Summer@ICERM 2017: Topological Data Analysis
Mini-course: Theoretical Aspects of Computational Topology

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Description: This course is devoted to theoretical foundations of computational topology. We will start with a short overview of the field and show some applications that students will learn more about by the end of the class. We will define simplicial complexes, homology groups, discuss triangulations of point clouds and learn the theory behind one of the most popular tools in applied topology, persistent homology.

Format: The course consists of four blocks of 3 hours each. Each class will be split into a lecture of at most 120 minutes, followed by a short break. The rest of the time will be devoted to working on problem sets.

Lecture 1: A short introduction to Applied Topology, Simplicial Complexes.

We introduce applied topology and give a short overview of some of the popular applications before continuing with the first important topic: simplicial complexes.

Introductory Talk: A Short Introduction to Applied Topology
Notes: SimplicialComplexes.pdf

Exercises 1: Simplicial complexes

Problem Set: SimplicialComplexesExercises.pdf

Lecture 2: Homology Groups. Reduction techniques.

We define simplicial homology and learn techniques that make computations simpler.

Notes: SimplicialHomologyTriangulations.pdf

Exercises 2: Compute homology groups by hand.

Problem Set: SimplicialHomologyGroupsExercises.pdf

Lecture 3: Point Cloud Triangulations: Čech, Vietoris–Rips, and witness simplicial complexes. Persistent Homology.

In this class we study simplicial complexes in more detail, including Čech complexes, alpha complexes, Vietoris–Rips complexes, and witness complexes. We describe the Nerve Lemma for coverings of a topological space. We also define persistence vector spaces.

Notes: SimplicialComplexesTriangulation.pdf

Exercises 3: Prove properties of various complexes.

Problem Set: TriangulationsExercises.pdf

Lecture 4: Persistent Homology

We discuss the decomposition theorem for finitely presented persistence vector spaces and define persistent homology of a point cloud or a filtrations of simplicial complexes.

Notes: PersistentHomology.pdf

Exercises 4: Compute persistent homology groups by hand.

Problem Set: PersistentHomologyExercises.pdf

Reading materials

We will not follow a single textbook for the entire mini-course. For topology, Algebraic Topology by A. Hatcher will be useful. For algebra, Abstract Algebra by Dummitt and Foote is excellent. H. Edelsbrunner & J. Harer's Computational Topology contains sample problems at the end of each section. I also highly recommend Gunnar Carlsson's two introductory papers: Topology and Data and Topological Pattern Recognition for Point Cloud Data.

References

[1] Henry Adams and Gunnar Carlsson. Topological pattern recognition for point cloud data. *Acta Numerica*, 23 (2013): 289–368.

[2] Gunnar Carlsson. Topology and data. *Bulletin of the American Mathematical Society*, 46(2): 255–308, 2009.

[3] Herbert Edelsbrunner and John Harer. *Computational topology: An introduction.* American Mathematical Society, 2010.