ITERATIVE METHODS FOR ELECTROMAGNETIC PROBLEMS ON AXISYMMETRIC DOMAINS

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1. Project Overview

Finite Element Methods (FEMs) are well-known numerical methods one can use to find a good approximate solution to a system of partial differential equations (PDEs). An axisymmetric problem is a problem defined on a three-dimensional ($3D$) domain that is symmetric with respect to an axis. By using cylindrical coordinates and a Fourier series decomposition with respect to the rotational variable θ, an axisymmetric $3D$ problem can be reduced to a sequence of two-dimensional ($2D$) problems. Such dimension reduction is an attractive feature considering computation time, but due to the Jacobian arising from change of variables, the resulting $2D$ problems are posed in weighted function spaces where the weight function is the radial component r. These weighted spaces include functions with singularities at the axis of rotation, so the analysis of such weighted problems requires special attention. Furthermore, the resulting weighted $2D$ problems are quite different from the corresponding unweighted problems. The term Fourier Finite Element Methods (Fourier-FEMs) is used when a suitable FEM is applied to approximate each Fourier-mode of the solution. Fourier-FEMs can be used to efficiently solve axisymmetric problems with general data.

Maxwell equations form a system of PDEs that govern the behavior of electric and magnetic fields. Various applications, including induction heating and antenna design, require one to solve the Maxwell equations on an axisymmetric domain, so we are interested in developing efficient numerical methods to solve the axisymmetric Maxwell equations.

2. Project Outlook

Fourier analysis and reduced rank approximations have similarities. Usually there are a few Fourier modes that capture most of the information of the original function, and the remaining Fourier modes can be discarded with little effect. Furthermore, multigrid may be viewed as an iterative method that uses a “low rank” approximation of finite dimensional subspaces as well. In this project, we will study Fourier-FEMs and an iterative method called multigrid to approximate the solution to the axisymmetric time-harmonic Maxwell equations. We will also study how the multigrid convergence rate changes for different Fourier-modes in the Fourier-FEMs.

Suggested readings:

pages 1–21 of the following textbook:
`https://team-pancho.github.io/documents/anIntro2FEM_2015.pdf`

http://web.pdx.edu/~gjay/pub/ax.pdf
https://doi.org/10.1016/j.camwa.2015.08.020
https://doi.org/10.1016/j.jmaa.2020.124209