Negative Snell's Propagation

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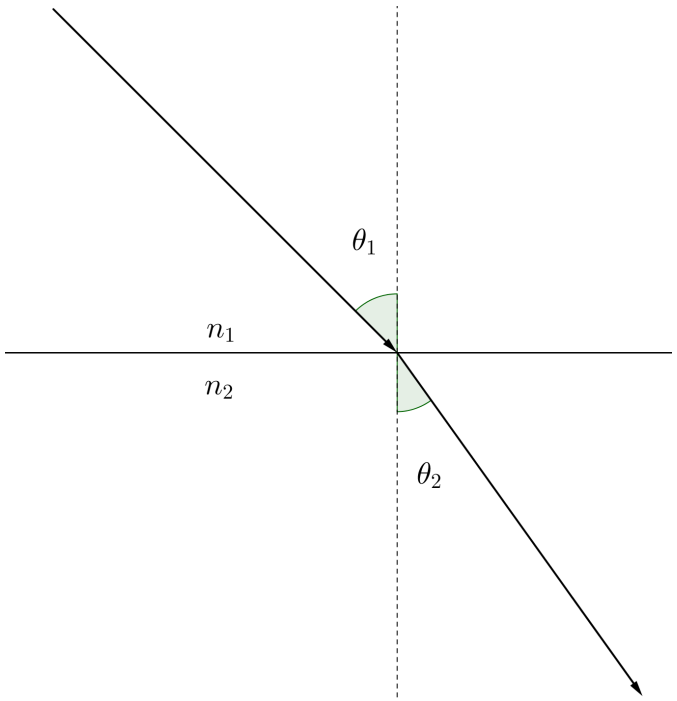
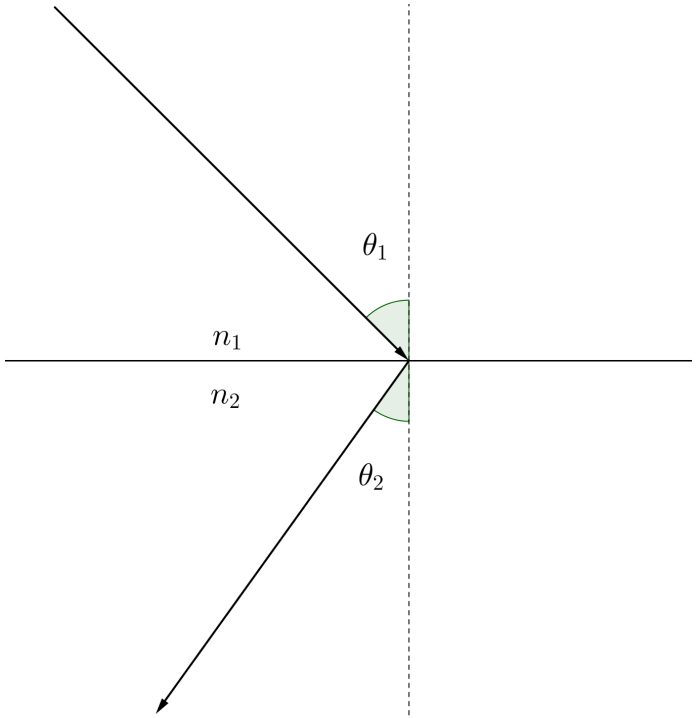
August 8, 2012

Snell's Law



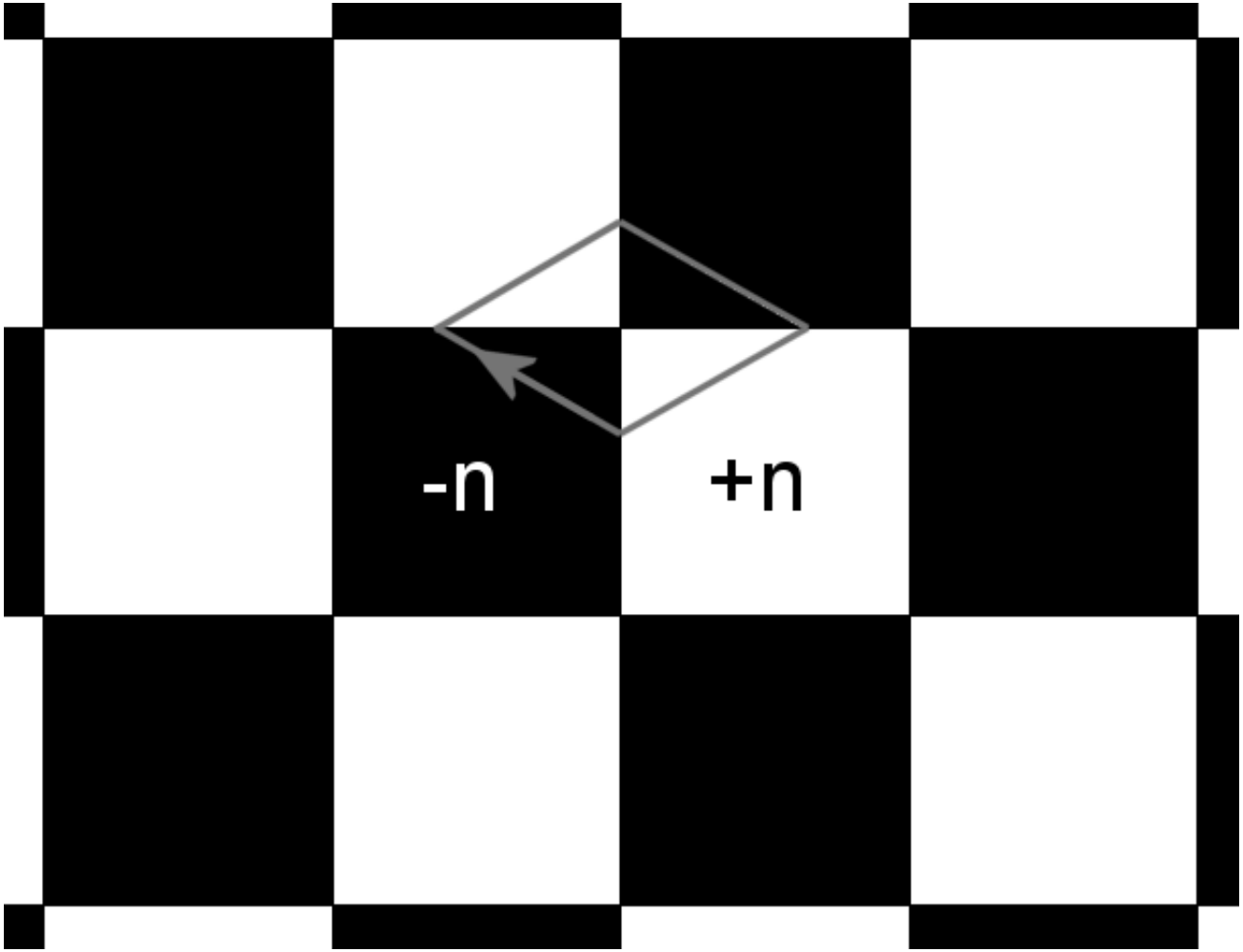
$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Negative Snell's Law

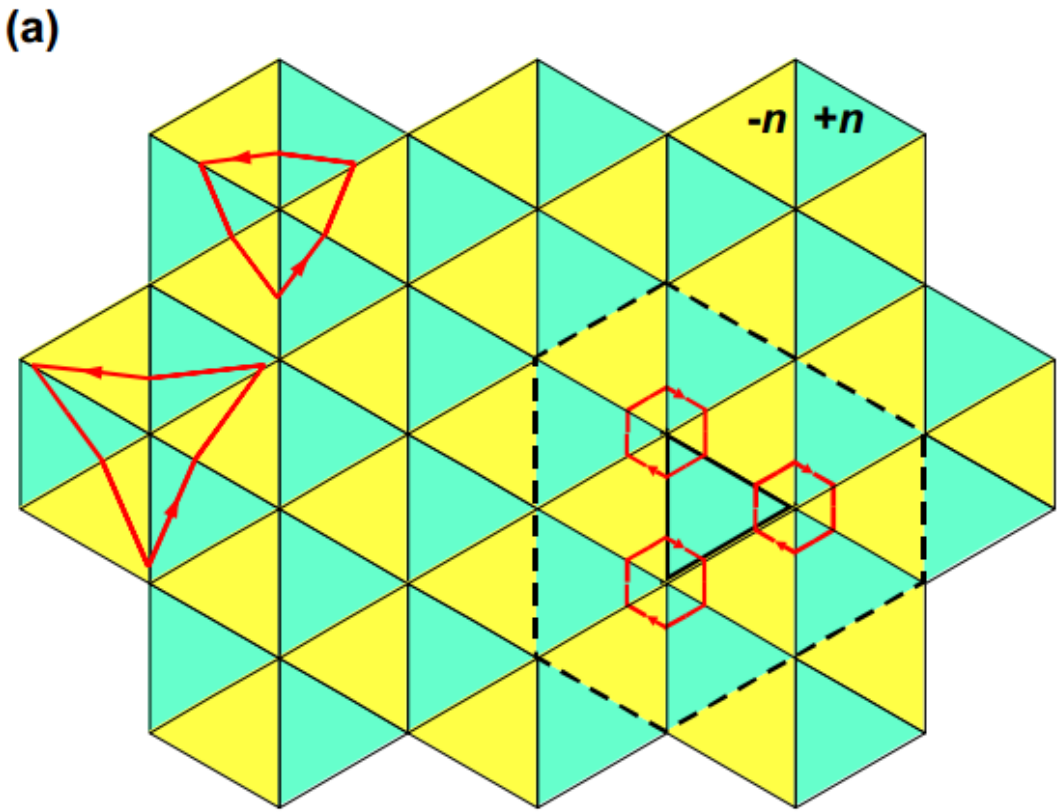


$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Previous Work

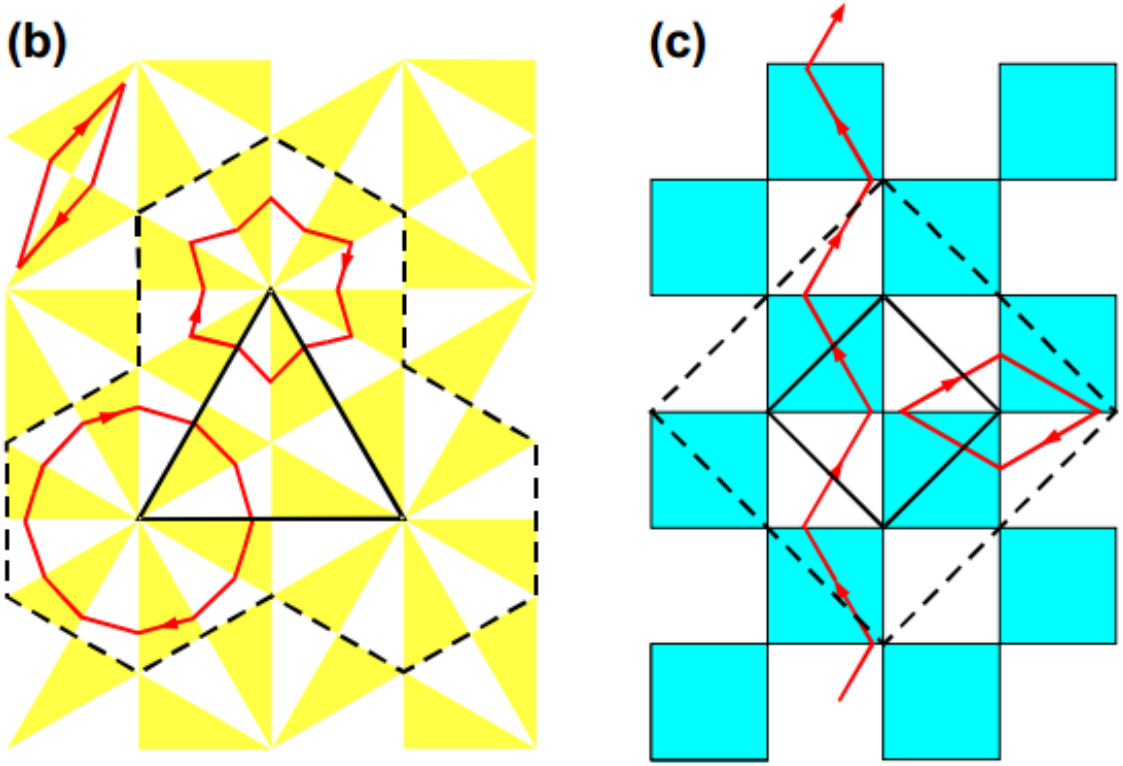


Previous Work



Mascarenhas A, Fluegel B. Antisymmetry and the breakdown of Blochs theorem for light. Unpublished.

Previous Work



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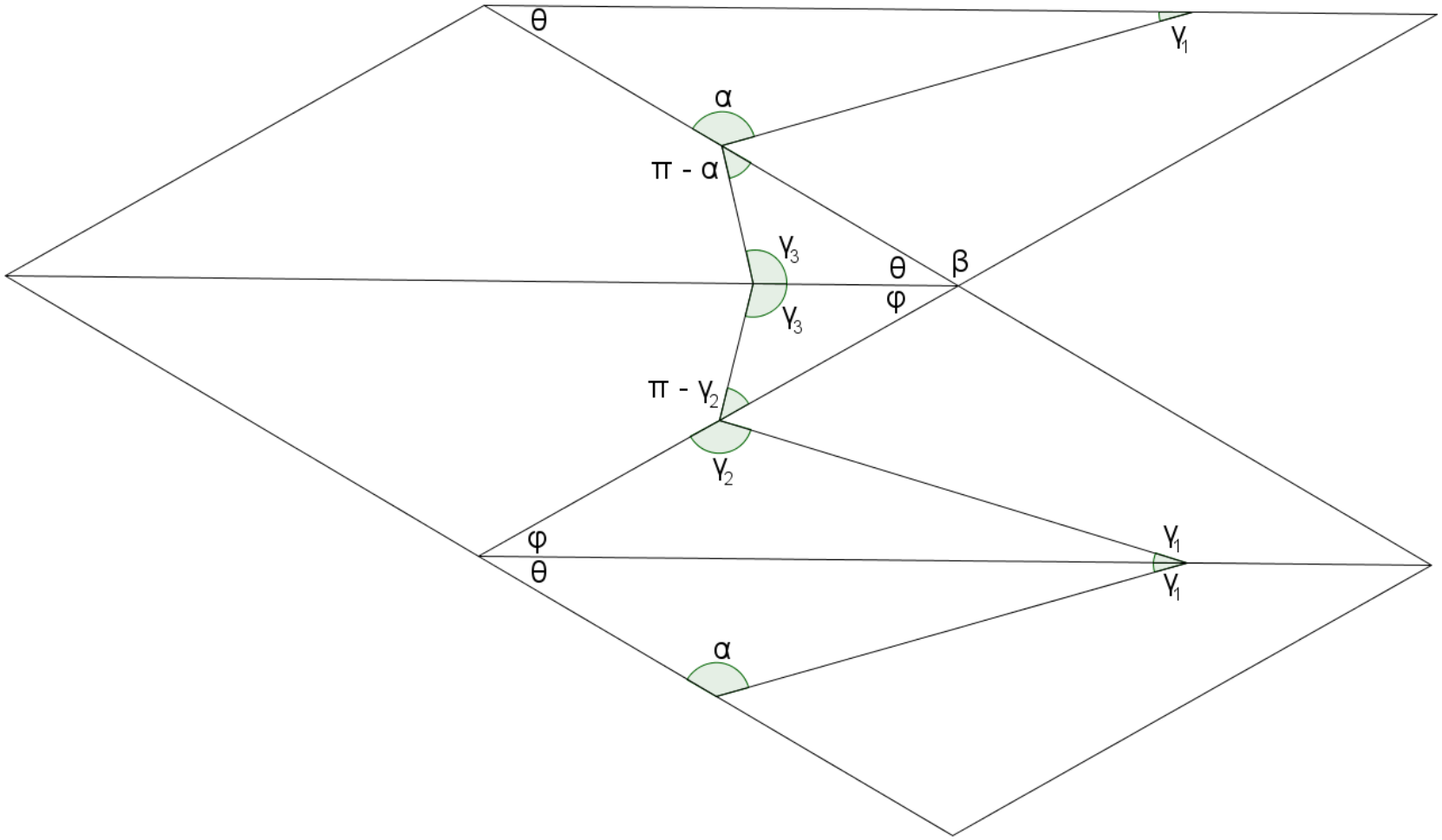
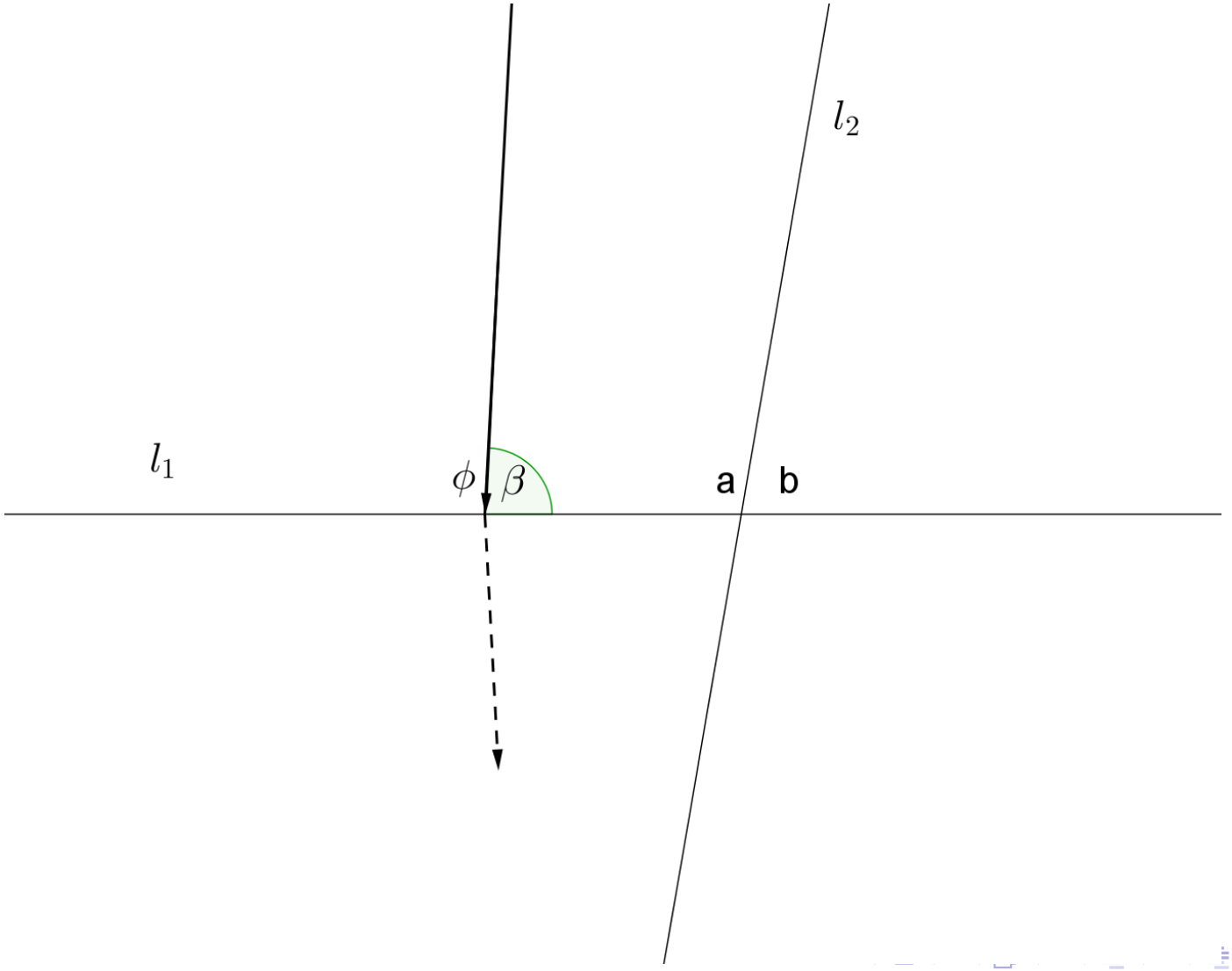
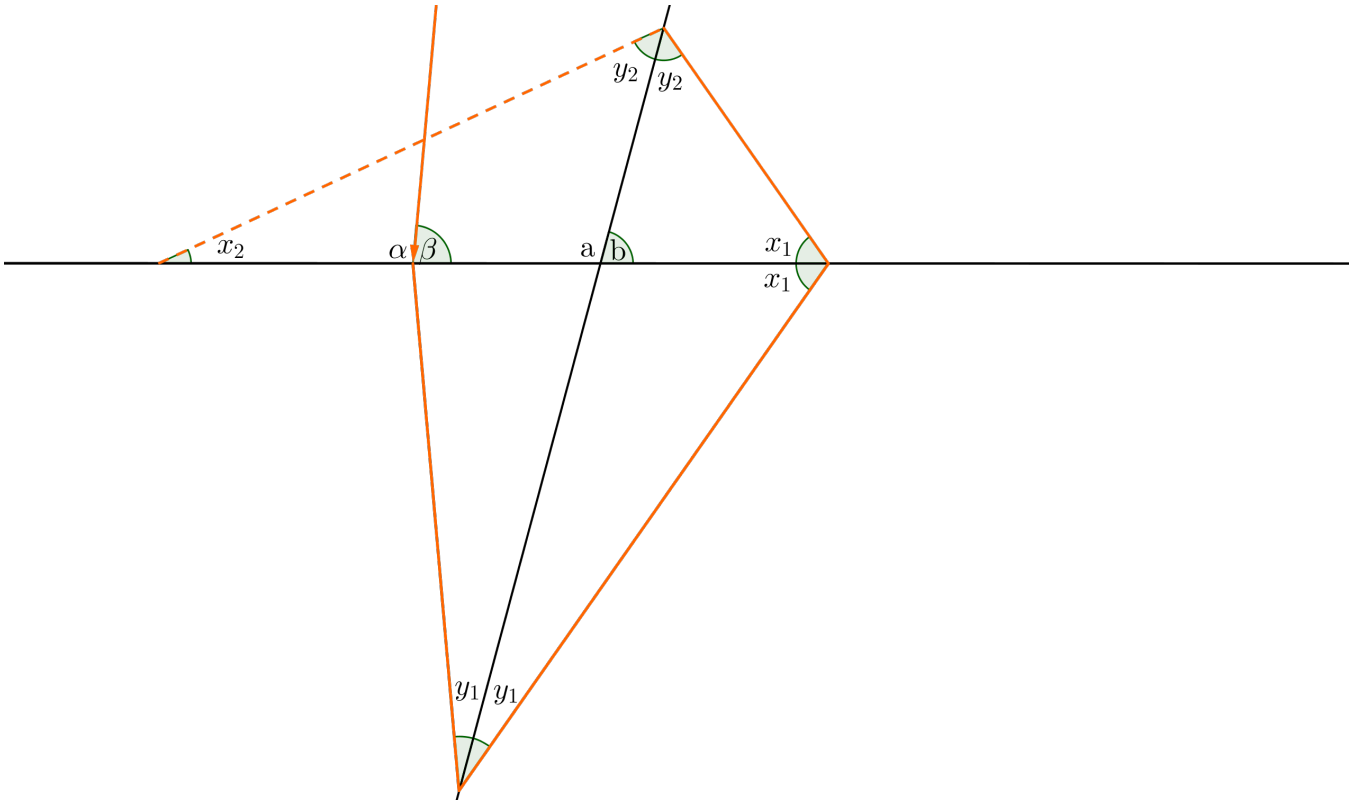
Escaping to Infinity with Triangle Tiling



2 Lines



2 Lines



$$y_n = b - \beta + n(a - b)$$

$$x_n = \beta + n(b - a)$$

Theorem

Theorem

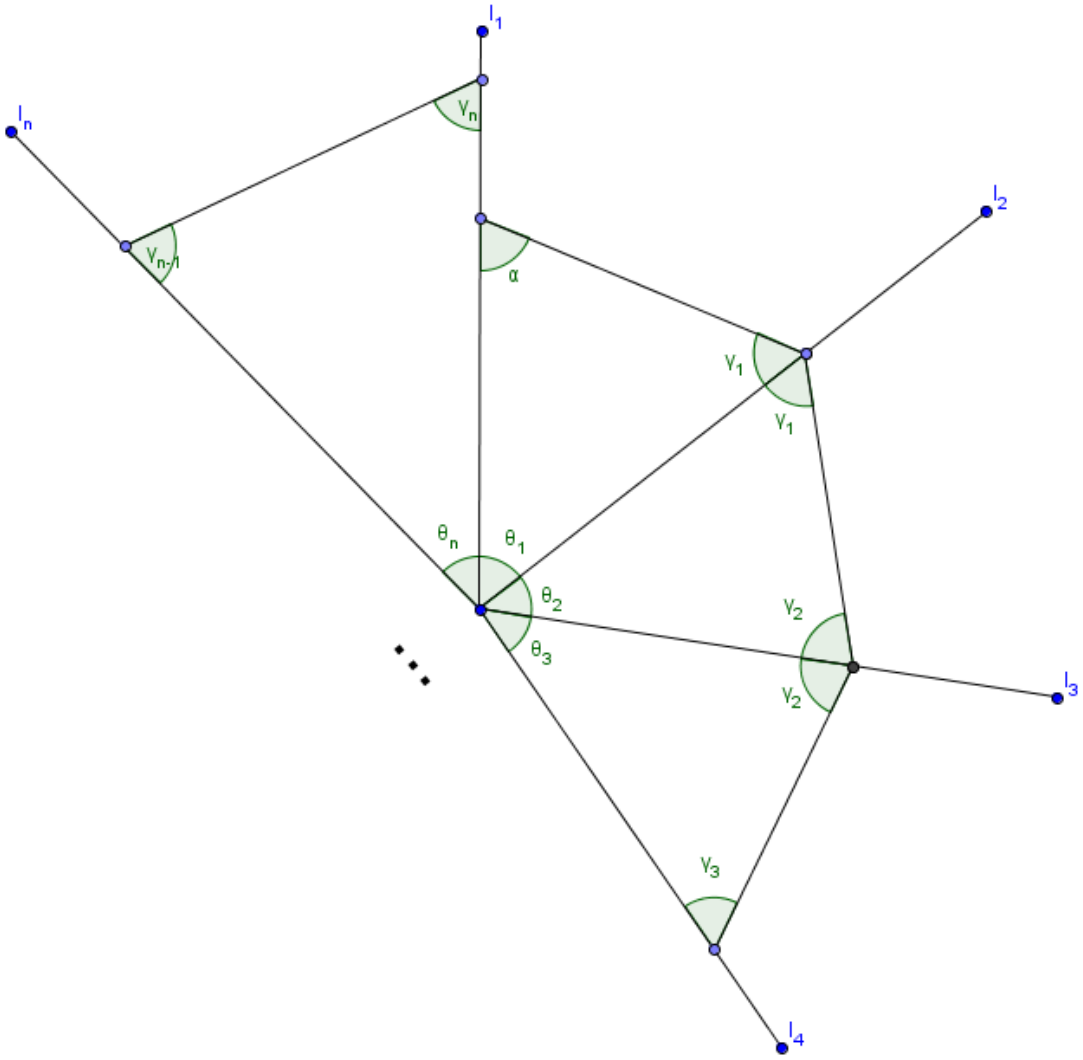
A particle can not spiral around the intersection of two lines infinitely.

Corollary

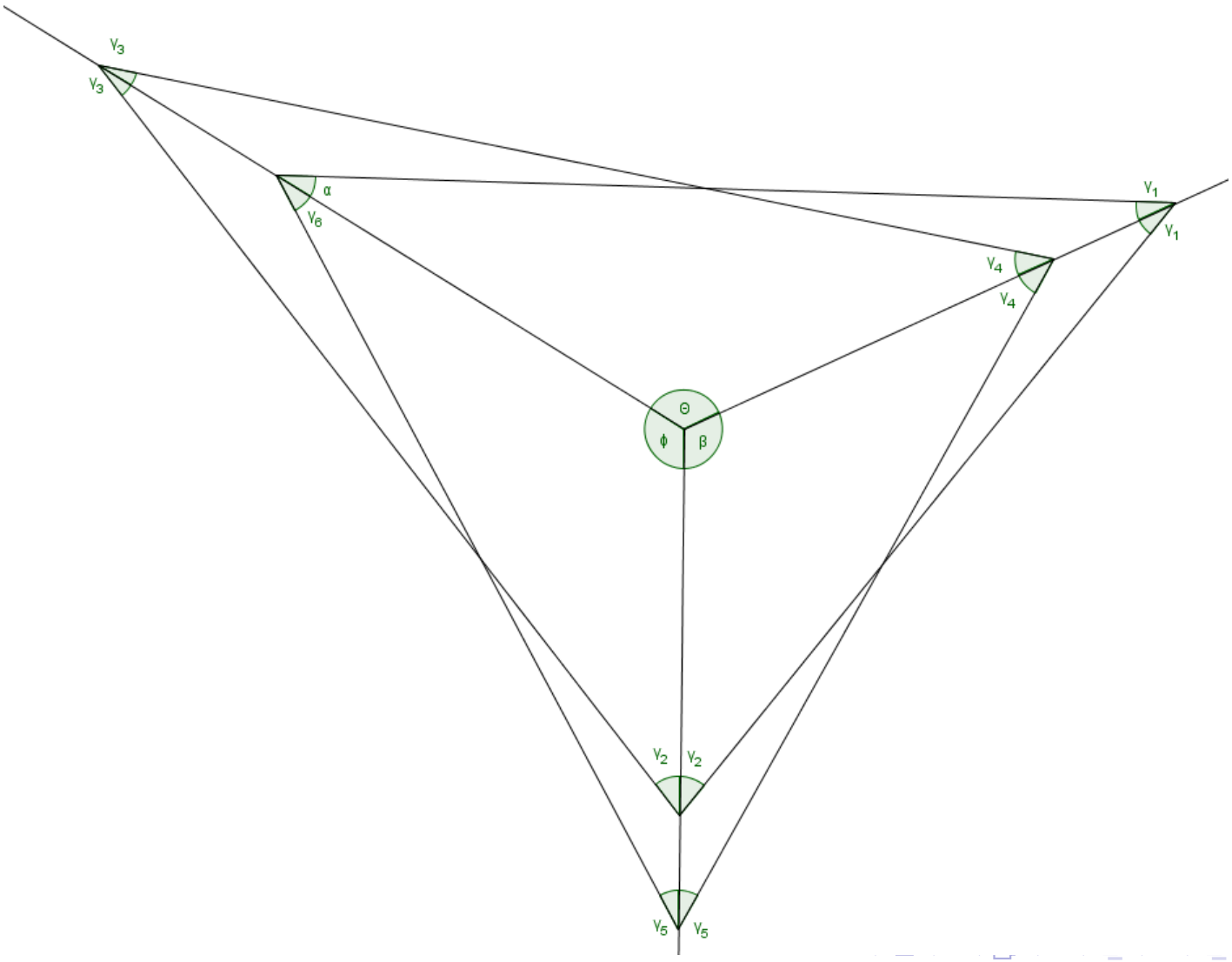
Corollary

The number of times the particle hits l_2 *is* $\left\lceil \frac{\beta}{a-b} \right\rceil$

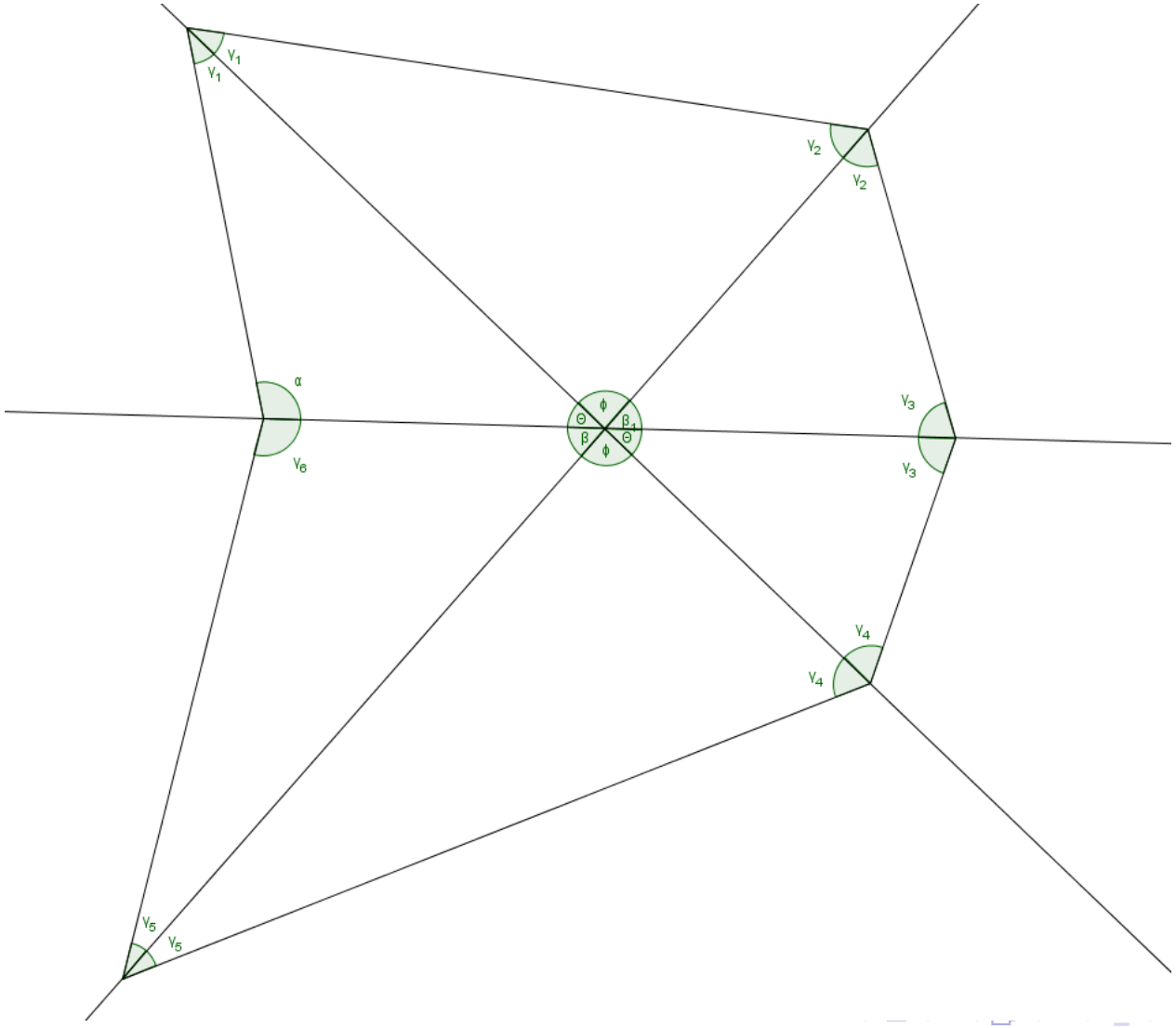
N-Rays


I_1
I_n
I_2
I_3
I_4
γ_n
γ_{n-1}
α
γ_1
γ_1
θ_n
θ_1
θ_2
θ_3
γ_2
γ_2
γ_3

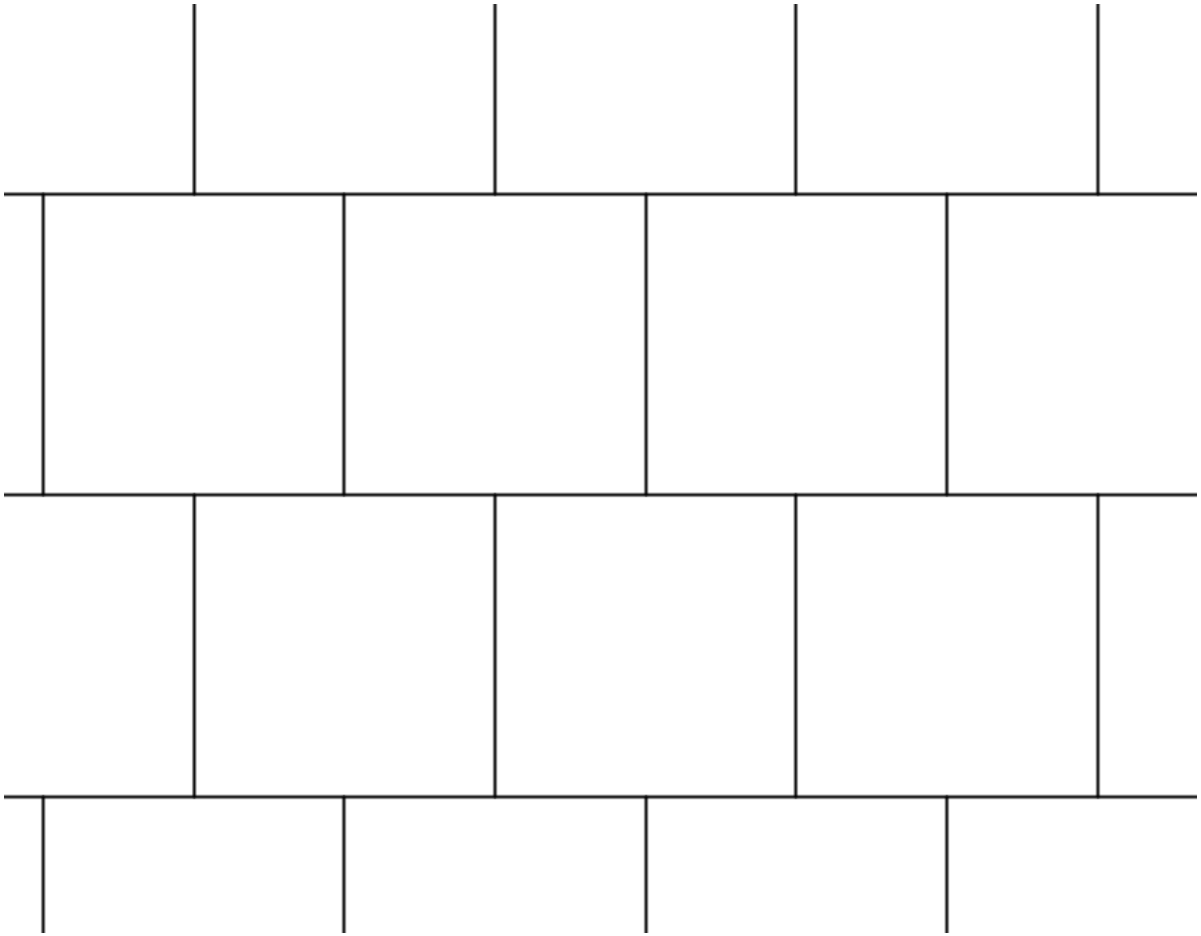
3-Rays



3-Lines



Square Skewed by $\frac{1}{2}$



Square Skewed by $\frac{1}{2}$

Theorem

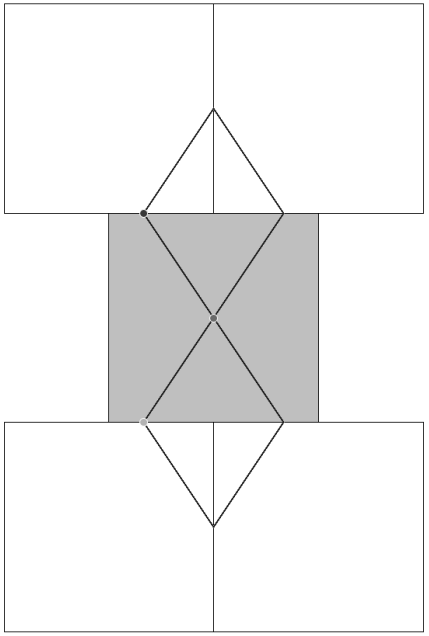
Every path in the square brick tiling skewed by $\frac{1}{2}$ is periodic within translation.

Square Skewed by $\frac{1}{2}$

	e_1	e_2	e_3	e_4	e_5	e_6
e_1	$\cdot$	$\cdot$	(3)	(1)	(2)	(5)
e_2	$\cdot$	$\cdot$	(5)	(2)	(1)	(3)
e_3	$\cdot$	$\cdot$	$\cdot$	(5)	(3)	(4)
e_4	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	(3)
e_5	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	(5)
e_6	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$

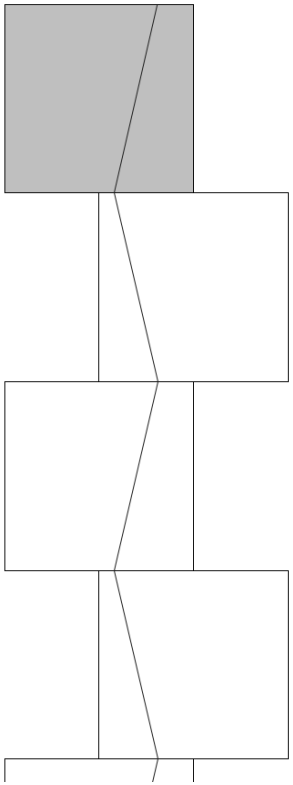
Square Skewed by $\frac{1}{2}$

Case 1:$e_1 \to e_4$.



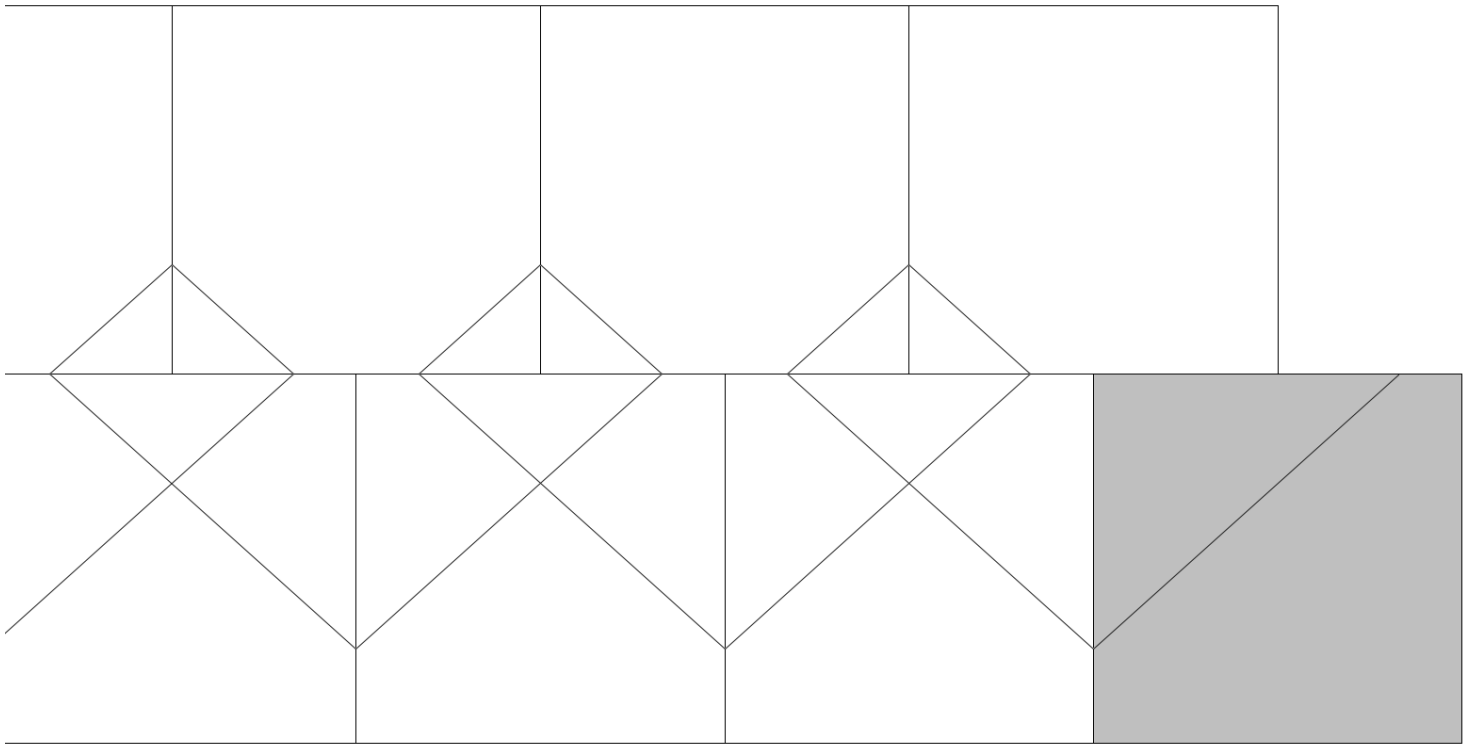
Square Skewed by $\frac{1}{2}$

Case 2: $e_1 \to e_5$.



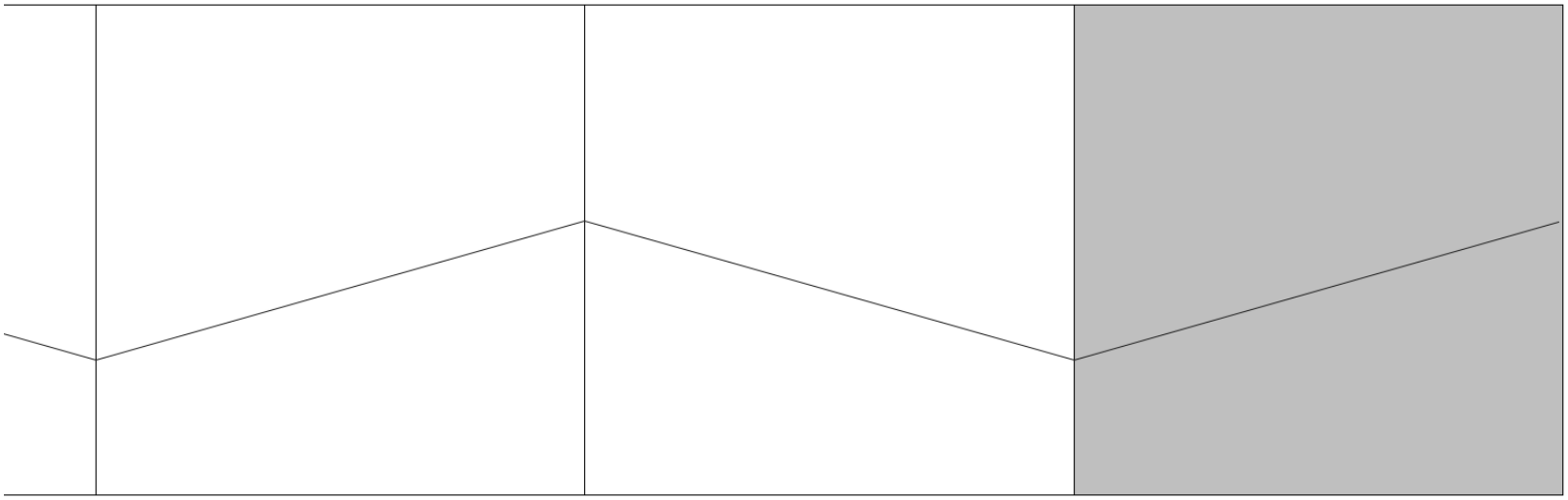
Square Skewed by $\frac{1}{2}$

Case 3: $e_2 \to e_6$.



Square Skewed by $\frac{1}{2}$

Case 4: $e_3 \to e_6$.

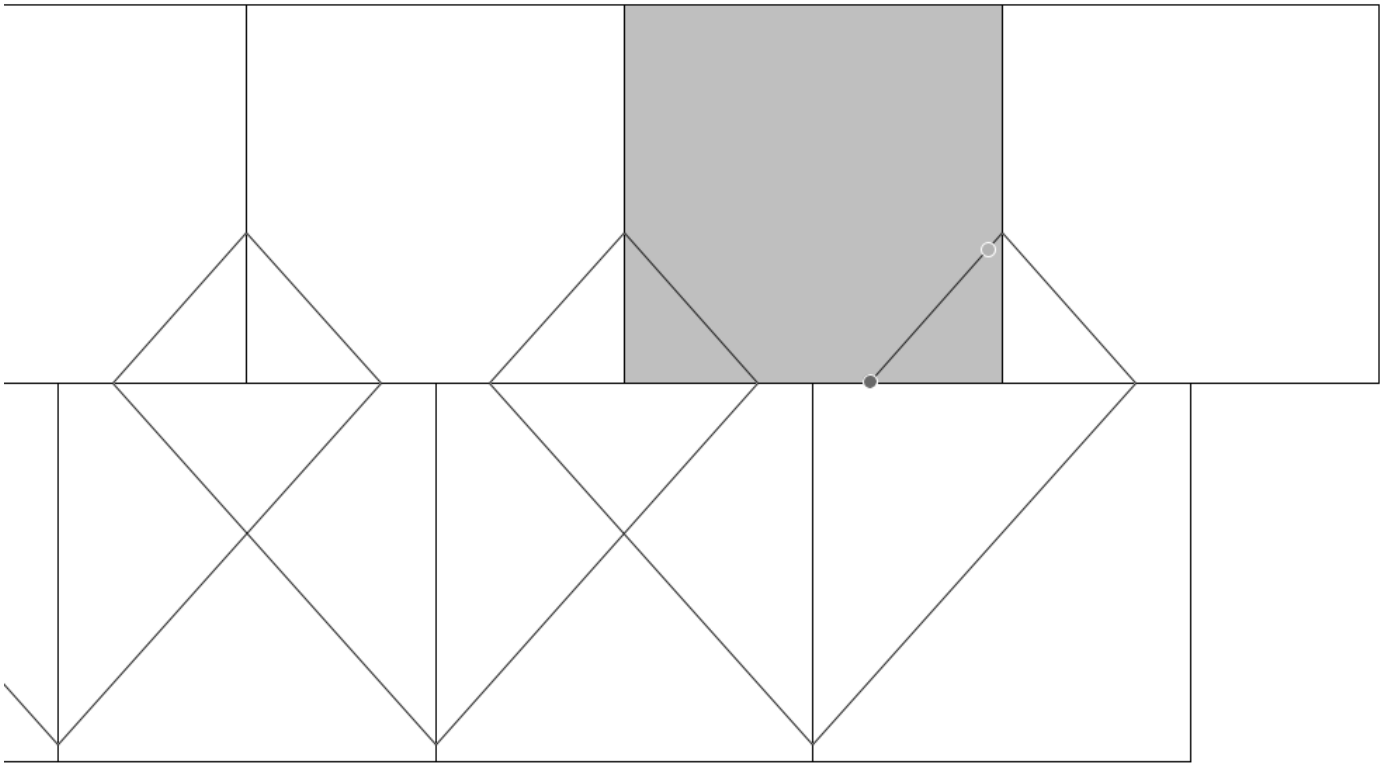


Square Skewed by $\frac{1}{2}$

Case 5: $e_5 \to e_6$.

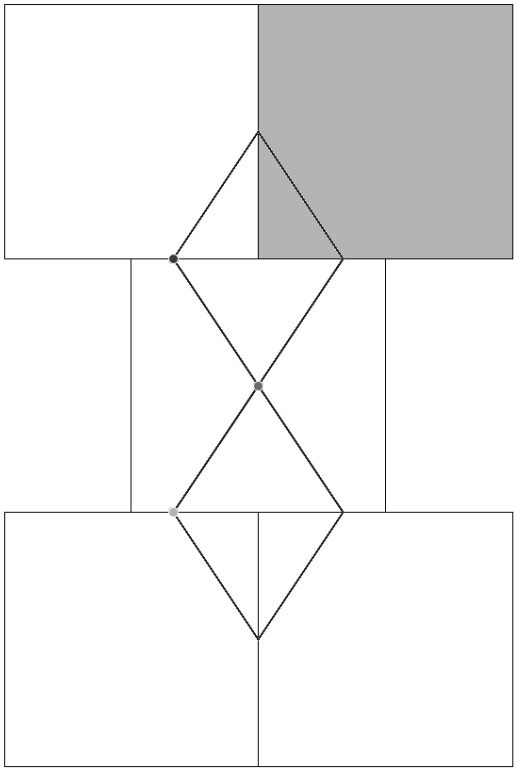
Square Skewed by $\frac{1}{2}$

Case 5: $e_5 \to e_6$. $\alpha \in \left(0, \arctan\left(\frac{1}{\frac{1}{2}+\lambda}\right)\right)$

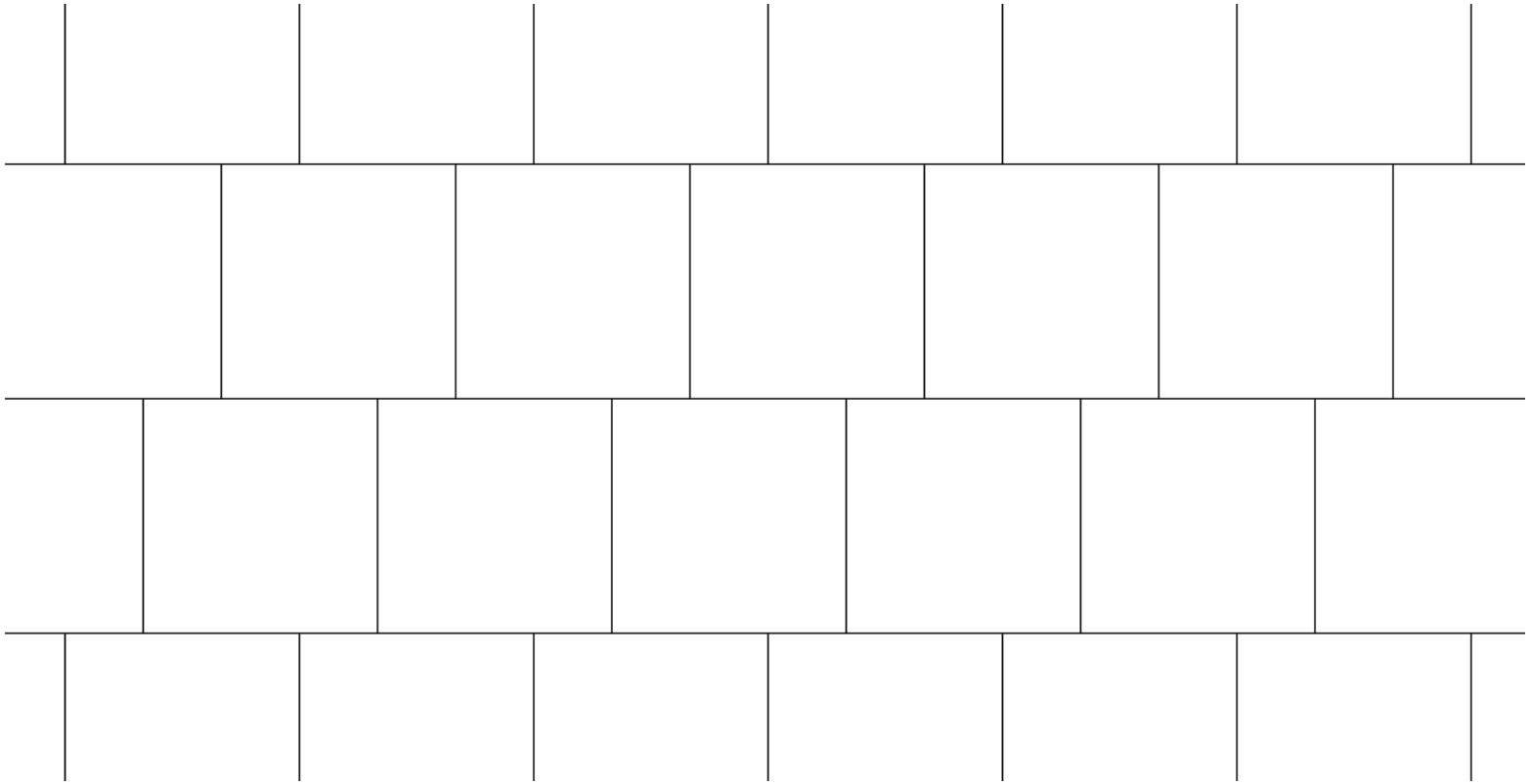


Square Skewed by $\frac{1}{2}$

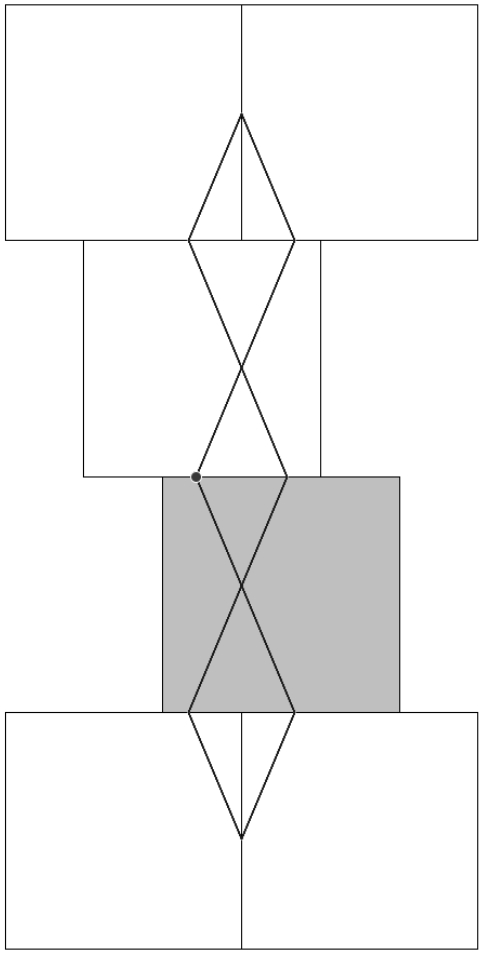
Case 5: $e_5 \to e_6$. $\alpha \in \left(\arctan\left(\frac{1}{\frac{1}{2}+\lambda}, \pi\right)\right)$



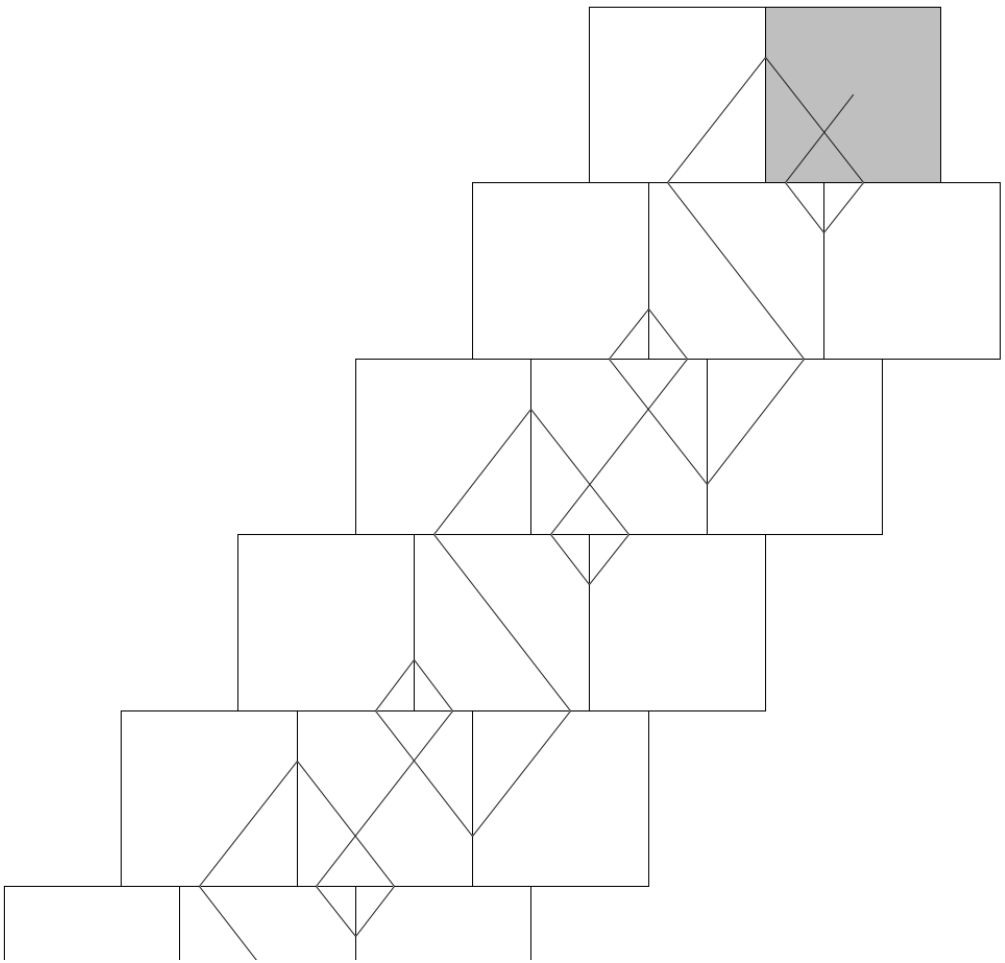
Square Skewed by $\frac{p}{q}$



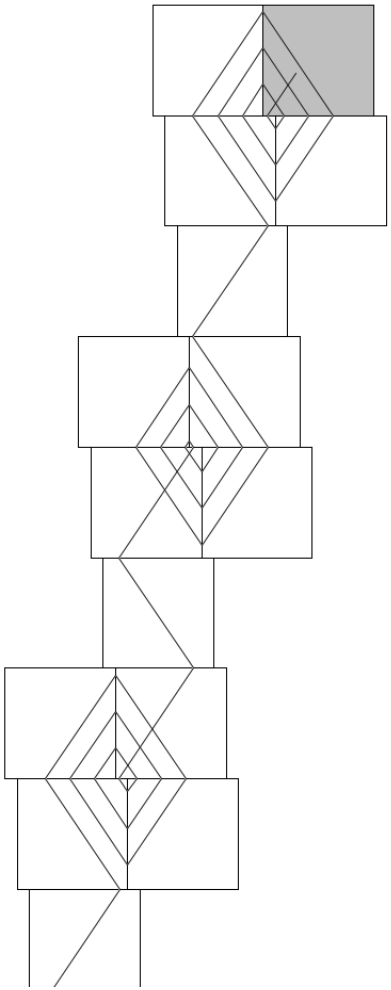
Square Skewed by $\frac{1}{3}$



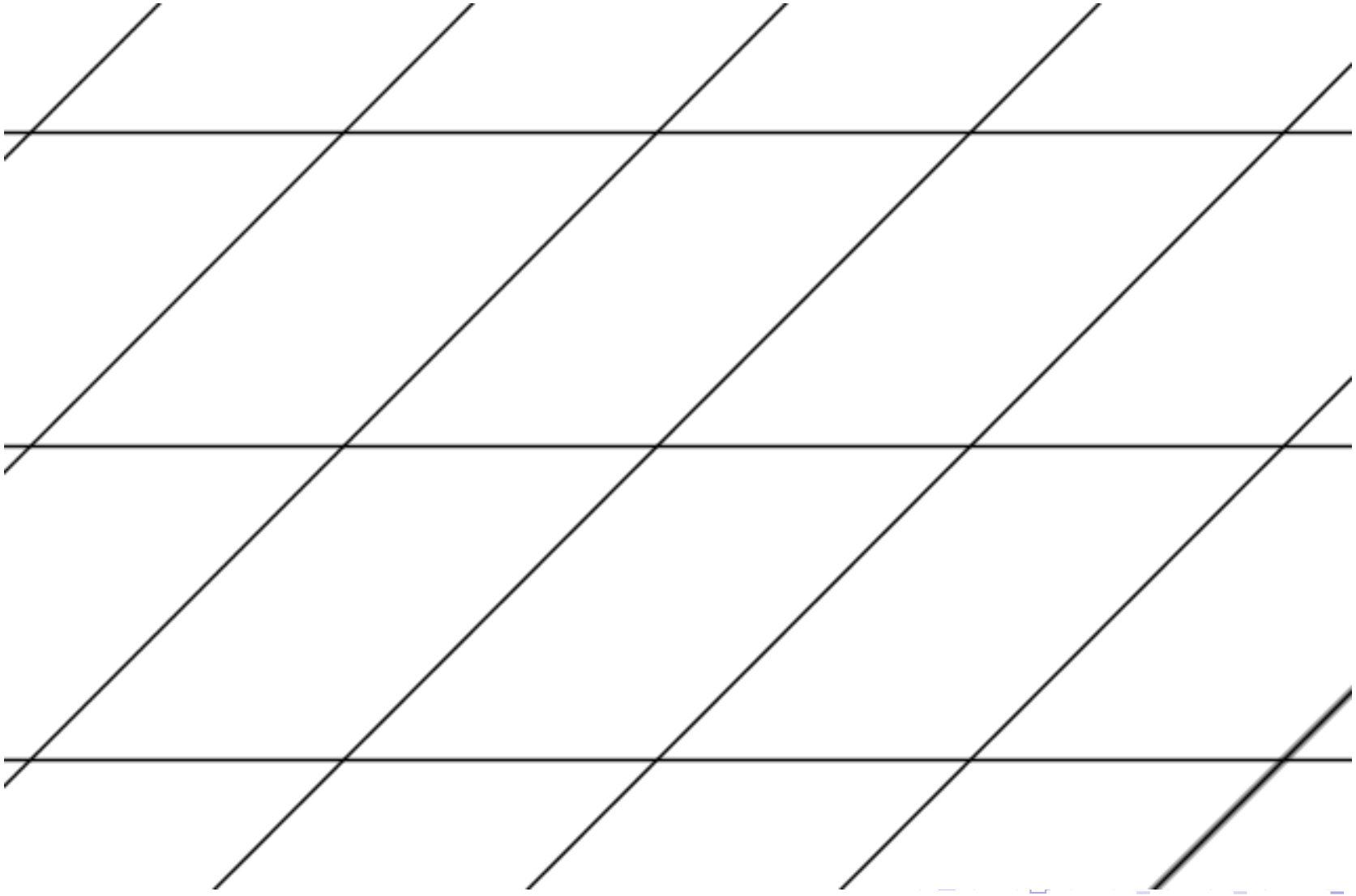
Square Skewed by $\frac{1}{3}$



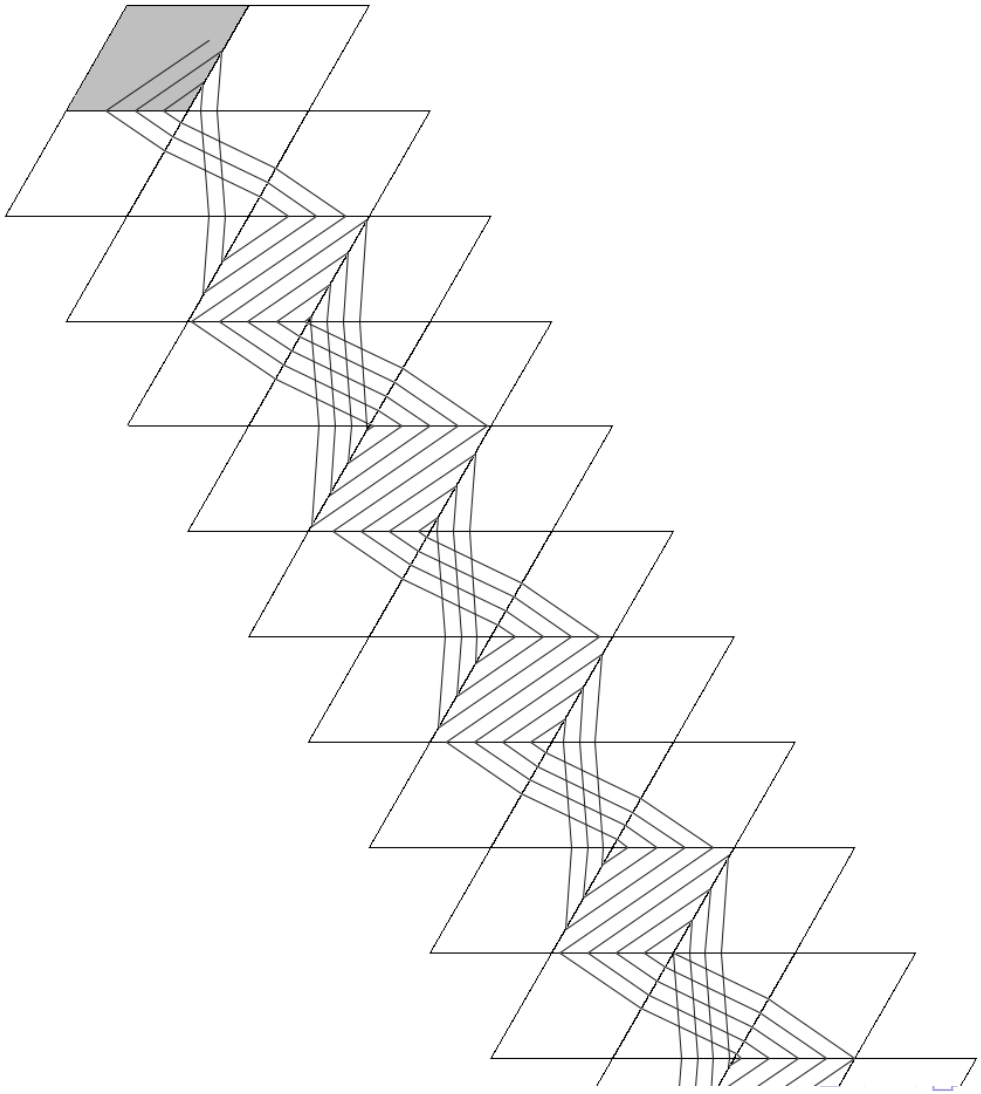
Square Skewed by $\frac{1}{9}$



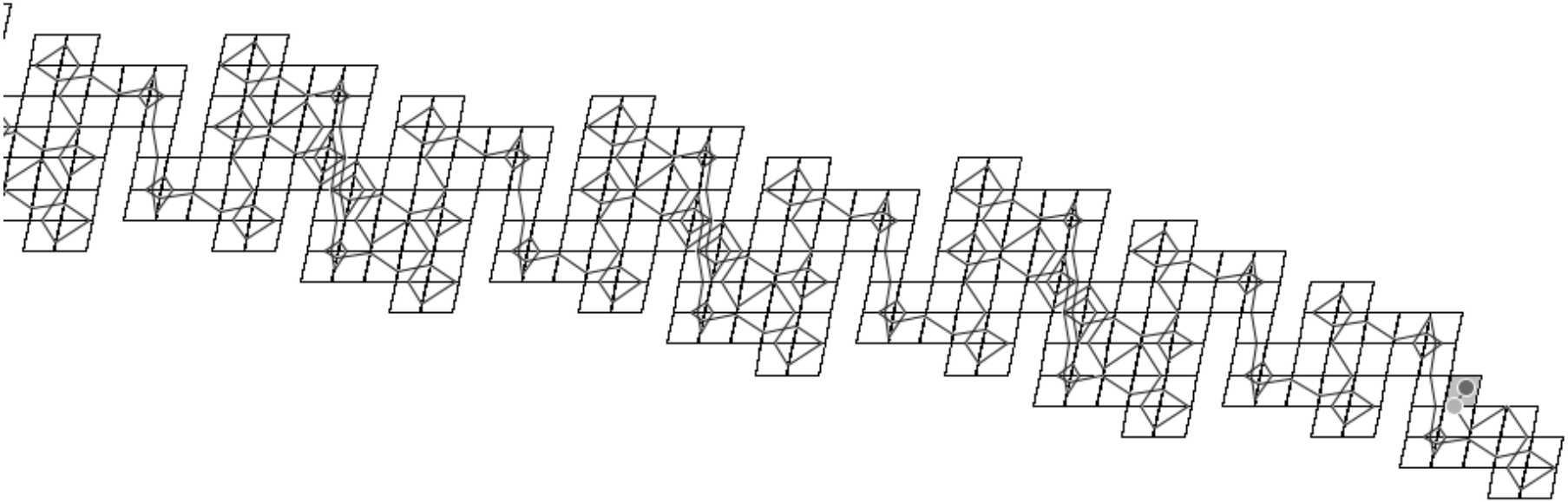
Parallelogram Tiling



Parallelogram Tiling



Parallelogram Tiling



Parallelogram Tiling

Acknowledgements

This research was done by funding through ICERM.
We would also like to thank Diana Davis, W. Patrick Hooper, and Sergei Tabachnikov for helping us with our research.
And Tarik Aougab for telling funny stories.