWS: distance between mirrors is 4 kilometers
- GW causes small, time-dependent changes \( \Delta L \approx 10^{-18} \text{km} \)

- Smaller than the size of a proton
- Gravitational wave detection requires complicated data analysis algorithms
Role of Computational Models

- Signal is weak, buried in detector noise
- Precise computer models are used to generate thousands / millions of “template” signals from likely sources
- Comparison to templates allow for detection and parameter estimation
- Computational modeling is absolutely essential in the discovery process!
Solving Einstein’s equation on a computer

(A brief history of computational relativity)
What is computational relativity?

- Use algorithms, mathematical tools, and computational resources to find solutions of general relativity equations.
- We specify the problem’s setup: masses of the black holes, spins of the black holes, etc.
- Computer simulations solve the equations for this configuration to get various outputs like the gravitational wave signal.
Two body problem (setup)

Input to the computer: black holes mass & spins
Just differential equations...

- “These are just differential equations, and people solve those all the time. Throw them on a computer. Just do it!“
- Weather simulations
- Airplane simulations
- Rocket re-entry simulations
(1957) Origins of computational relativity

- Bryce DeWitt spends time at Lawrence Livermore National Lab working on fluid simulations. He and Charles Misner suggest the following:

  “First we assume that you have a computing machine better than anything we have now, and many programmers and a lot of money, and you want to look at a nice pretty solution of the Einstein equations....”

(2005) First stable simulation of two black holes

- Strongly hyperbolic formulations with constraint damping
- 2nd order finite difference (still predominate use)
High-performance computing in 1964

- First attempt carried out by Hahn and Lindquist
- Hahn, a student of Peter Lax, had access to the IBM 7090 supercomputer
  - 1 MegaFLOPs
  - Cost 3 million
- 51x51 mesh points
- Crashes in 50 steps
- 4 minutes/step
An unsolicited proposal — 1983

- (1982) Peter Lax’s report on supercomputing in the US: Why are there no supercomputers available to US academics
  - Simulations done in Germany or classified project
- Larry Smarr, who works in computational relativity, is using supercomputers through a friend at Livermore Lab.
- Larry writes the first unsolicited NSF proposal “A center for Scientific & Engineering Supercomputing” to be funded
  - The first supercomputing center network is born
    - Cornell, NCSA, Pittsburgh, San Diego, Princeton
  - Many of the first simulations are computational relativity
Progress, but somethings wrong

• (early 1990s) Despite new supercomputing centers (thanks to Larry) and decades of research effort, no one can evolve a binary black hole system
  • Algorithms or formulations are unstable — codes crash
• (1993) LIGO is funded to detect gravitational waves; but we don’t know what they look like!
• (Late 1990s) NSF funds the Binary Black Hole Grand Challenge to support new mathematical, numerical, and HPC techniques to solve the problem
Grand Challenge: Why does the code “blow up”?

- Yvonne Choquet-Bruhat proved that solutions exist
- What is the right way to instruct a computer to find them?
- There are many wrong ways, which lead to uncontrolled errors; the computer stops working
- This “blue screen of death” is a familiar situation for anyone who has used a computer
Grand Challenge Collaborations

- Complex problems now on small scales: experiment on small scales
- NSF Black Hole Grand Challenge
  - Brought together computer science and relativity
    - University of Texas
    - NCSA/Illinois (Seidel)
    - North Carolina (Eichhorn)
    - Syracuse (G. Fox)
    - Cornell (Shapiro, Tichy)
    - Pittsburgh (Winicour)
    - Penn State (Laguna)
  - Many lessons learned, highly appreciated
- NASA Neutron Star Grand Challenge
  - Illinois, Argonne, Washington
  - Deliverables focused
- Later: EU Network in Gravitational Relativity
The 2005 Breakthrough

- Frans Pretorius' talk at the Banff mathematical research institute
  - First stable simulations of binary black holes; first numerical gravitational waves
- Key reasons it worked:
  - Constraint damping; originally proposed by mathematician Heinz-Otto Kriess
  - Adaptive mesh refinement
  - Removed the black hole singularity from the grid
  - Numerical dissipation
- Today: many research groups have their own codes
Spectral Einstein Code (SpEC)

- SpEC uses a multi-domain grid
- High-order basis functions
- Parallelization by domain
Brief history of computational relativity

- Typical simulation
  - 100 cores; 2 - 4 weeks
  - Runs on Blue Waters, Stampede, Comet, etc…
Key contributions to gravitational wave science

- Two-body binary black hole problem
  - 8D parameter space (each hole has mass and spin)
- Simulations are used to...
  - Building high-fidelity gravitational wave models
  - Compare directly to observed GW datasets
- Need a good model for answering science questions
  - Final mass and spin of the merged black holes
Ongoing Work* and Future Directions

* Biased towards U. Mass Dartmouth
Over the next decade, gravitational-wave detectors will be observing more events, with higher signal-to-noise ratios, and longer durations.

Heavy demands will be placed on simulation codes, models, and data analysis efforts.
Multi-disciplinary Approach

• Research groups...
  • Gravity theory
  • Computational astrophysics
  • Numerical analysis
  • Data science

• PhD students...
  • Yun Hao, Rahul Kashyap, Caroline Mallary, Ed McClain, Alec Yonika, Gustavo Reynoso, Vishal Tiwari

• Masters students ...
  • Joel Baer, Gabriel Casabona, Connor Kenyon, Nishad Muhammed, Nur Rifat, Feroz Shaik

• Undergraduate students...
  • Dwyer Deighan, Chris Gilbert, Kim Matsuda, Owen Tower
White dwarf mergers and explosions

Ongoing work & challenges

Rahul Kashyap

Gabriel Casaba

Robert Fisher
Novel HPC solutions

- Using GPUs and playstation to accelerate simulations of perturbations of rotating black holes

Alec Yonika

Gaurav Khanna
Ongoing work & challenges

Fast computational models

• Recall a single computational relativity simulations takes 2 - 4 weeks. Can we use these in real-time data analysis studies?
• Yes! Train a fast-to-evaluate model directly from the numerical data
• Evaluation of model is fast (<< 1 second) and as accurate as the numerical gravitational wave model

Nur Rifat
Feroz Sheik
Additional Projects

• Classifying gravitational waves with convolutional neural networks
• Accurate models for gravitational wave propagation
• Discontinuous Galerkin methods for extreme mass ratio binary black holes and relativistic hydrodynamics
Final thoughts

• The computation of gravitational waves has a rich history, deeply interconnected with (applied) mathematics and high-performance computing.

• Recent set of gravitational wave detections has underscored the need for fast, accurate numerical computations.

• Meeting observational demands will rely heavily on the efficient usage of HPC resources & improved numerical methods.

  • ICERM will host a semester long program addressing some of these issues in 2020 (Fall).
SpECTRE: A new code

Ongoing work & challenges

- Total amount of time spent executing each task summed across all processors in a given time interval. The vertical axis shows the combined processor utilization (from 0% to 100%) and the horizontal axis shows the wall time.
- Black: Charm++ RTS
- white: idle cores
- The additional colors show SpECTRE tasks.
Fast computational models

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- Yes! Train a fast-to-evaluate model directly from the numerical data

Ongoing work & challenges

Parameter sampler

N parameters \( \{\alpha_i\}_{i=1}^N \)

SpEC Solver

Training Data
\( \{h(t; \alpha_i)\}_{i=1}^N \)

No

Future Work

Bad parameter values

Accurate surrogate?

Yes

h_S(t)

No

Build Surrogate

Waveform Alignment

Decompose Data

Approximate (decomposed) Data

h_{Sur}(t; \alpha)

SpECTRE: A new code

Issues

• Moore’s law is dead
• Next-generation systems will have millions of cores
• Cores idle during communication (waiting for data)
• Load balancing & synchronization with current codes

Proposed solution

• Discontinuous Galerkin method
• Task-based parallelism (using the Charm++ library)
SpECTRE: A new code

- Strong scaling on Blue Waters; fixed grid size
- Problem: Orszag-Tang vortex test case
Challenges and Opportunities

• When matter fields are included (stars), simulations are often neither accurate nor efficient enough to meet observational needs
  • Essentially no use of modern techniques like discontinuous Galerkin methods, finite element, reduced basis, ...
  • Star detonation is a multi-scale, multi-physics process. Needed for even qualitatively correct results
• Simulations are too slow for direct use in, say, Bayesian parameter estimation studies
• Long duration GW signals are inaccessible to current codes
• Building accurate computational models to enable high-precision science
• Future space-based detectors will lead to new opportunities and challenges