Dynamics of an n-body Crowd Model

What can simple physical models tell us about densities in crowds?

Introduction

- Motivation
- Past Crowd Models
  - Helbing’s work
  - Open Questions Remain
    - How does pedestrian density build over time?

Motivation

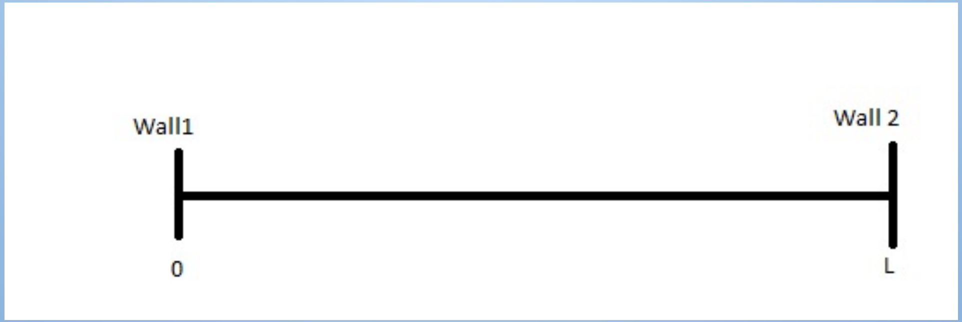
Predictable patterns in high density?

Can we control it?

General description

One-dimensional model.

- Finite interval [0,L]
- Two walls: Wall 1 at 0 and Wall 2 at L



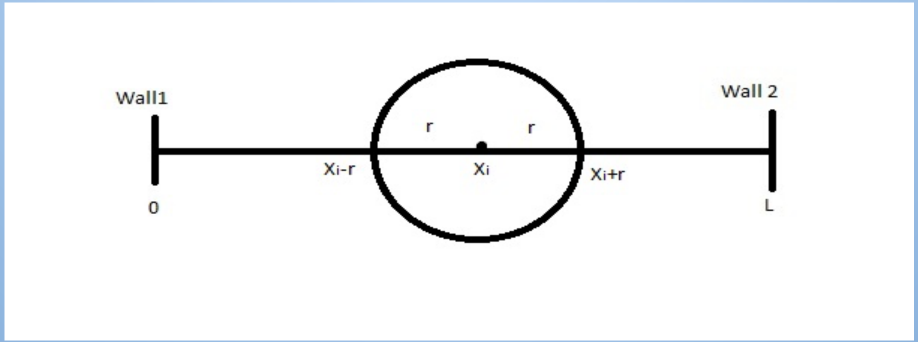
Pedestrians

n people with position $x_i \in (0,L), i \in \{1,\ldots,n\}$.

Space occupied by pedestrian: (x_i-r, x_i+r),

$X_i < X_{i+1}$ for all time t.

How to measure density?



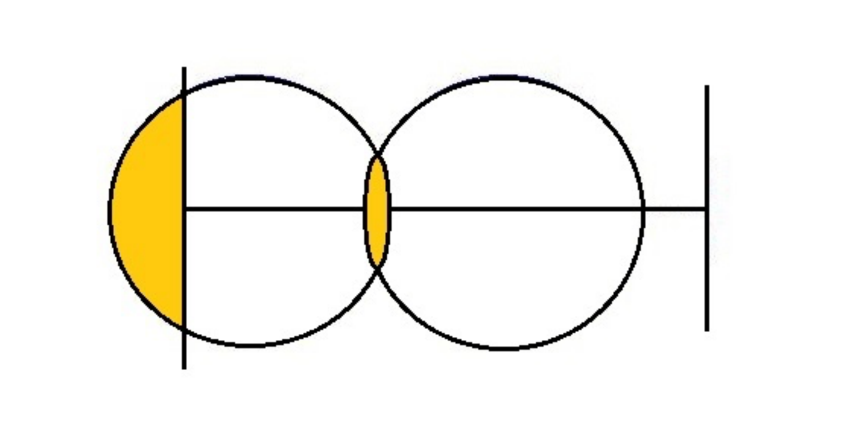
Overlaps (assumed greater than zero)

- Overlap between person i and person i+1:

$$\Delta_i = (x_i + r) - (x_{i+1} - r) = x_i - x_{i+1} + 2 \cdot r$$

- Overlaps with the walls:

$$\Delta_{W_L} := -x_i + r \qquad \Delta_{W_R} := x_n + r - L$$



Forces

Motivation: Normal Force

Recall: $\boxed{x_i < x_{i+1}}$ for all time t

- No crossing between people
- Obvious force: Normal Force

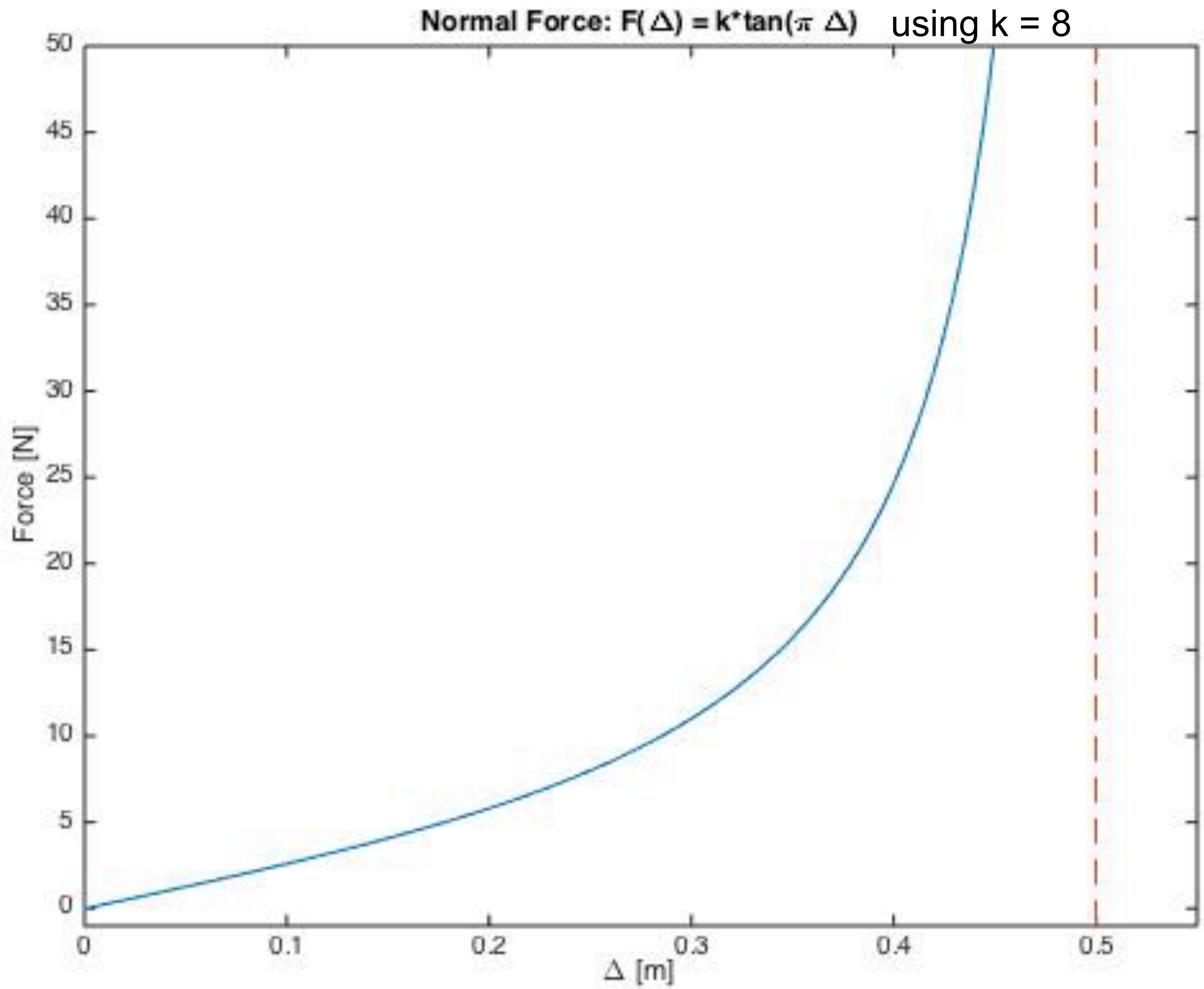
When two pedestrians are in contact, the normal force is:

$$F_i^N(\Delta_i) := \kappa \tan\left(\frac{\pi}{2}\frac{\Delta_i}{2r}\right)$$

- When a pedestrian is in contact with the wall, the normal force is:

$$F^N_{W_L} := \kappa \tan(\tfrac{\pi}{2}\tfrac{\Delta_L}{r})$$

$$F^N_{W_R} := \kappa \tan(\frac{\pi}{2}\frac{\Delta_R}{r})$$


Normal Force: F(Δ) = k*tan(π Δ)
using k = 8
Force [N]
Δ [m]
0
5
10
15
20
25
30
35
40
45
50
0
0.1
0.2
0.3
0.4
0.5

Motivation: Pushing force

Necessary to depict human behavior in tightly packed conditions and need for personal space.

We expect to observe density peaks and hope to find patterns.

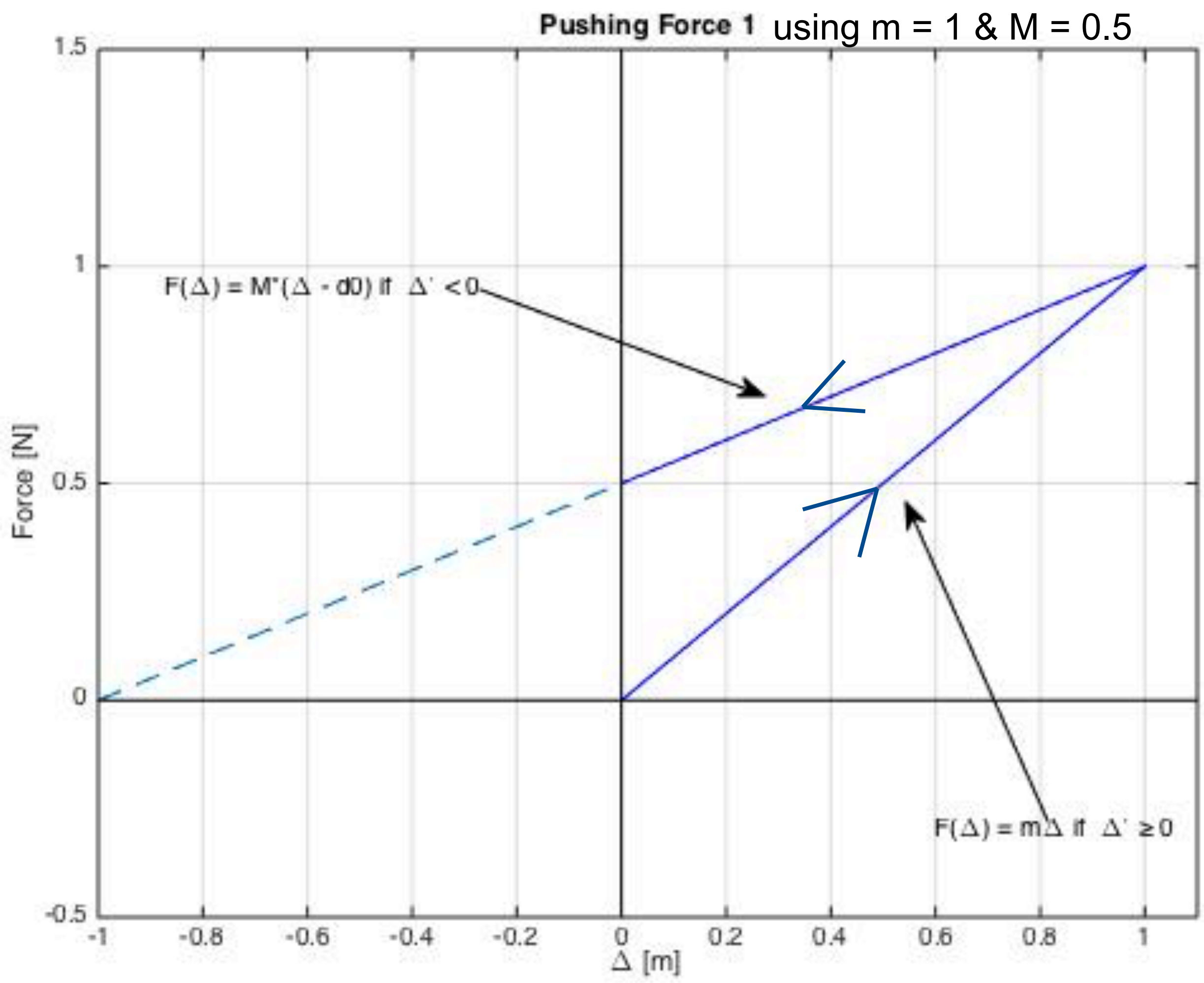
Pushing force I

Scenario: Reactive Pushing Force

It seems natural to consider that the intensity of the force depends on the level of overlap.

This results in a force of the form:

$$F^A(\Delta_i, \dot{\Delta}_i) = \begin{cases} m\Delta_i & \text{if } 0 \geq \dot{\Delta}_i \\ M(\Delta_i - d0) & \text{if } 0 < \dot{\Delta}_i \end{cases}$$


Pushing Force 1 using m = 1 & M = 0.5
F(Δ) = M*(Δ - d0) if Δ' < 0
F(Δ) = mΔ if Δ' ≥ 0
Force [N]
Δ [m]
1.5
1
0.5
0
-0.5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Equations (Newton’s 2nd Law)

$$m_1\ddot{x}_1 = F^N(\Delta_{W_L}) - F^N(\Delta_1) - F^P(\Delta_1, \dot{\Delta}_1)$$

$$m_i\ddot{x}_i = F^N(\Delta_{i_1}) - F^N(\Delta_i) + F^P(\Delta_{i-1}, \dot{\Delta}_{i-1}) - F^P(\Delta_i, \dot{\Delta})$$

$$m_n\ddot{x}_n = -F^N(\Delta_{W_R}) + F^N(\Delta_{n-1}) + F^P(\Delta_{n-1}, \dot{\Delta}_{n-1})$$

Observable Data

Energy

Can we verify Consistency?

Kinetic energy:

$$\sum_{i=1}^{n} KE_i = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2$$

Potential energy:

$$\int F_L^N(\Delta_L)d\Delta_L + \int F_R^N(\Delta_R)d\Delta_R + \sum_{i=1}^{n} \int F_i^N(\Delta_i)d\Delta_i$$

Total energy:

$$E = \frac{1}{2}\sum_{i=1}^{n} m_i \dot{x}_i{}^2 + \int F_L^N(\Delta_L)d\Delta_L + \int F_R^N(\Delta_R)d\Delta_R + \sum_{i=1}^{n} \int F_i^N(\Delta_i)d\Delta_i$$

Density

Consider an open interval J centered at $X_j \in [0,L]$

$$\rho(J,t) := \sum_{i=1}^{n} m[J \cap (x_i - r, x_i + r)]$$

An option is to study the whole interval [0,L] first partitioning it by:

$$0 = X_0 < X_1 < X_2 < . . . . < X_n = L$$

considering the density p_j,

$$\rho_j(t) := \rho((X_j - \delta, X_j + \delta); t)$$

$$= \sum_{i=1}^{n} m[(X_j - \delta, X_j + \delta) \cap (x_i - r, x_i + r)]$$

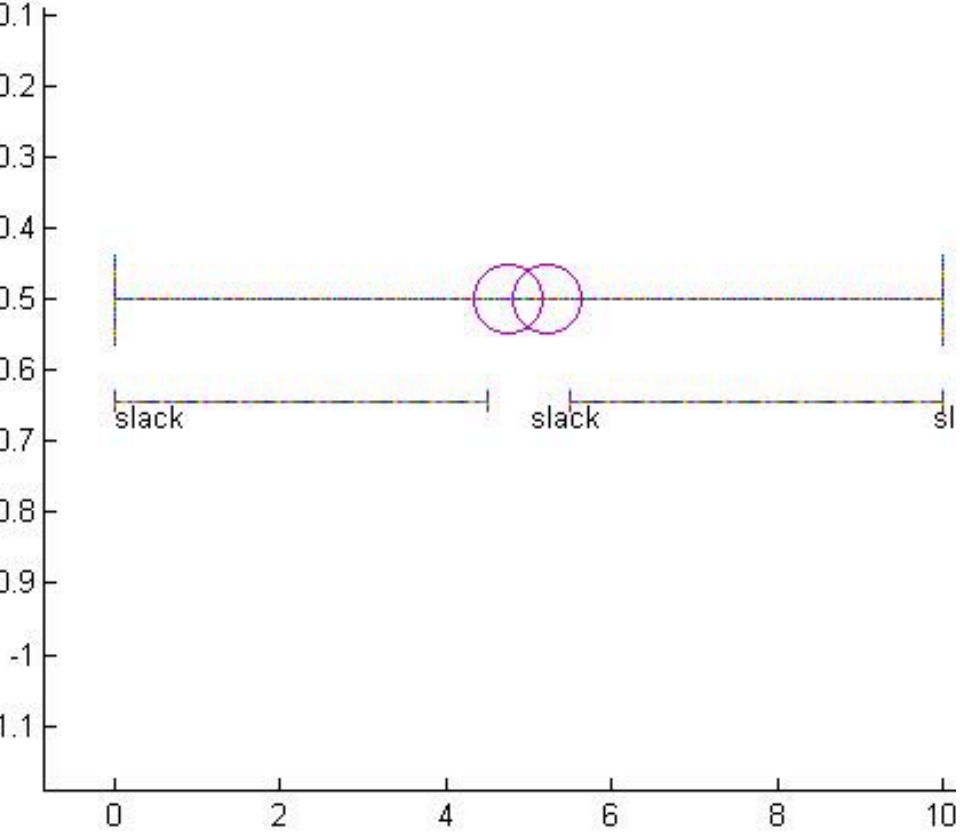
Spacial Organization

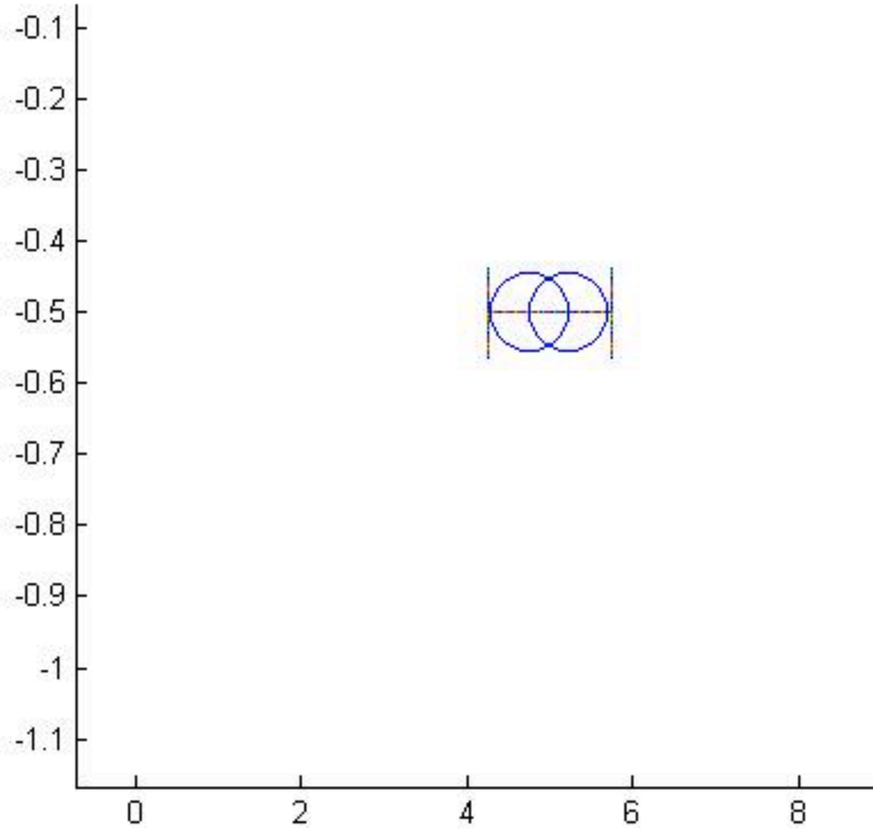
Length & slack

If we consider the length $L \geq 2rn$, with initial overlaps and velocities equal to 0, it provides a stationary state of the system.

Reducing the length of the interval by $\varepsilon > 0$ we can induce movement.

Does this affect the results?


slack
slack
sl
0
2
4
6
8
10
-1


-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
0
2
4
6
8

Initial configurations

Initial velocities are 0 m/s

We simulated with differing initial profiles

Example Simulations

Example I

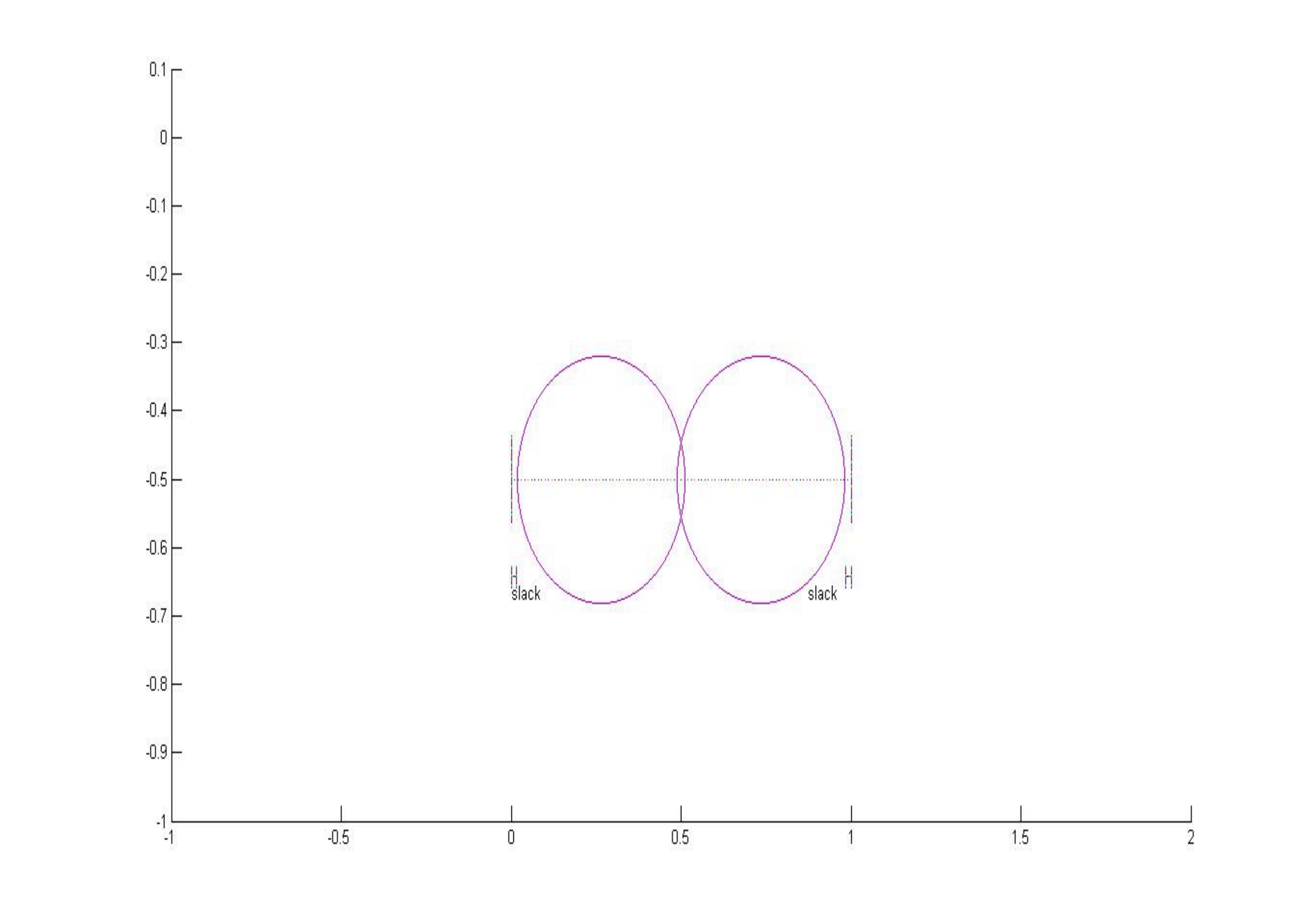
Initial configuration: III

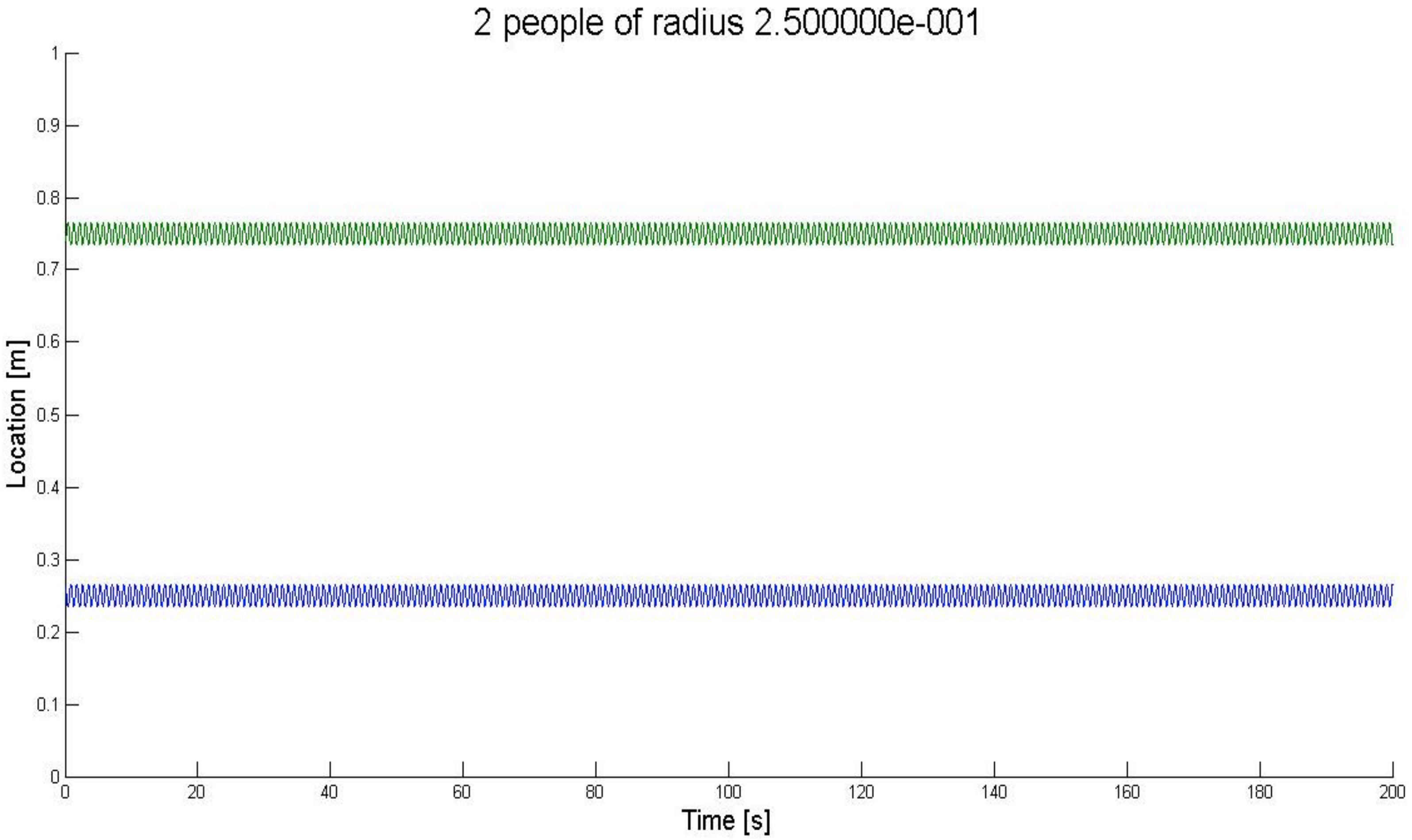
Number of people: n = 2

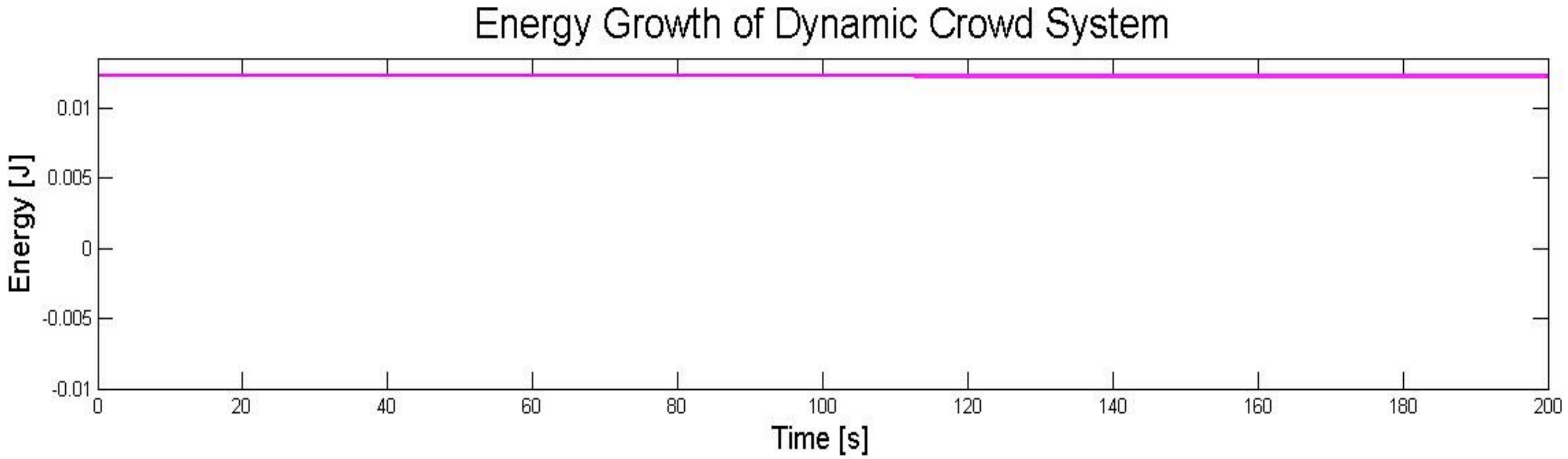
Forces: Normal Force Only

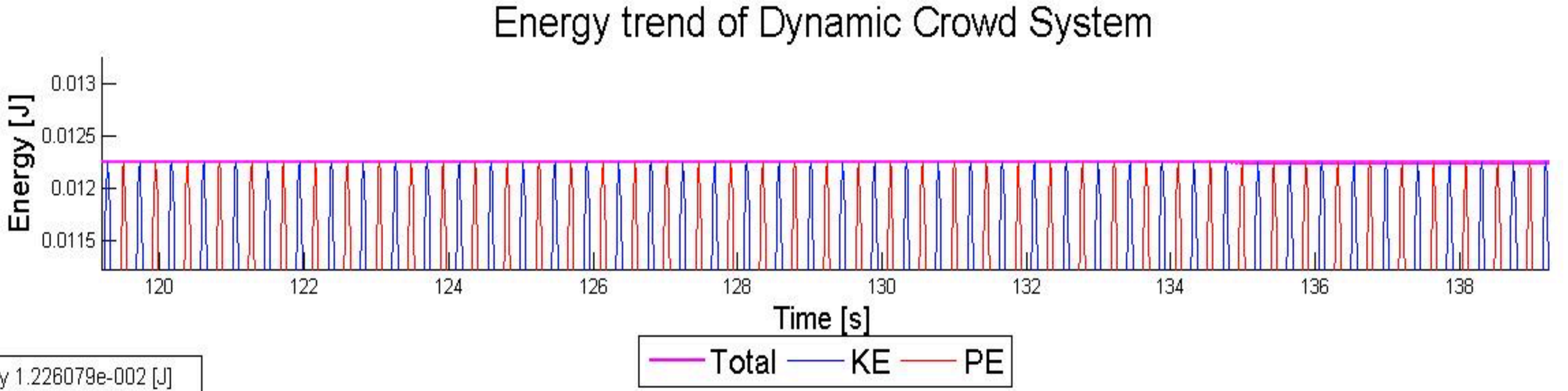
Length: 2rn

Initial configuration III

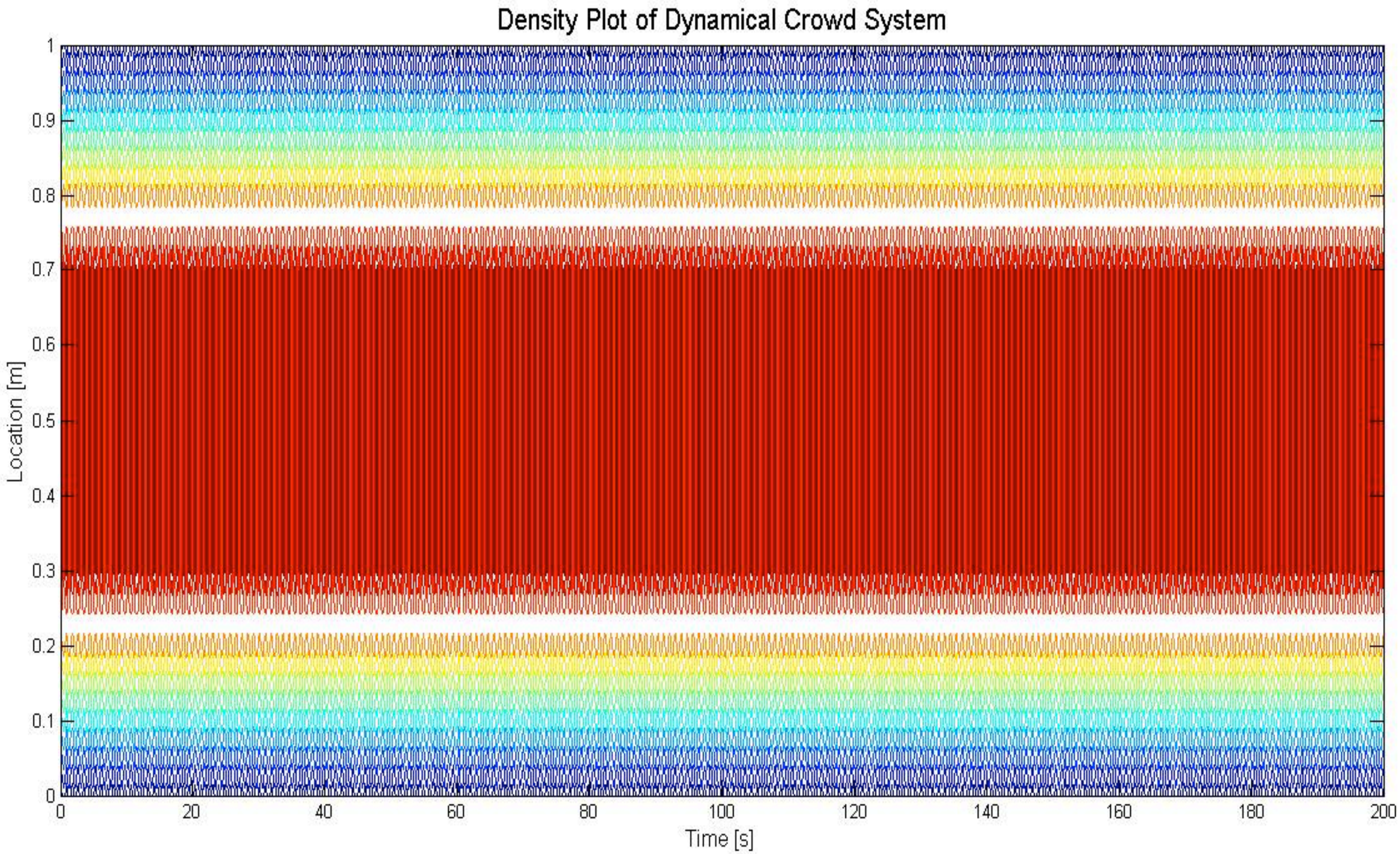


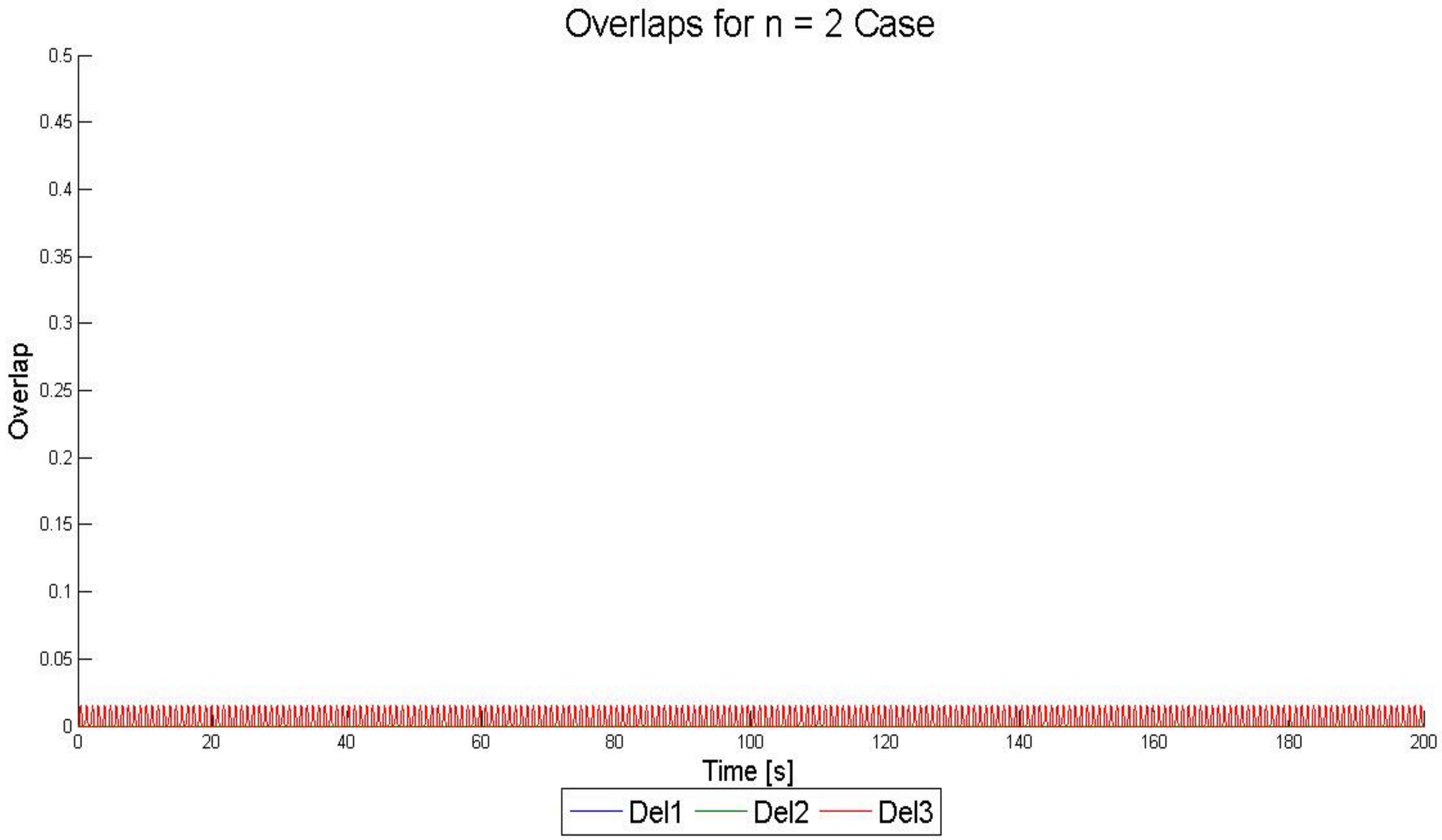

2 people of radius 2.500000e-001
Location [m]
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200

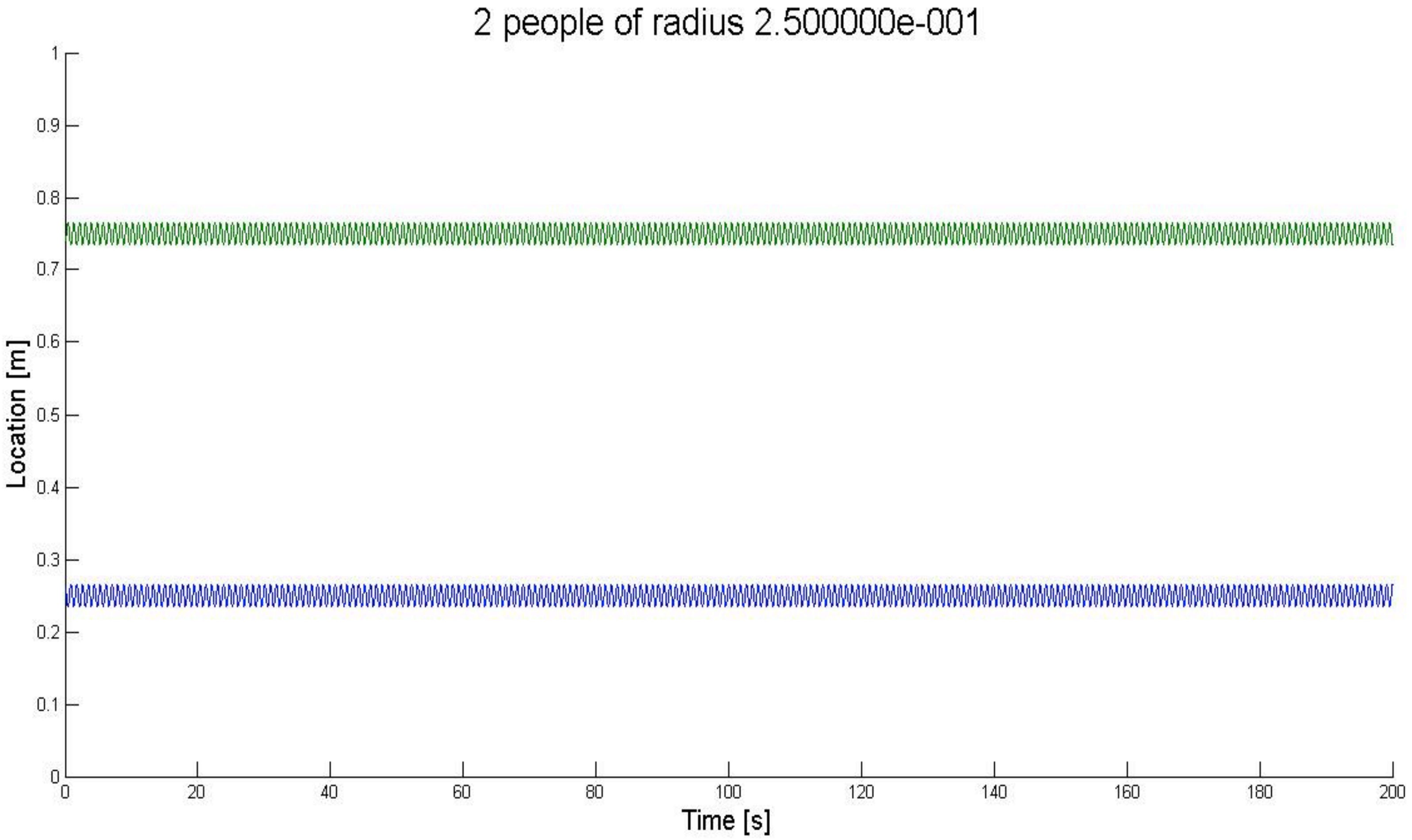




Average Energy 1.226079e-002 [J]


Density Plot of Dynamical Crowd System
Location [m]
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200


Overlaps for n = 2 Case
Overlap
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
20
40
60
80
100
120
140
160
180
200
Time [s]
Del1
Del2
Del3


2 people of radius 2.500000e-001
Location [m]
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200

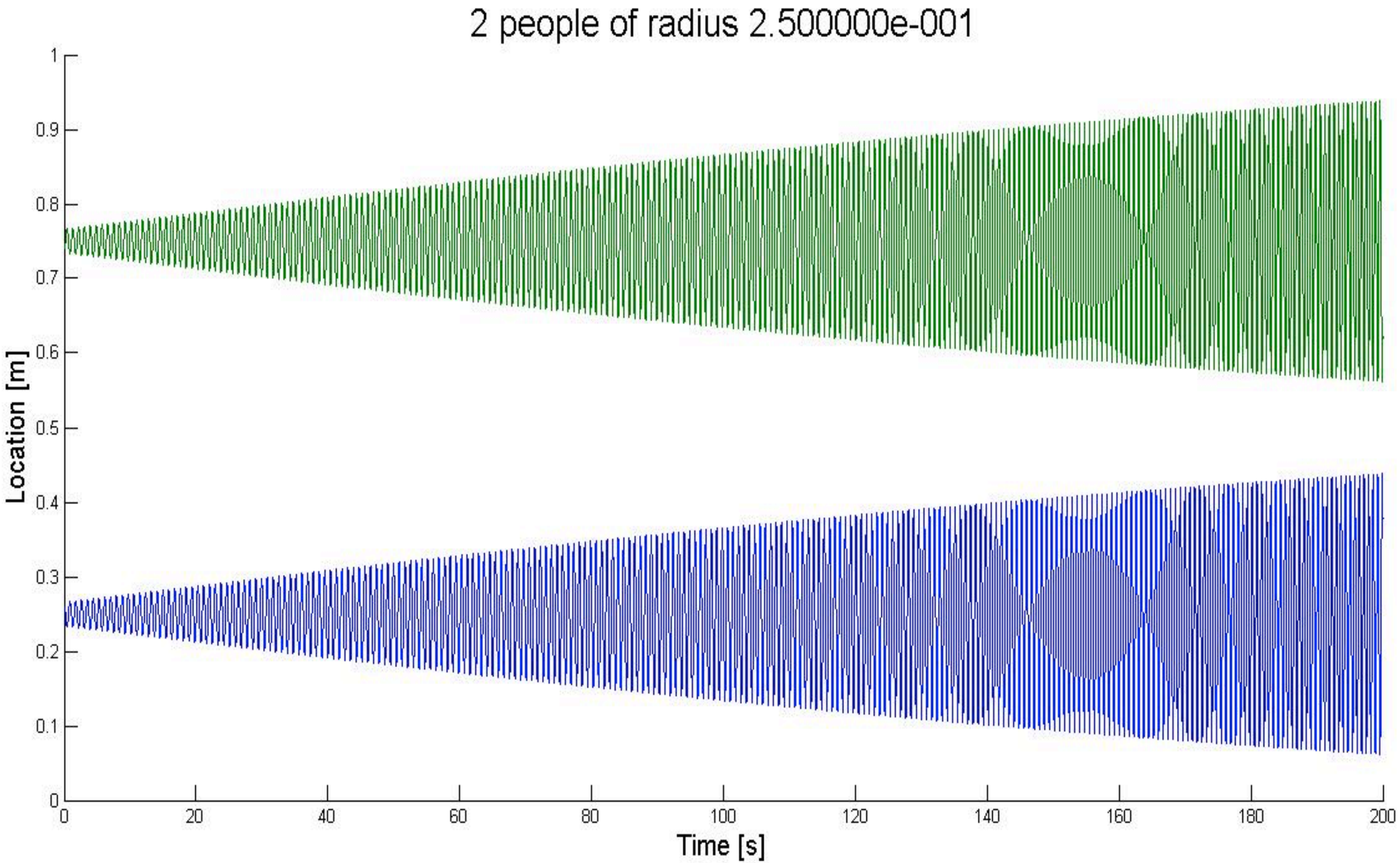
Example II

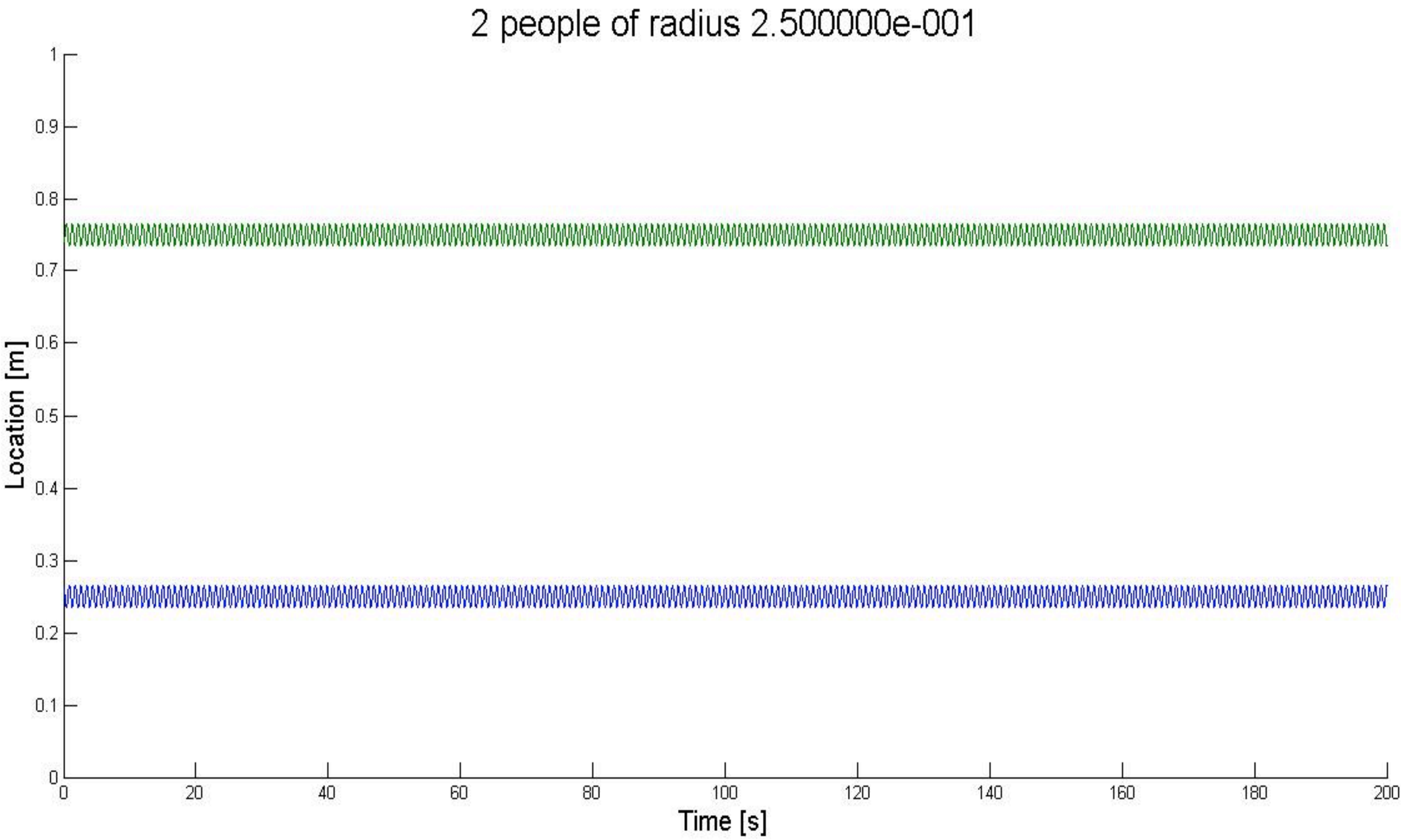
Initial configuration: III

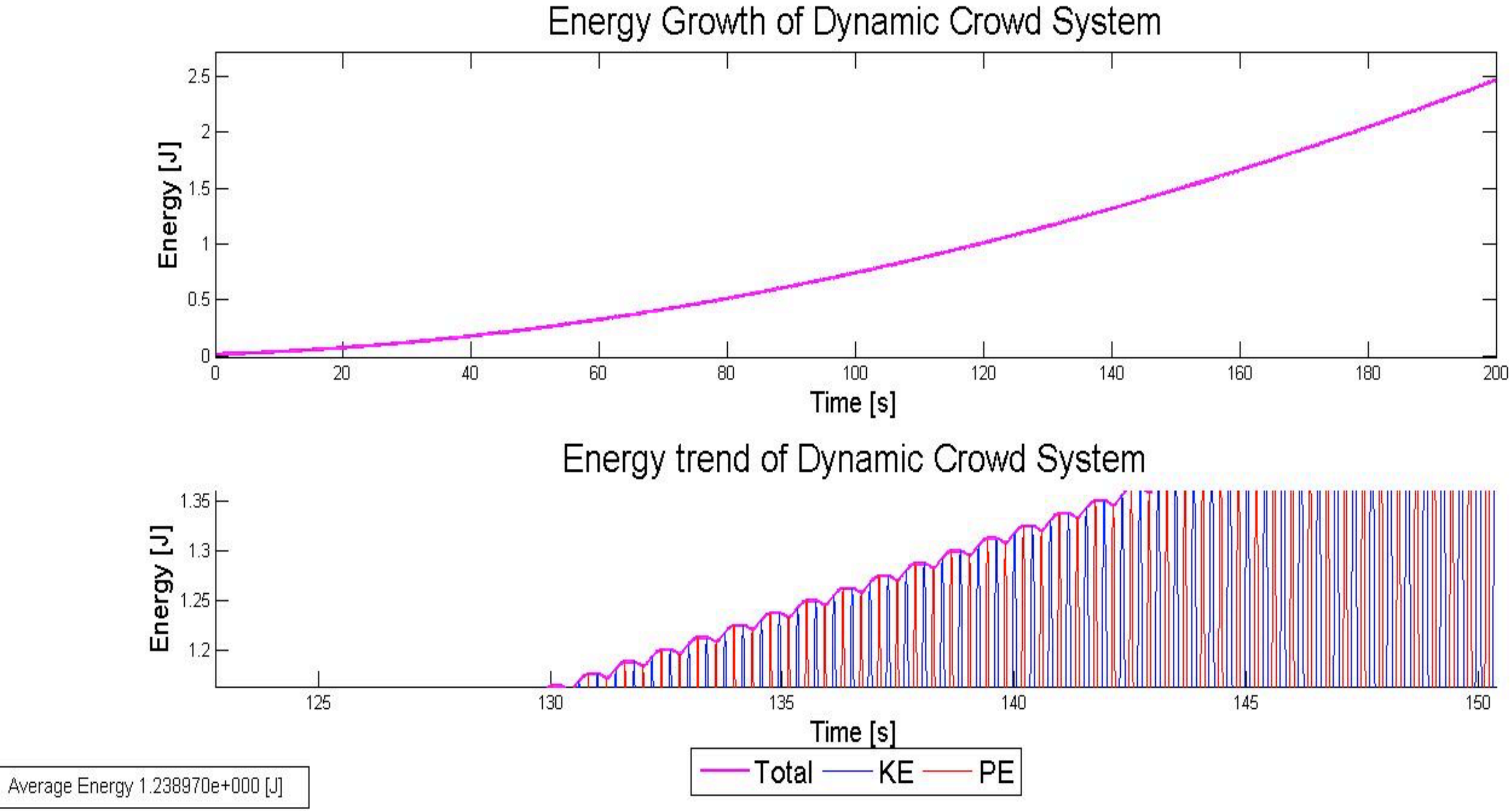
Number of people: $n = 2$

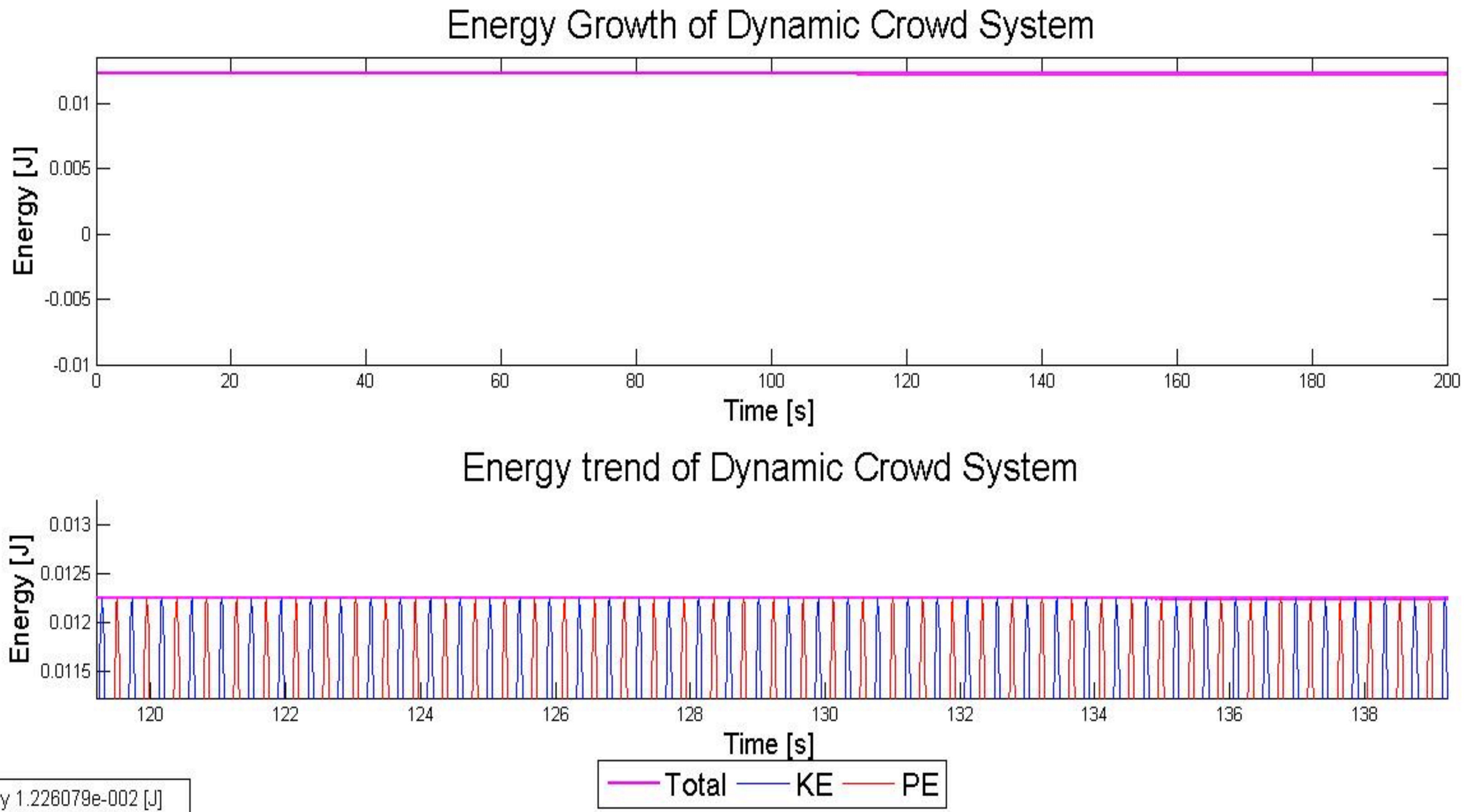
Forces: Normal Force & Pushing Force I

Length: $2rn$

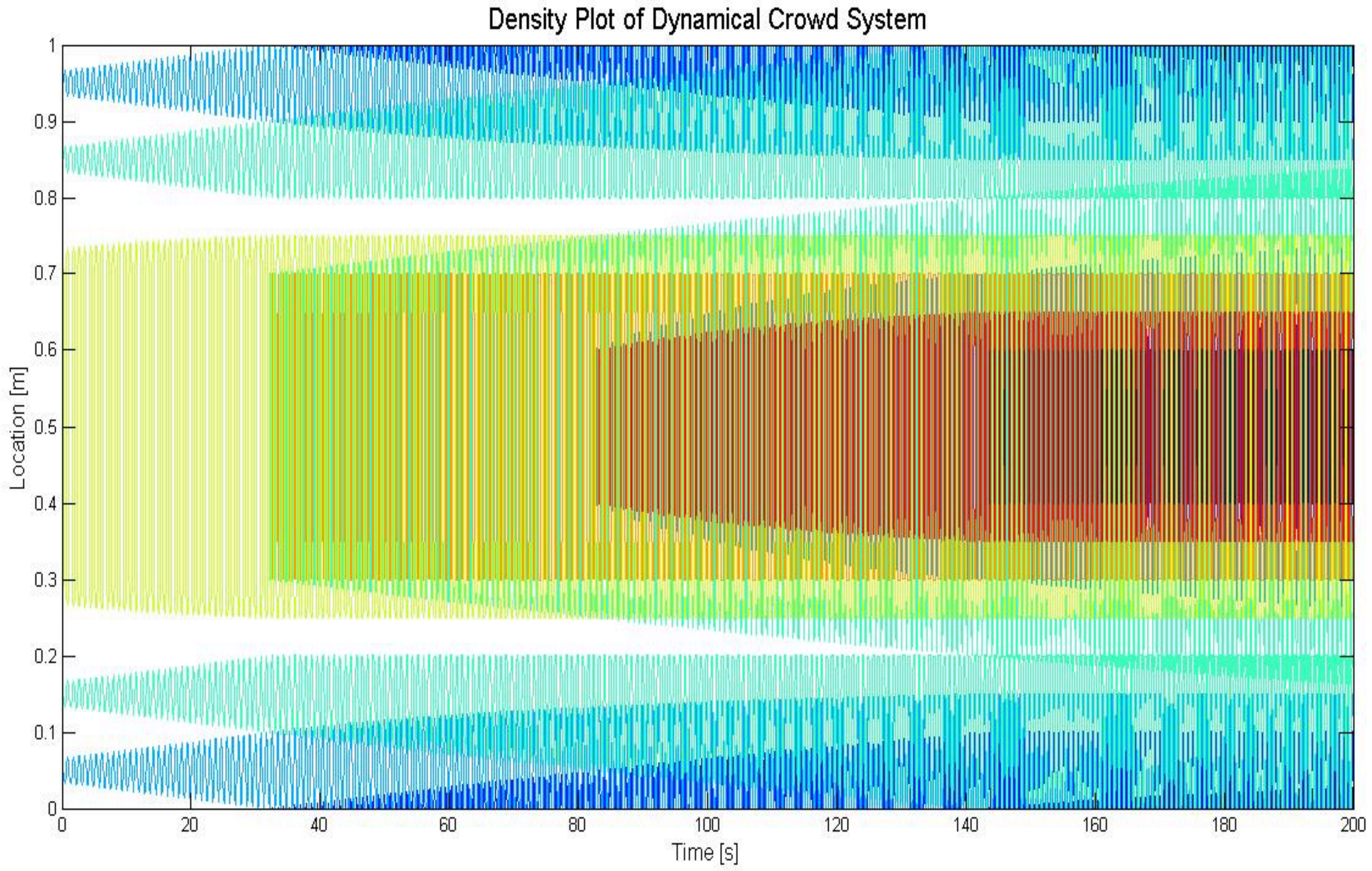

2 people of radius 2.500000e-001
Location [m]
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200

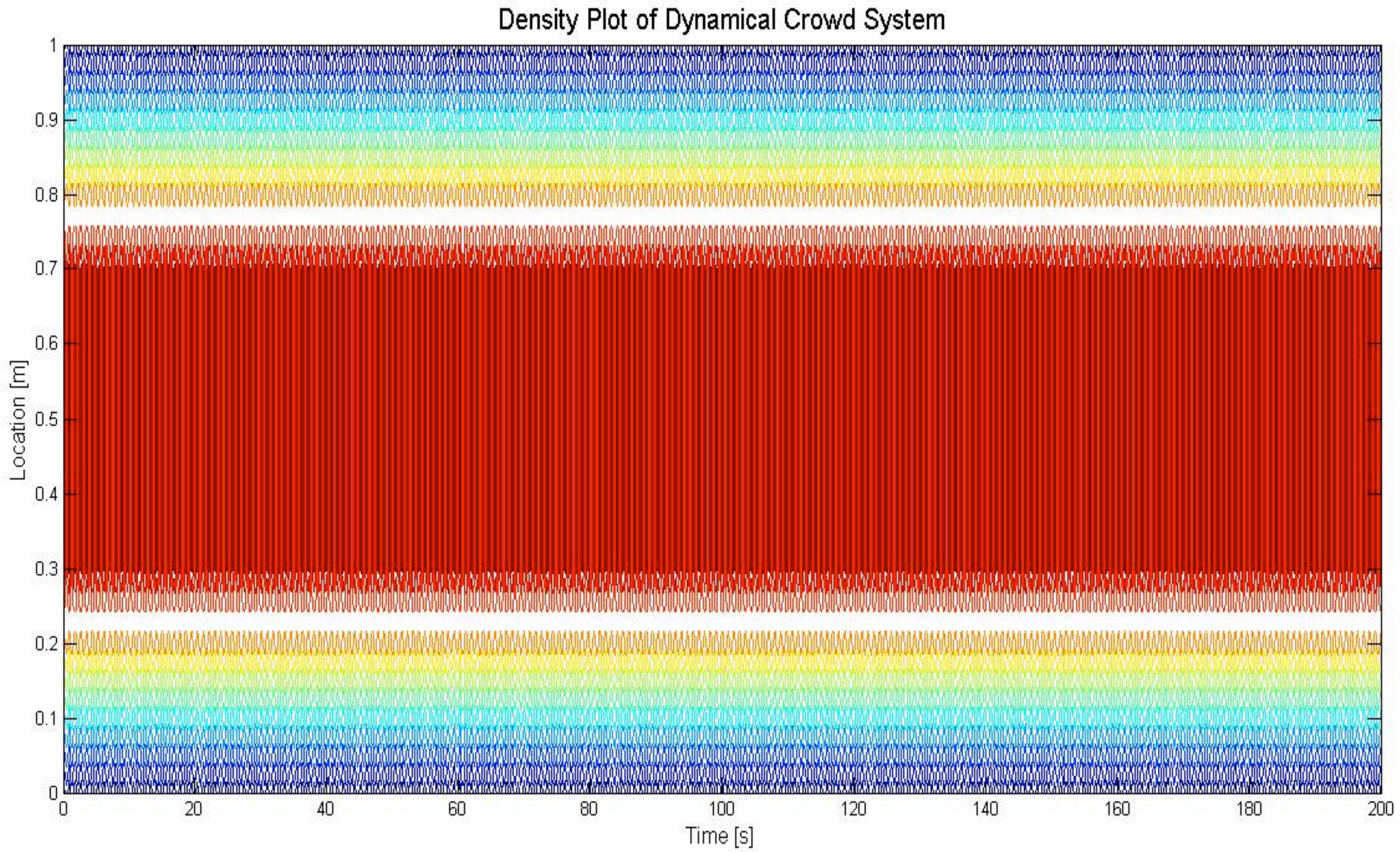

2 people of radius 2.500000e-001
Location [m]
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200

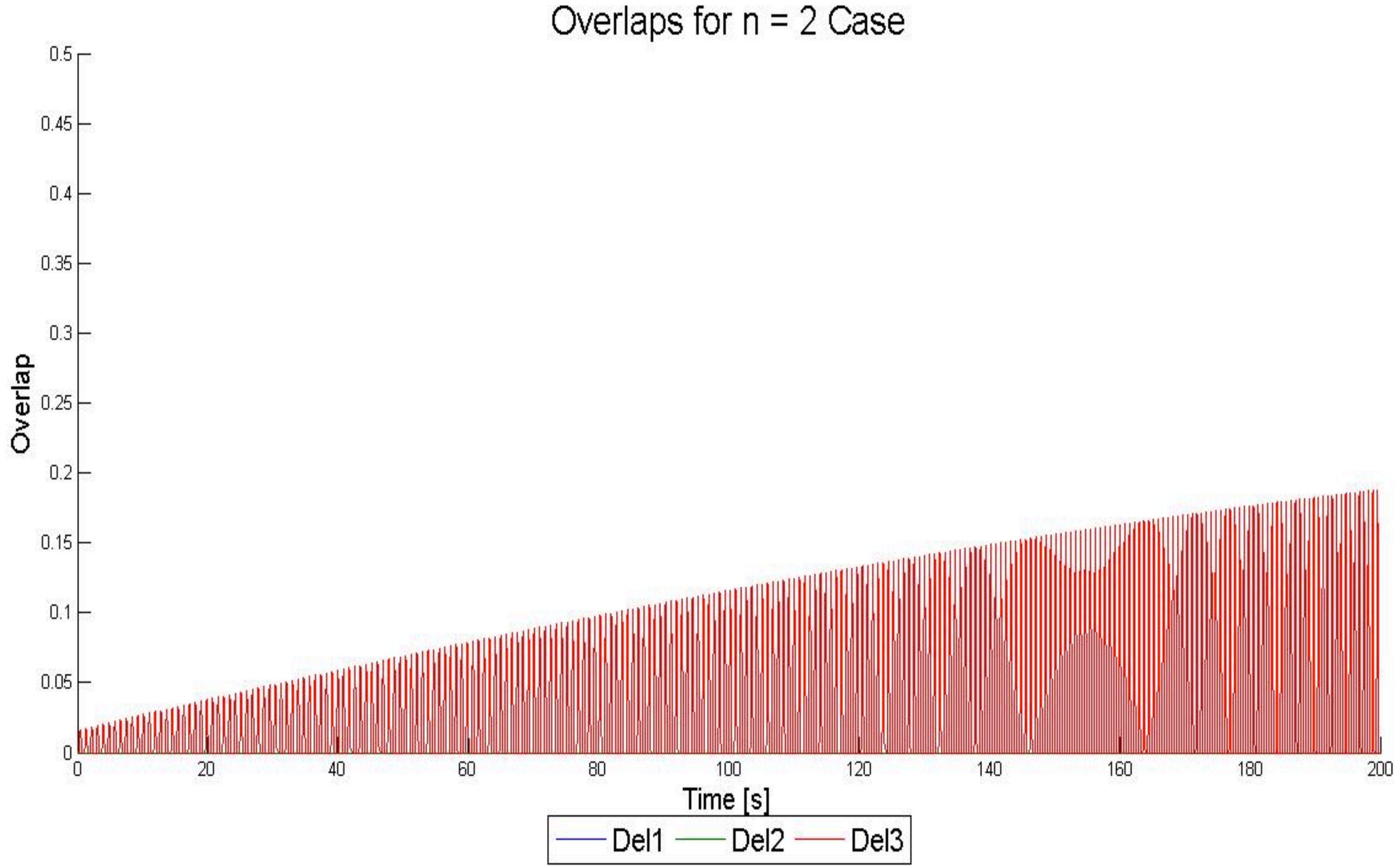

Energy Growth of Dynamic Crowd System
Energy [J]
0
0.5
1
1.5
2
2.5
0 20 40 60 80 100 120 140 160 180 200
Time [s]
Energy trend of Dynamic Crowd System
Energy [J]
1.2
1.25
1.3
1.35
125 130 135 140 145 150
Time [s]
Total
KE
PE
Average Energy 1.238970e+000 [J]



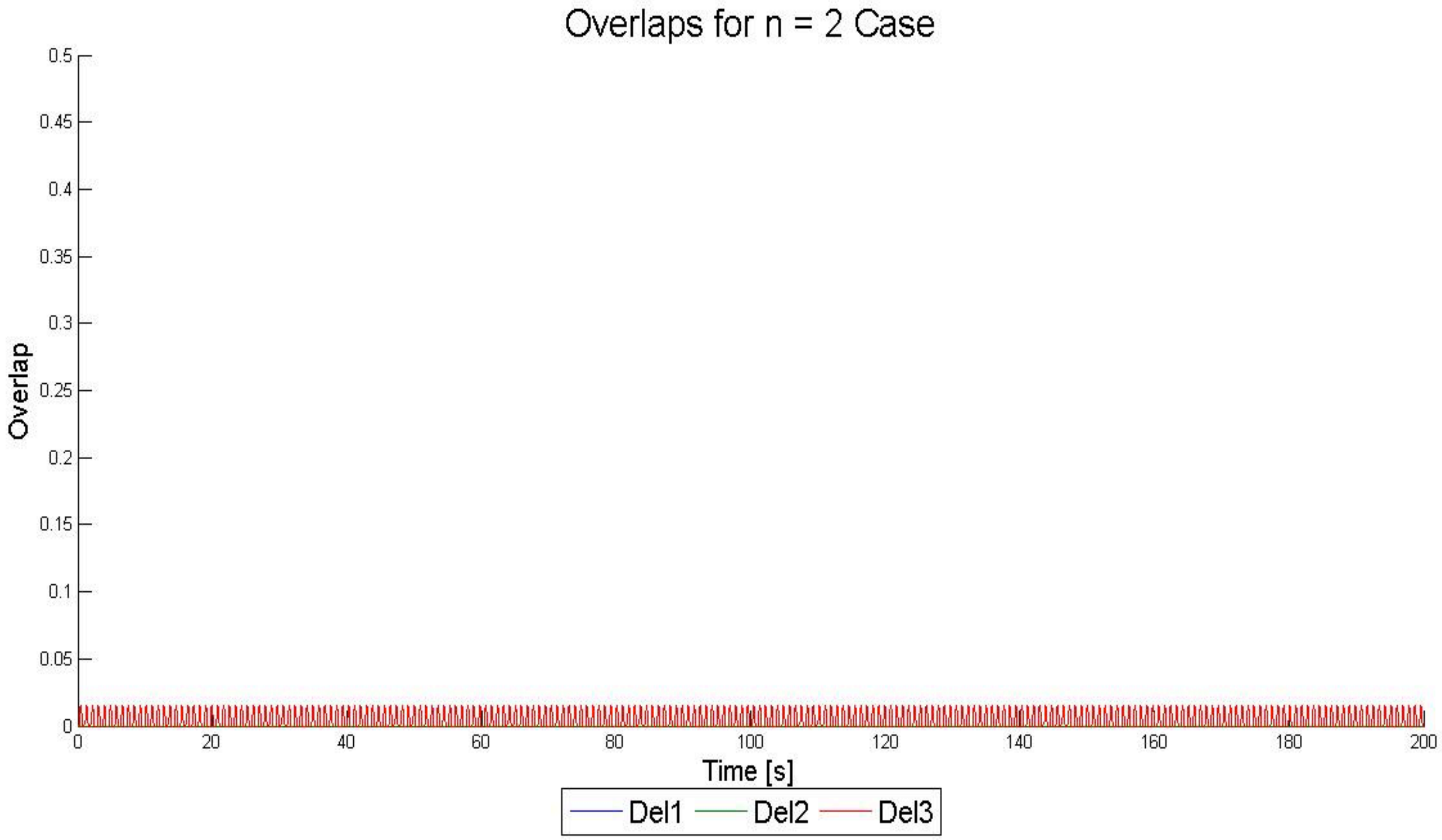
Average Energy 1.226079e-002 [J]


Density Plot of Dynamical Crowd System
Location [m]
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200


Density Plot of Dynamical Crowd System
Location [m]
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200



Overlaps for n = 2 Case
Overlap
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
20
40
60
80
100
120
140
160
180
200
Time [s]
Del1
Del2
Del3


Overlaps for n = 2 Case
Overlap
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
20
40
60
80
100
120
140
160
180
200
Time [s]
Del1
Del2
Del3

Example III

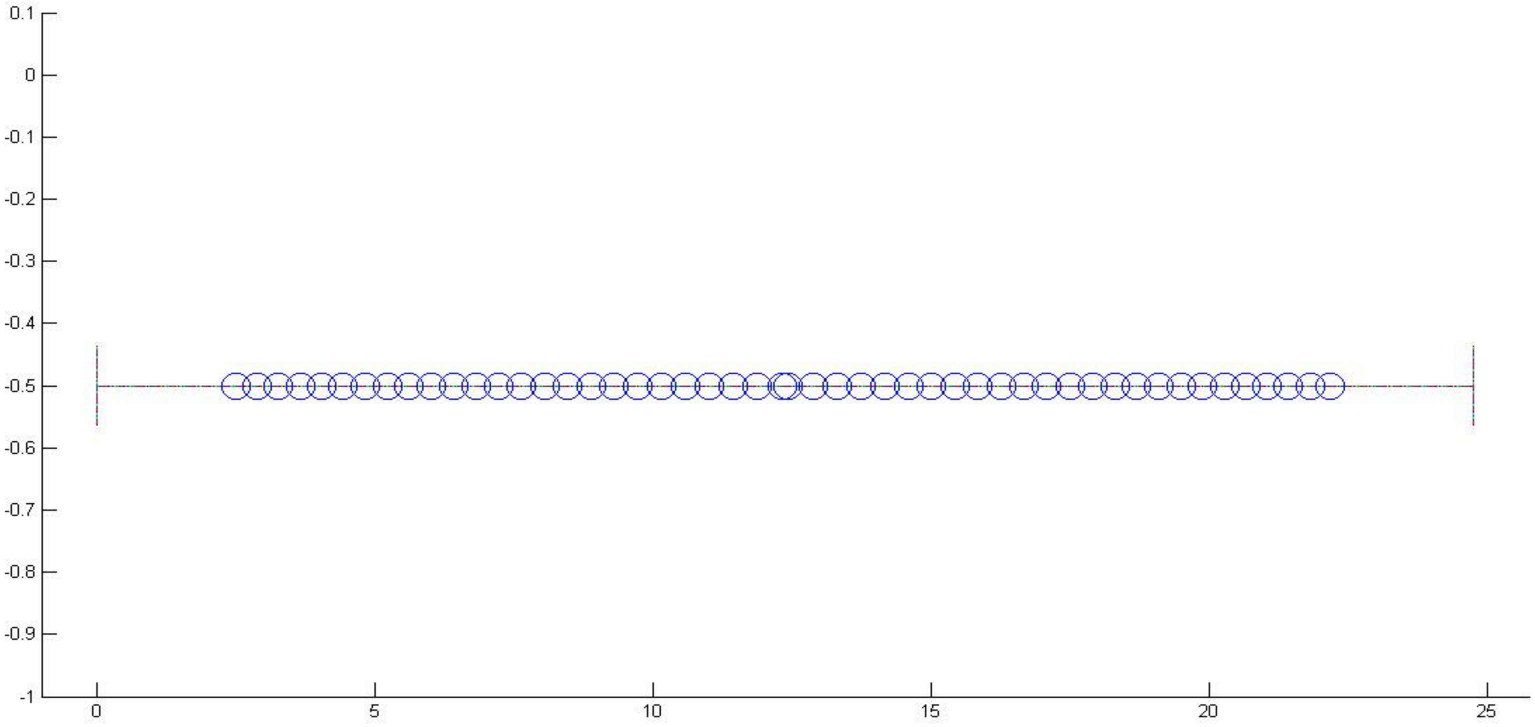
Initial configuration: Bell Curve

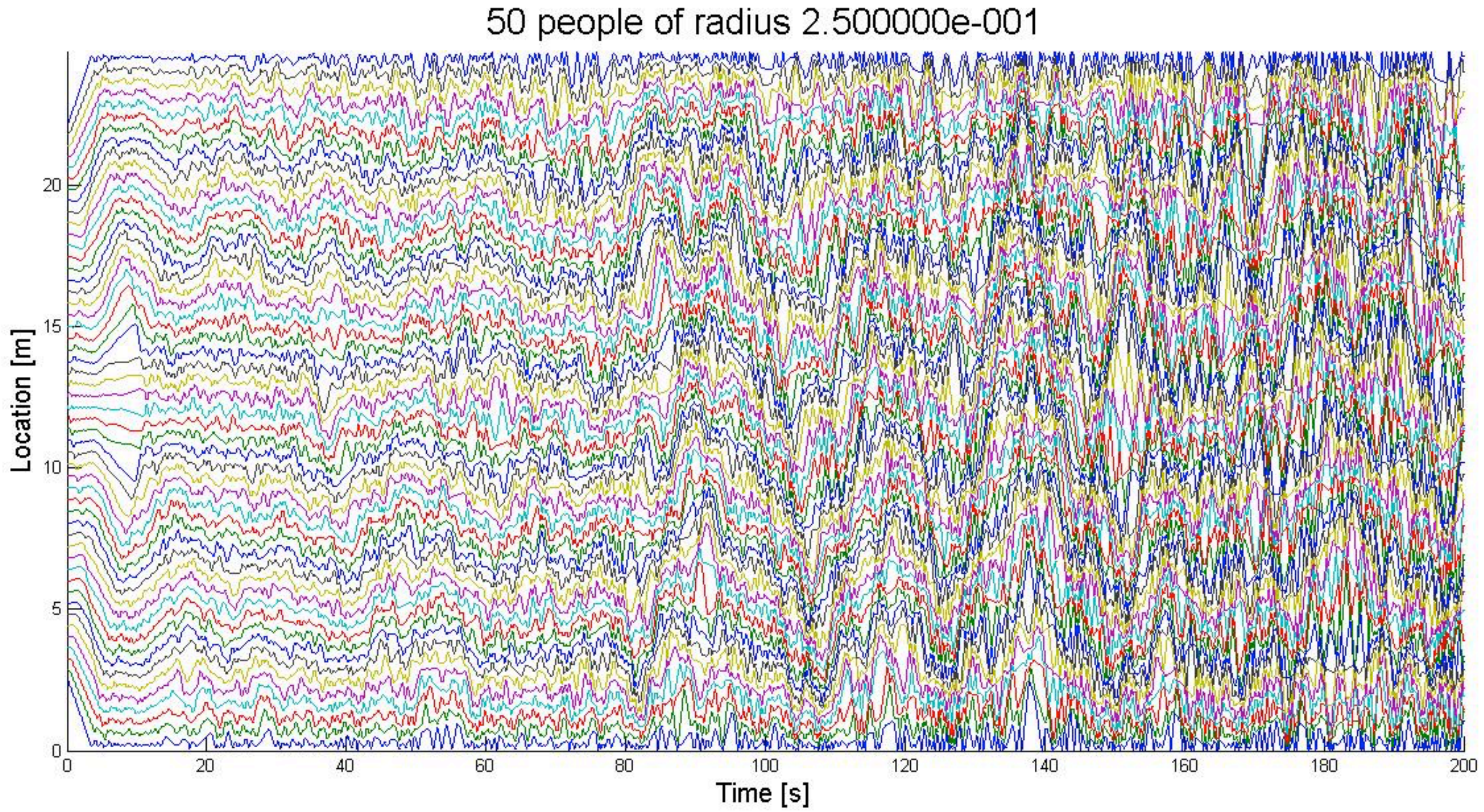
Number of people: $n = 50$

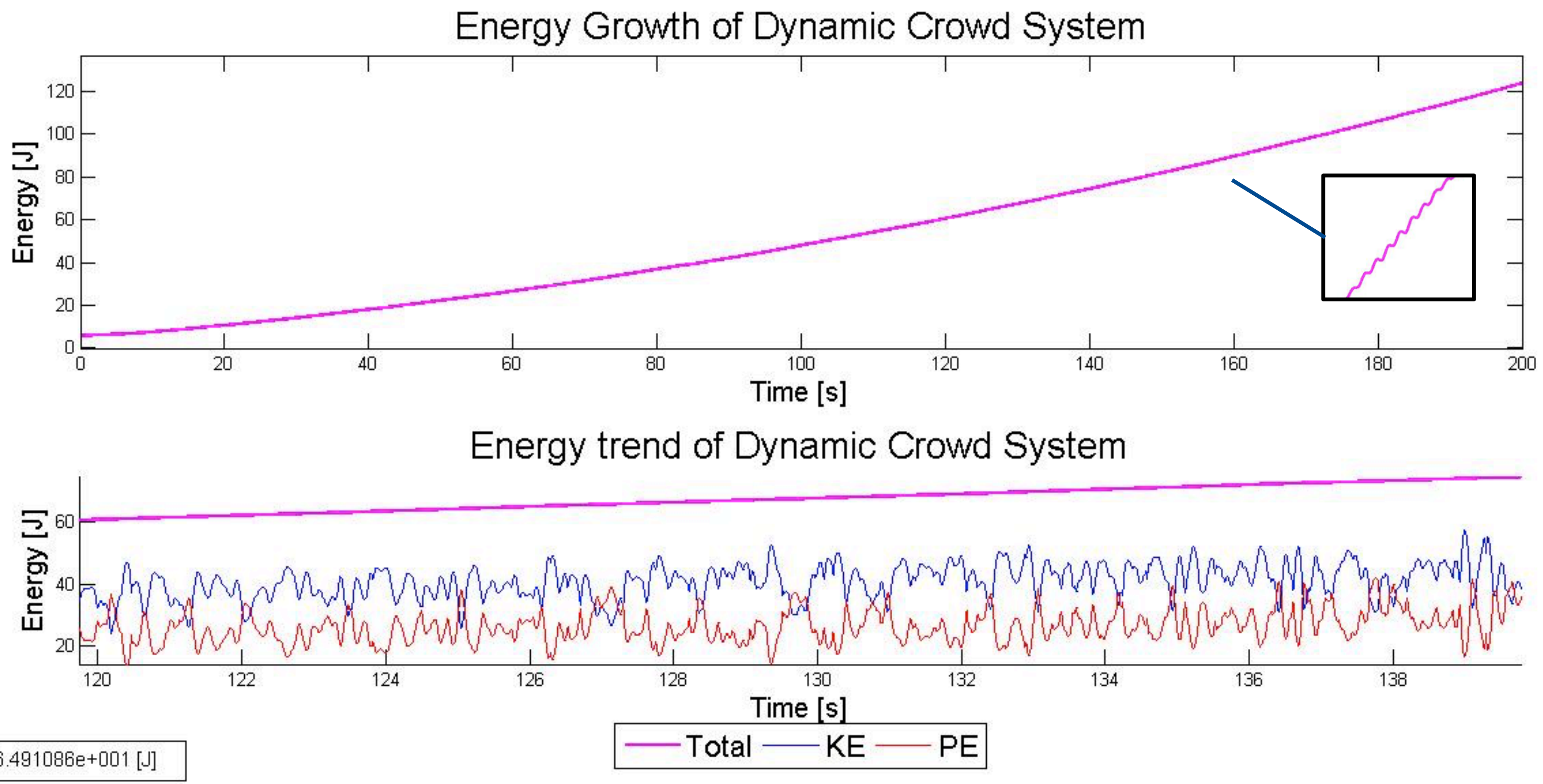
Forces: Pushing Force I & Normal Force

Slack: Reduced

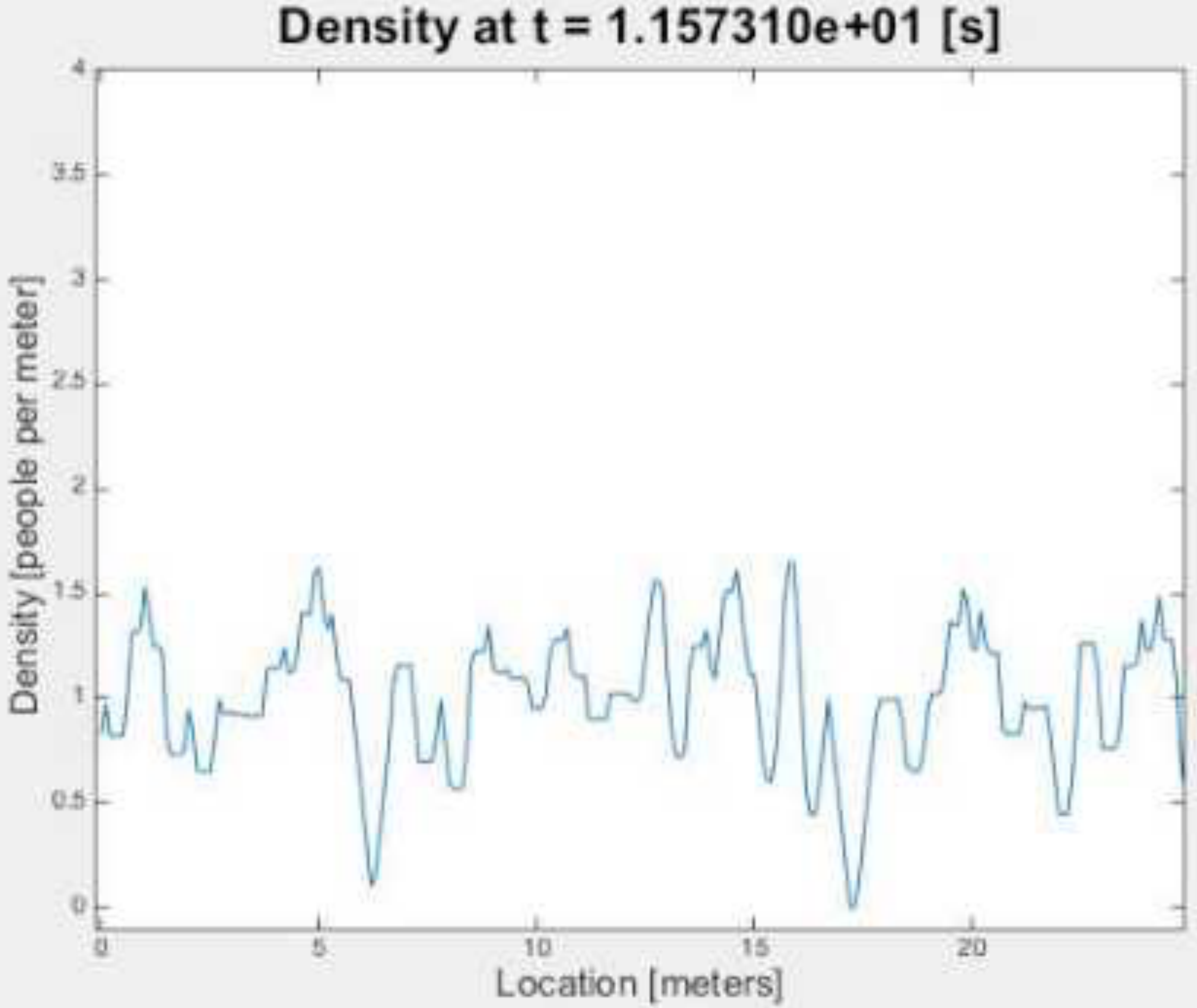
Initial configuration I (slack reduced)



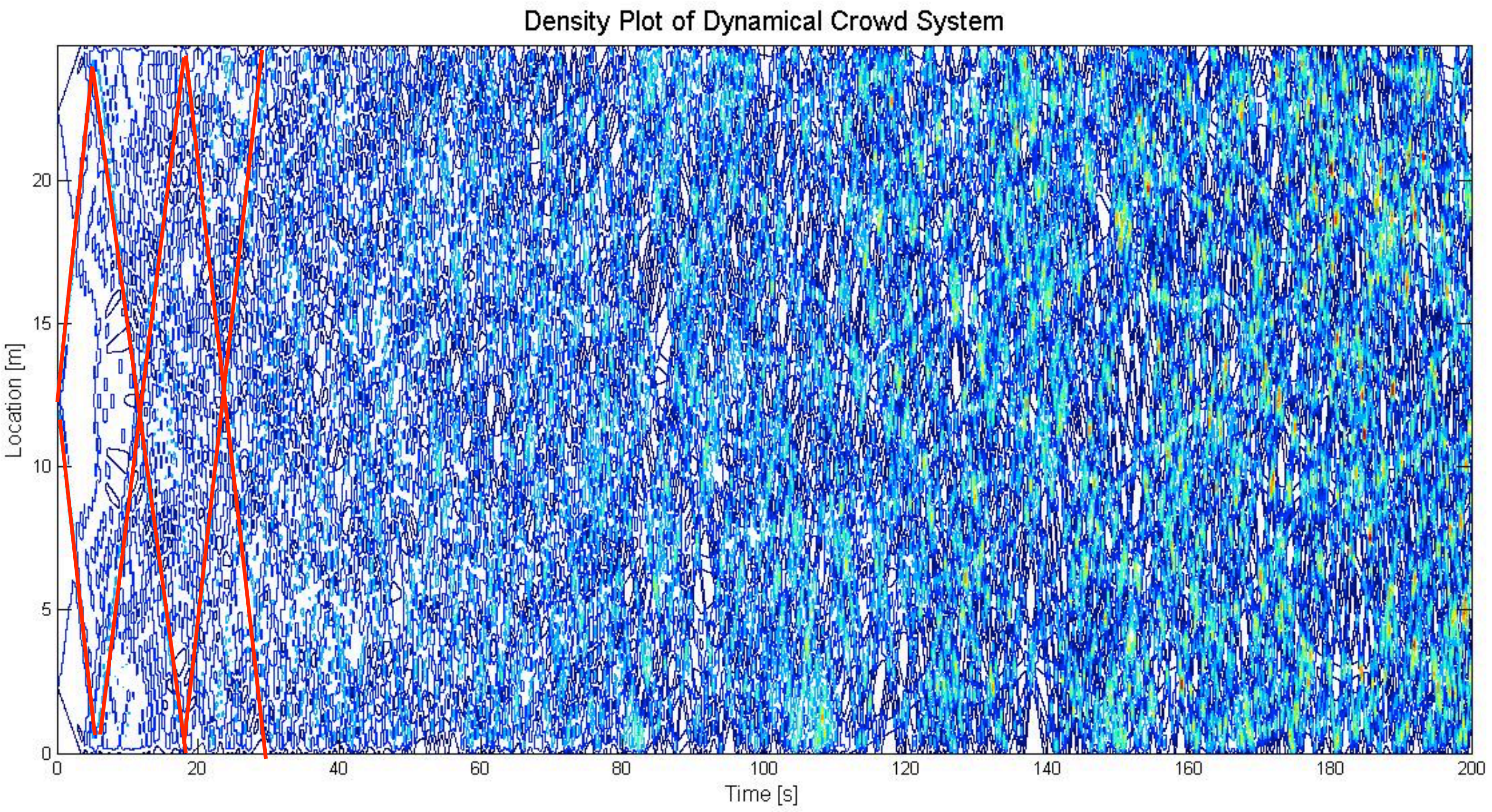

50 people of radius 2.500000e-001
Location [m]
Time [s]
0
5
10
15
20
0
20
40
60
80
100
120
140
160
180
200



Average Energy 6.491086e+001 [J]



Density at t = 1.157310e+01 [s]
Density [people per meter]
Location [meters]
0
0.5
1
1.5
2
2.5
3
3.5
4
0
5
10
15
20



Density Plot of Dynamical Crowd System
Location [m]
0
5
10
15
20
Time [s]
0
20
40
60
80
100
120
140
160
180
200

Example IV

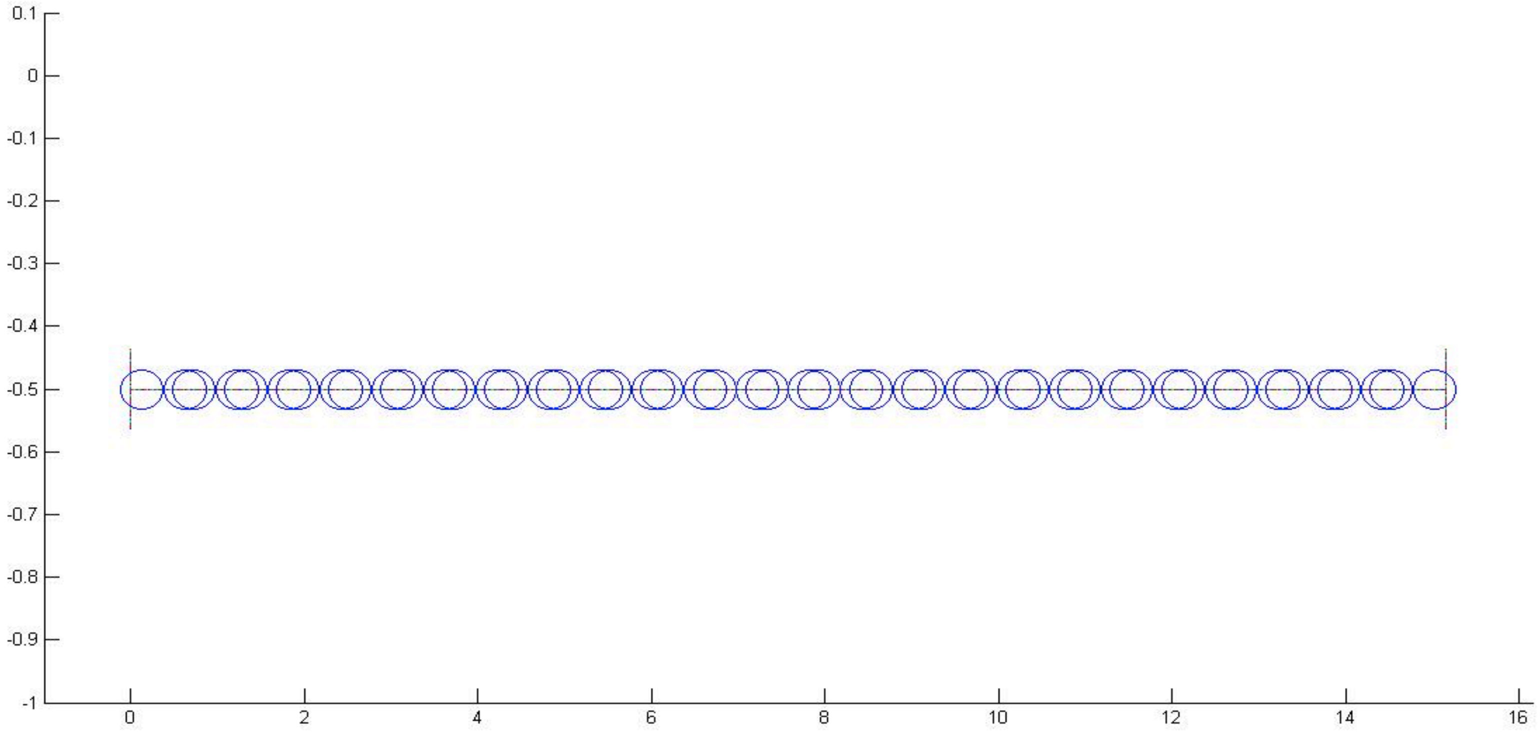
Initial configuration: Wavy

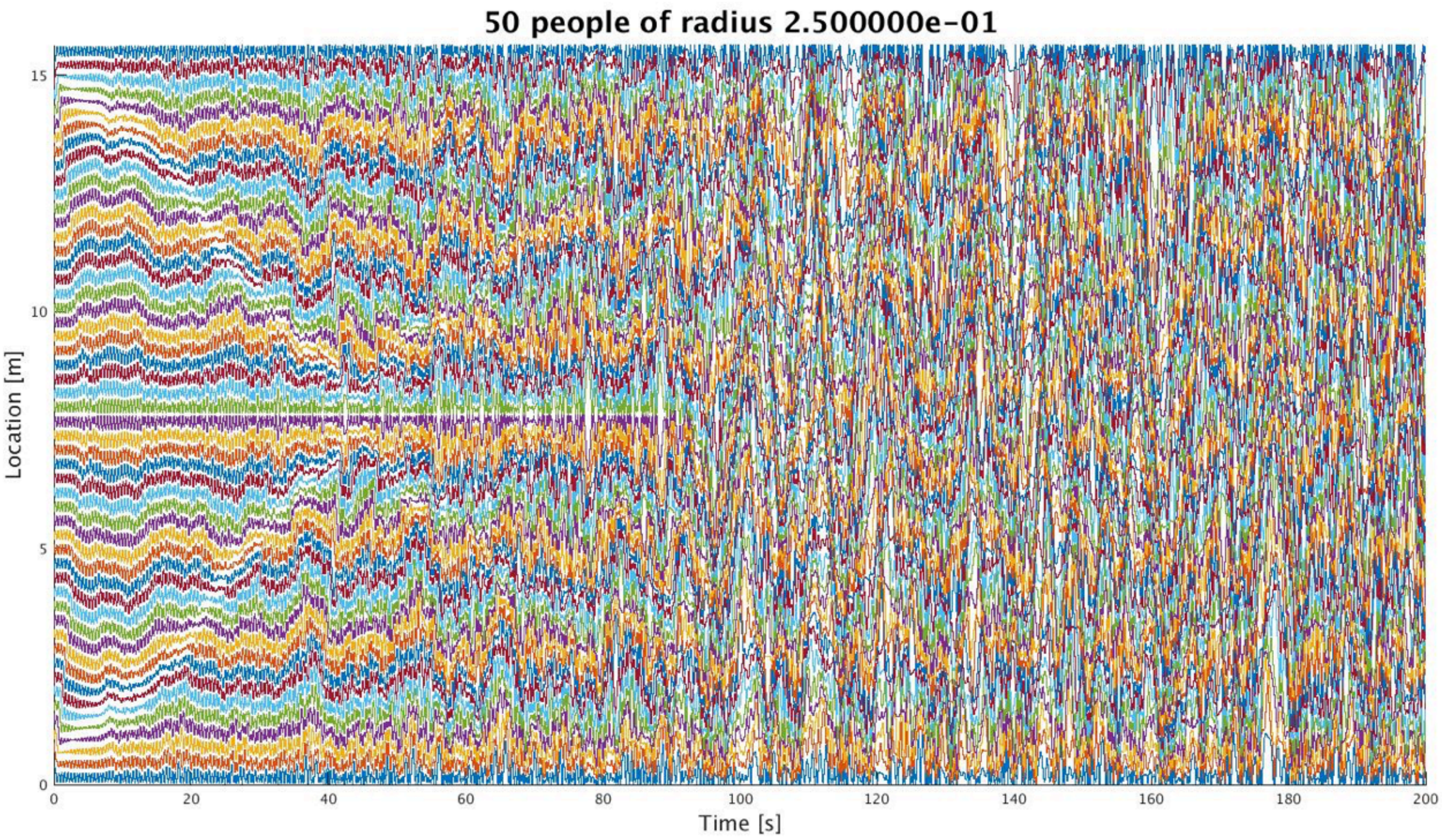
Number of people: n = 50

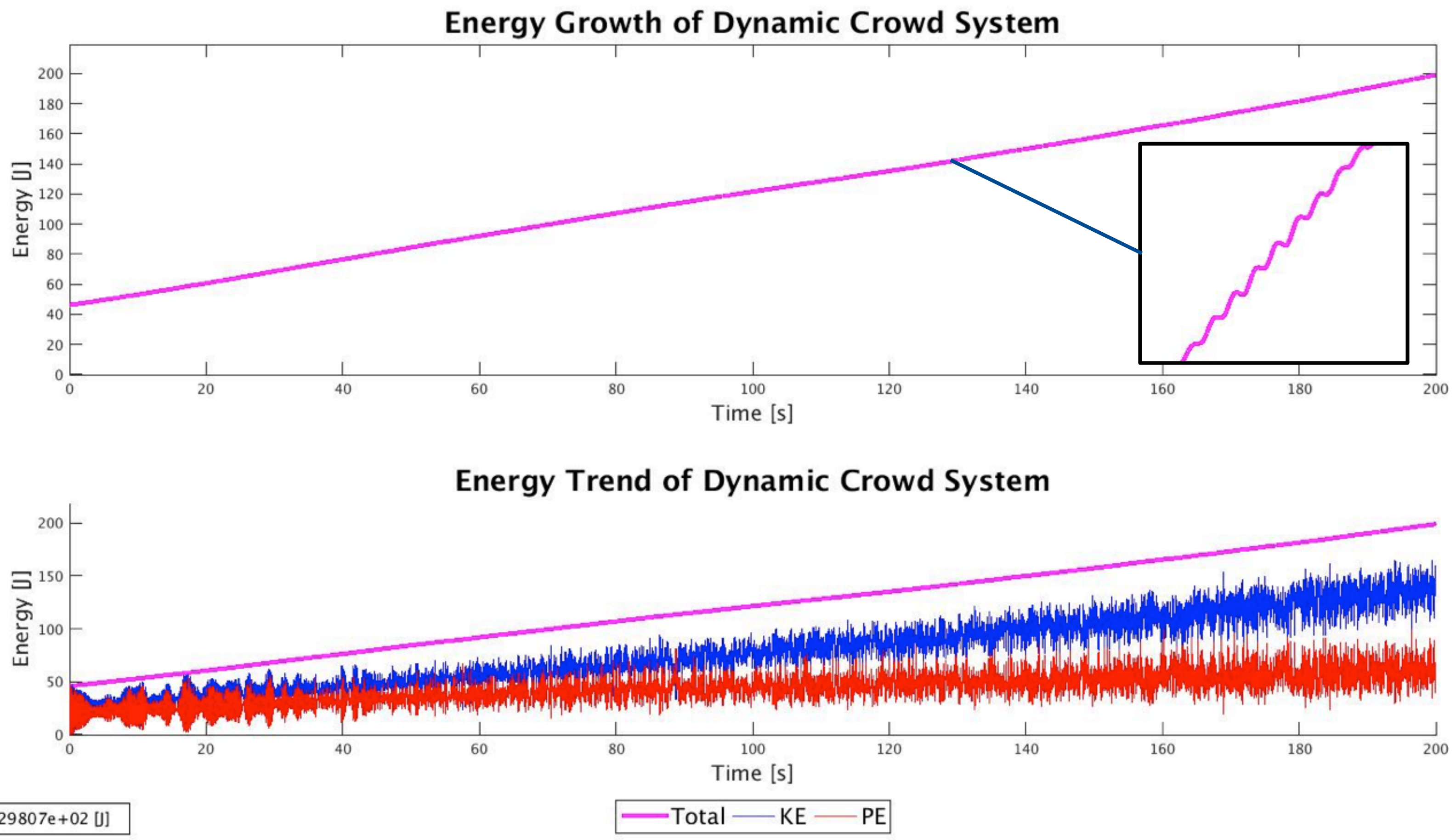
Forces: Pushing Force I & Normal Force

Slack: Reduced

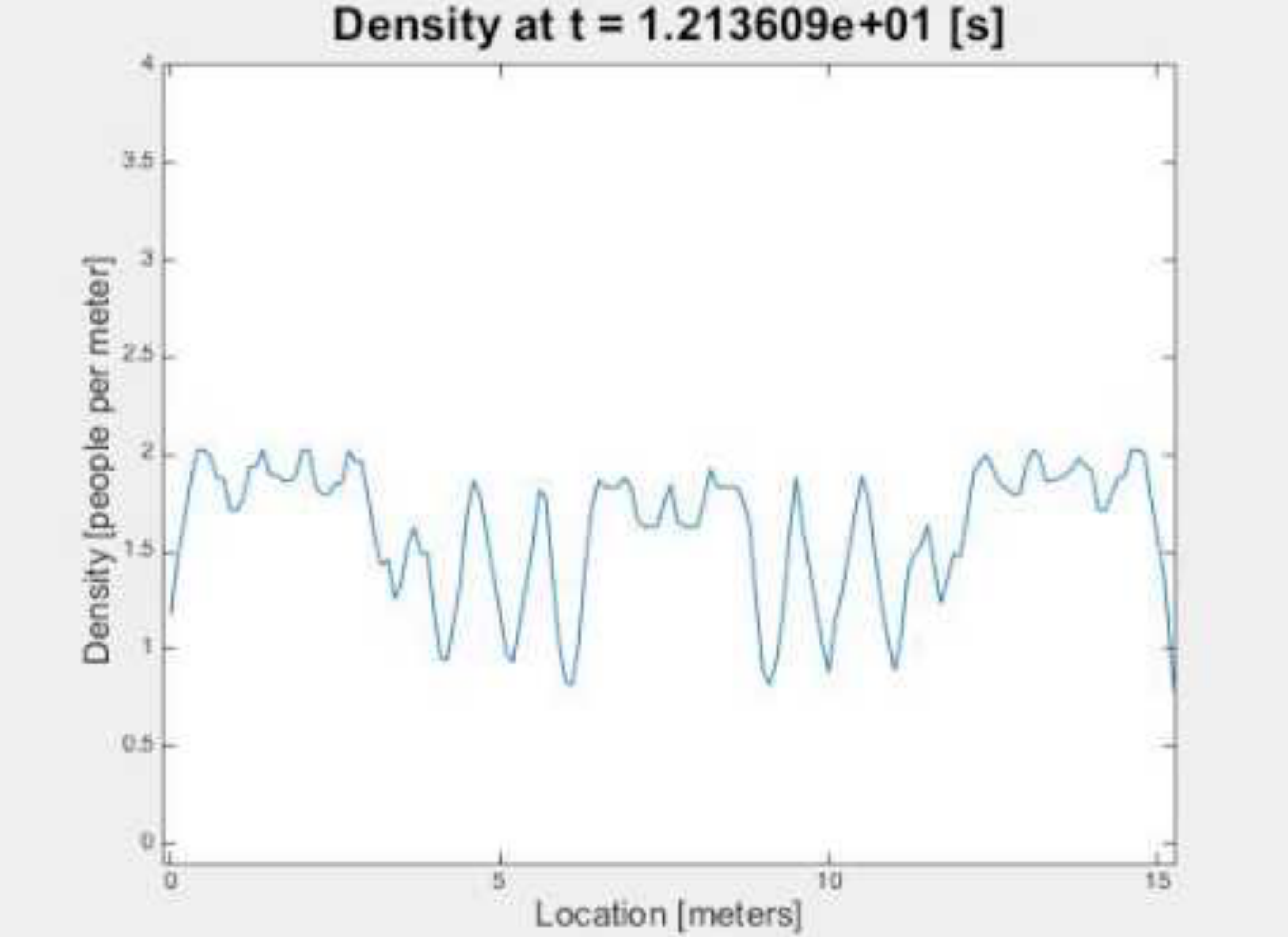
Initial configuration II (slack reduced)

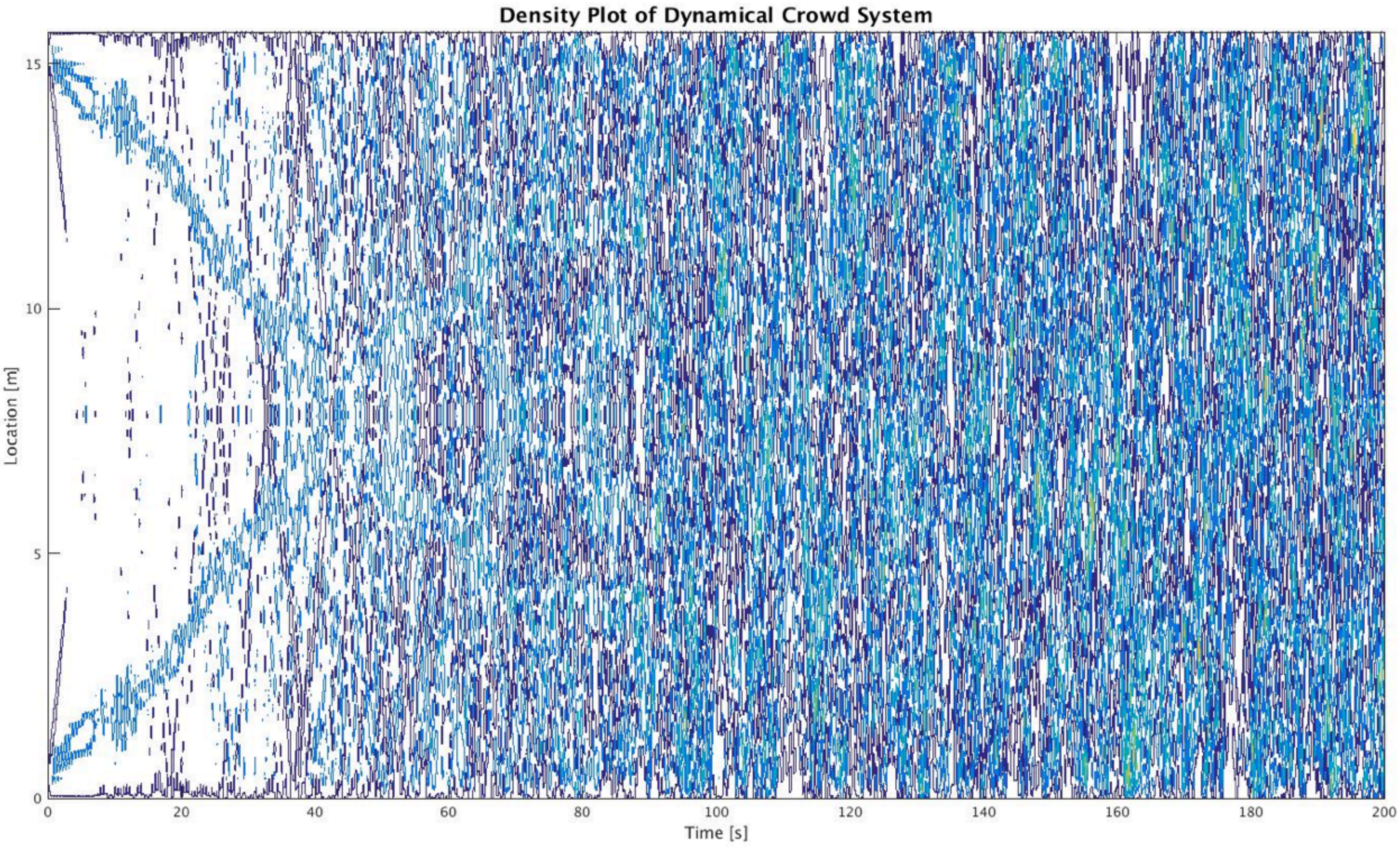



50 people of radius 2.500000e-01
Location [m]
Time [s]
0
5
10
15
0
20
40
60
80
100
120
140
160
180
200



Average Energy 1.229807e+02 [J]


Density at t = 1.213609e+01 [s]
Density [people per meter]
Location [meters]
0
0.5
1
1.5
2
2.5
3
3.5
4
0
5
10
15



Density Plot of Dynamical Crowd System
Location [m]
Time [s]
0
5
10
15
0
20
40
60
80
100
120
140
160
180
200

Conclusions

Relation between energy and overlaps.

Patterns in density.

Dependence on configuration.

Energy Growth from Δ

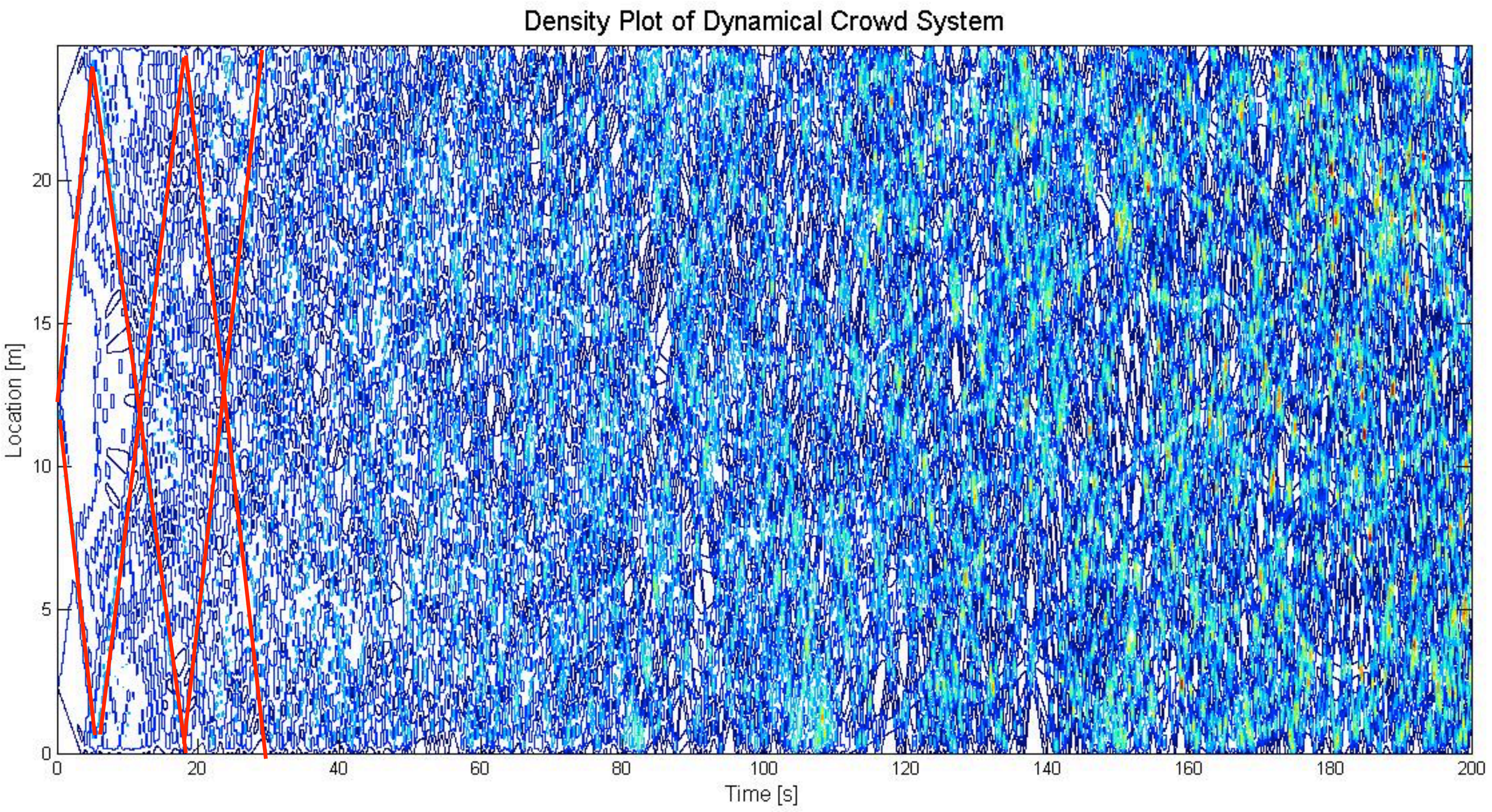
$$dE = -\sum_{i=1}^{n} P(\Delta_i, \dot{\Delta}_i)\, d\Delta_i$$

$$E(t) = \int_0^t dE = -\int_0^t \sum_{i=1}^{n} P(\Delta_i, \dot{\Delta}_i)\, d\Delta_i$$

Density Waves and Dependance on Configurations

pulses form due to configurations

extrema occur along pulses


Density Plot of Dynamical Crowd System
Location [m]
0
5
10
15
20
Time [s]
0
20
40
60
80
100
120
140
160
180
200

Future work

Understand phenomena we have already observed.

New simulations with different number of people and different initial conditions.

Introduce new forces.

Introduce concept of pressure.

Develop a continuous model.

2-D model.

References

- [1] D. Helbing et al, Simulation of Pedestrian Crowds in Normal and Evacuation Situations, Pedestr. Evac. Dyn. 21(2002)
- [2] D. Helbing et al, Simulating Dynamical Features of Escape Panic, Nature. 407(2000)
- [3] Moussaid et al, How Simple Rules Determine Pedestrian Behavior and Crowd Disasters, PNAS, 108 (2011)
- [4] R. Hughes: A Continuum Theory for The Flow of Pedestrians, Transportation Research Part B 507-535(2002)

Questions or Suggestions?