Convergence of strong shock waves in a non-ideal gas

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Abstract

We studied the problem of converging cylindrical and spherical strong shock waves collapsing at the axis/center of symmetry for a non-ideal gas with constant density. We have applied the perturbation series technique which provides us a global solution to the implosion shock wave problem yielding the results of Guderley's local self similar solution. We analyzed the flow parameters by expanding the solution in powers of time and found the similarity exponents as well as the corresponding amplitudes in the vicinity of the shock-collapse. The flow parameters and the shock trajectory have been drawn in the region extending from the piston to the center of collapse for different values of adiabatic coefficient and the non-ideal parameter.

Introduction

- Convergence of strong shock wave in gas has been a problem of great interest from both mathematical and physical points of view.
- The converging shock waves offer interesting possibilities of attaining extremely high temperature, pressure and density.
- Applications: thermonuclear fusion, synthesizing materials, the phenomenon of sonoluminescence and treatment of stones in the human Body.
- The collapse of an imploding shock wave, which is a self-similar solution of the second kind, was first studied by Guderley [1].
- In 1945, Landau and Stanyukovich [2] independently solved this particular problem.
- Among the extensive work that followed, we mention the work of Whitham [3], Lazarus and Richtmyer [4], Van Dyke and Guttmann [5], Sakurai [7], Hafner [6], Zeldovich [8], Madhumita and Sharma [10] and Arora and Sharma [11] who calculated the numerical value of similarity exponents to several significant figures using various techniques.
- In the present paper, we successfully demonstrated the application of perturbation series technique proposed by Van Dyke and Guttmann [5] to the implosion shock wave problem in a non-ideal gas of constant density, which provides a global solution to the imploding shock problem.

Basic equations

We consider a cylindrical or spherical piston of initial radius R_0 which is filled with non-ideal gas of constant density ρ_0. Let the initial conditions be as follows

$$v = 0,\ \rho = \rho_0,\ p = p_0,$$

The basic equations for the unsteady one-dimensional adiabatic motion of a non-ideal gas can be written in the following form (Madhumita and Sharma [10])

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho v)}{\partial r} + \frac{m\rho v}{r} = 0,$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0,$$

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial r} - \frac{\gamma p}{(\gamma-1)\rho}\left(\gamma - 1 + \alpha\frac{\rho}{\rho_0}\right)\left(\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r}\right) = 0,$$

Hugoniot (R-H) relations:

$$v = (1-\beta^{-1})\dot{R},\ \rho = \beta\rho_0,\ p = (1-\beta^{-1})\rho_0(\dot{R})^2, \quad \text{at } r = R(t),$$

where $\dot{R}(t)$ and β denotes the shock speed and the measure of the shock strength, respectively which is given by

$$\beta = \frac{\gamma+1}{\gamma-1}\left(1 - \frac{2\alpha}{(\gamma-1)^2}\right).$$

Solution in time

The no flow condition through the piston gives

$$v = -V \text{ at } r = R_0 - Vt.$$

For convenience, we evaluate the distance inward; let $x = R_0 - r$ and let $u = -v$ be the corresponding velocity directed inward. We introduce a new variable

All the variables are non-dimensionalized by referring lengths to R_0, speed to V, density to ρ_0, pressure to $\rho_0 V^2$, internal volume of the gas molecules b to $1/\rho_0$ and time to R_0/V. Then, the system of differential equations

$$\left(1 - \left(k + \tfrac{1}{2}(\gamma-1)z\right)t\right)\left(\rho\tfrac{\partial u}{\partial z} + \left(u - k - \tfrac{1}{2}(\gamma-1)z\right)\tfrac{\partial\rho}{\partial z} + \tfrac{1}{2}(\gamma-1)t\tfrac{\partial\rho}{\partial t}\right) = \tfrac{1}{2}(\gamma-1)mt\rho u,$$

$$\rho\left(u - k - \frac{1}{2}(\gamma-1)z\right)\frac{\partial u}{\partial z} + \frac{1}{2}(\gamma-1)t\rho\frac{\partial u}{\partial t} + \frac{\partial p}{\partial z} = 0,$$

$$\left(u - k - \frac{1}{2}(\gamma-1)z\right)\left(\rho\frac{\partial p}{\partial z} - \gamma p\left(1 + \frac{\alpha\rho}{(\gamma-1)}\right)\frac{\partial\rho}{\partial z}\right) + \frac{1}{2}(\gamma-1)t\left(\rho\frac{\partial p}{\partial t} - \gamma p\left(1 + \frac{\alpha\rho}{(\gamma-1)}\right)\frac{\partial\rho}{\partial t}\right) = 0,$$

$$u = (1-\beta^{-1})\dot{X},\ \rho = \beta,\ p = (1-\beta^{-1})(\dot{X})^2$$

at $z = (2/(\gamma-1))(X/t - k)$; and

$$u = 1 \text{ at } z = \frac{-2(k-1)}{(\gamma-1)}.$$

Assuming that the solution is analytic in time, we expand the unknown position of the shock wave in a Taylor series as

$$X(t) = \sum_{n=1}^{\infty} X_n t^n,$$

and similarly we expand the flow variables as

$$u = \sum_{n=1}^{\infty} U_n(z)t^{n-1},\ \rho = \sum_{n=1}^{\infty} R_n(z)t^{n-1},\ p = \sum_{n=1}^{\infty} P_n(z)t^{n-1}.$$

$$U_1 = 1,\ R_1 = \beta,\ P_1 = \frac{1}{1-\beta^{-1}},$$

$$X_1 = \frac{1}{1-\beta^{-1}},\ k = \frac{3-\gamma+(\gamma-1)\beta^{-1}}{2(1-\beta^{-1})}.$$

$$U_2 = (1-\beta^{-1})[\beta(1-\gamma) + \gamma + 1 + (\gamma-1)(\beta-1)z]X_2,$$

$$R_2 = (\beta-1)\left[\frac{(\gamma-1)m\beta}{2} - (\beta-1)^2(\gamma-1)X_2\right](1-z),$$

$$P_2 = 4X_2,$$

The forms of U_2, R_2 and P_2 suggest that in higher approximations the coefficients U_n, R_n and P_n are polynomials in z of degree $n-1$, of the following forms

$$U_n(z) = \sum_{j=1}^{n} U_{nj}z^{j-1},\ R_n(z) = \sum_{j=1}^{n} R_{nj}z^{j-1},\ P_n(z) = \sum_{j=1}^{n} P_{nj}z^{j-1}.$$

we obtain the position of the shock wave

$$X(t) = t\frac{1}{(1-\beta^{-1})} + t^2\frac{\gamma(\gamma-1)^2m\beta^2\left(1+\frac{\alpha\beta}{(\gamma-1)}\right) + (\beta-\beta\gamma+\gamma+1)m\gamma(\gamma-1)\beta^2\left(1+\frac{\alpha\beta}{(\gamma-1)}\right)(\beta-1)^{-1}}{2\left[4\beta(\gamma-1)+\gamma(\gamma-1)^2\left(1+\frac{\alpha\beta}{(\gamma-1)}\right)\beta(\beta-1)^2 + (\beta-\beta\gamma+\gamma+1)\left(1+\frac{\alpha\beta}{(\gamma-1)}\right)(\beta-1)(\gamma-1)\beta\gamma\right]} + \cdots.$$

Radius of convergence

We obtain the radius of convergence assuming that the nearest singularity has the form

$$R(t) = 1 - X(t) = 1 - \sum X_n t^n$$

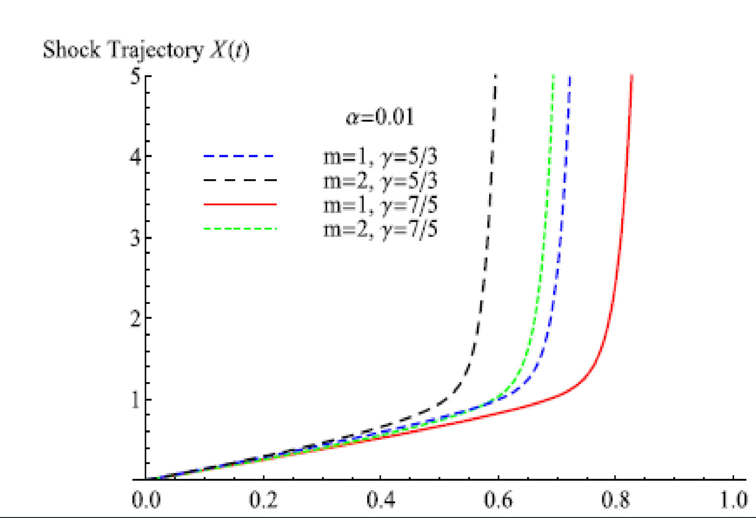
$$\sim A_1\left(1 - \frac{t}{t_c}\right)^{\delta_1} \text{ as } t \to t_c,$$

where δ_1 and A_1 are the leading exponent and amplitude, respectively. Hence,

$$\frac{X_n}{X_{n-1}} \sim \frac{1}{t_c}\left(1 - \frac{1+\delta_1}{n}\right) \text{as } n \to \infty.$$

The computed values of the leading similarity exponent δ_1 and the results obtained by other authors.

γ	α	m	Computed δ_1	Guderley [1]	Madhumita [10]
7/5	0.001	1	0.834101762807714	0.832673	0.833820
7/5	0.001	2	0.715049924946400	0.713119	0.714644
7/5	0.003	1	0.830984382936880	0.829148	0.830894
7/5	0.003	2	0.709960029250350	0.704639	0.709833
7/5	0.005	1	0.828395435962383	0.825328	0.828024
7/5	0.005	2	0.705190307059380	0.702517	0.705248
7/5	0.007	1	0.825365368494210	0.823800	0.825178
7/5	0.007	2	0.700825539005120	0.697289	0.700797
5/3	0.001	1	0.815373347298886	0.815446	–
5/3	0.001	2	0.688284672614547	0.686881	–
5/3	0.005	1	0.813056251131820	0.810752	–
5/3	0.005	2	0.684777928295827	0.681246	–
5/3	0.01	1	0.810358893451119	0.805848	–
5/3	0.01	2	0.680805997140480	0.673952	–
6/5	0.001	1	0.857156055878090	0.853984	–
6/5	0.01	2	0.694970424980506	0.686955	–



RESULTS & CONCLUSION

- An increase in non-ideal parameter α causes the velocity, density to decrease while pressure to increase.
- The velocity and pressure decreases monotonically behind the shock
- An increase in non-ideal parameter α causes the time of shock collapse to increase.
- Guderley's local self similar solution gives only the first dominant similarity exponent while by using this method, we obtained other less dominant similarity exponents and the corresponding amplitudes.

REFERENCES

[1] G. Guderley, Starke kugelige und zylindrische Verdichtungsstosse in der Nahe des Kugelmittelpunktes bzw der Zylinderachse, Luftfahrtforschung 19 (1942) 302–312.
[2] L.D. Landau, K.P. Stanyukovich, Studying the detonation of condensed explosives, DAN SSSR 46 (9) (1945).
[3] G.B. Whitham, Linear and Nonlinear Waves, Wiley-Interscience, New York, 1974.
[4] R.B. Lazarus, R.D. Richtmyer, Similarity Solutions for Converging Shocks, Los Alamos Scientific Lab. Rep. LA-6823-MS, 1977.
[5] M. Vandyke, A.J. Guttmann, The converging shock wave from a spherical or cylindrical piston, J. Fluid Mech. 120 (1982) 451–462.
[6] P. Hafner, Strong convergent shock waves near the center of convergence: a power series solution, SIAM J. Appl. Math. 48 (1988) 1244–1261.
[7] A. Sakurai, Propagation of spherical shock waves in stars, J. Fluid Mech. 1 (1956) 436–453.
[8] Y.B. Zeldovich, Y.P. Raizer, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena Vol II Academic, New York, 1967, pp. 465–916.
[9] G. Madhumita, V.D. Sharma, Propagation of strong converging shock waves in a gas of variable density, J. Eng. Math. 46 (1) (2003) 55–68.
[10] G. Madhumita, V.D. Sharma, Imploding cylindrical and spherical shock waves in a non-ideal medium, J. Hyperbolic Differ. Equ. 1 (3) (2004) 521–530.